Optical Engineering
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Lecture - 38
Matlab tutorial on interference

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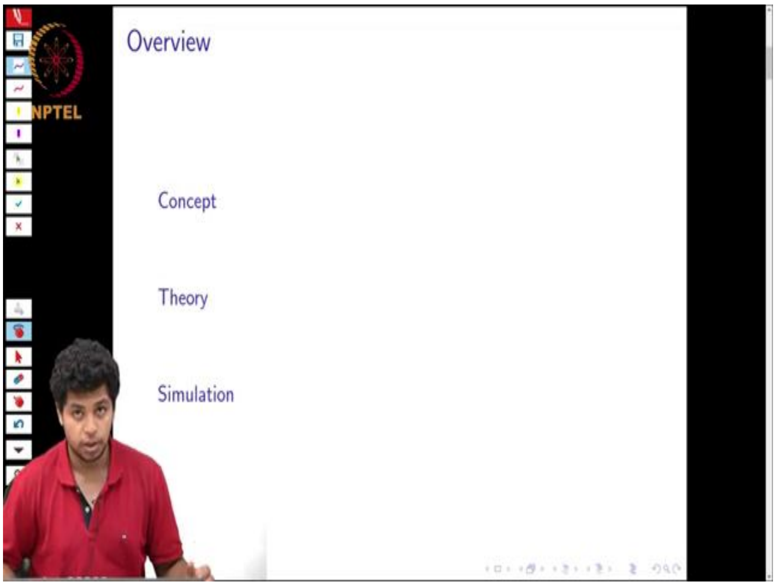

Hello, my name is Amogh Manthalkar and I will be taking a tutorial on the use of MATLAB to simulate interference. We will go by the concept of interference once then about the theory of what equations we are going to use and then I will show you how to simulate using MATLAB.

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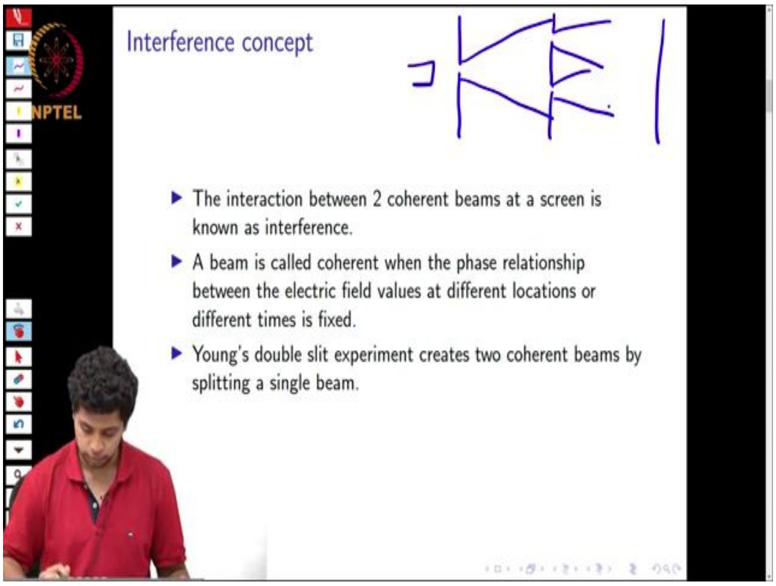

The concept of interference is fairly simple. The interaction of 2 coherent beams at a screen is called an interference. Now, what does this mean? A beam is called coherent when the phase relationship between the electric fields at different values of location or different values of time is fixed then it is called as coherence; which basically means if the value of phase at two different locations has a tight correlation or has a fixed relationship then it means then the beams are coherent in space or are spatially coherent.

If the phase relationship between the beams at different times for the same location has a tight correlation or has a fixed relationship, it is called temporal coherence. Now, the best way to imagine it is in terms of a Young's double slit experiment. As you might know in the Young's double slit experiment we have first screen after source and then we have two slits and then a screen. So, what this experiment essentially does is it takes one single source and splits it into two coherent sources. So, this is the best way to imagine it.

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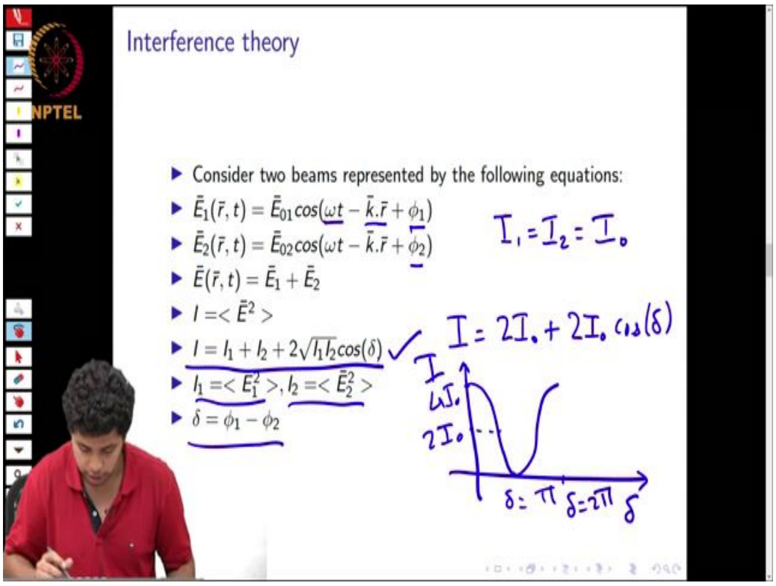

Now, we will go over the theory, what we want is two beams which we want to interfere. So, consider the following equations. $E_1(r)$ and t is given by this first equation right here, we use an $\omega(t)$ term to show its frequency, this $k.r$ is the direction and ϕ_1 is the initial phase of beam number 1.

For the second beam we have the same frequency and same direction, but a different phase. Interference is basically the sum of both of these electric fields at a screen. Now, intensity is the square of the electric field amplitude as we all know, then the interference equation is this one. I am the total intensity, I_1 is the intensity for beam 1 which is $E_1{}^2$ I_1 is the intensity for beam 2 which is E^2

And then there is a term Δ in this equation which is basically the difference between the initial phase of beam 1 and beam 2. Now, con let us consider this example here. Consider that $I_1 = I_2$ is equal to I naught which means; both the beams have equal intensity. And let us also consider that ωt and $k.r$ for both beams are the same.

So, the only thing different for both the beams is ϕ_1 and ϕ_2 which means there will be a nonzero Δ term, but everything else will be the same. So, what does the interference equation look like? It is $I = I_1 + I_2 = I_0$. So, $2I_0$ plus again $2I_0$ into I_0 its whole root.

So, again $2I_0 \cos(\Delta)$ this is what it would look like. We will see what happens when we plot I with respect to Δ there is a at $\Delta = 0$ the value is $4I_0$ keeps dropping as Δ increases it goes to 0 at $\Delta = \pi$ for which cost is -1 and then again it increases and when $\Delta = 2\pi$ and again goes to $4I_0$ that is the variation. So, the variation is essentially around $2I_0$ and it goes from 0 to $4I_0$. Now, we will have a look at how to simulate this particular example that we just discussed.

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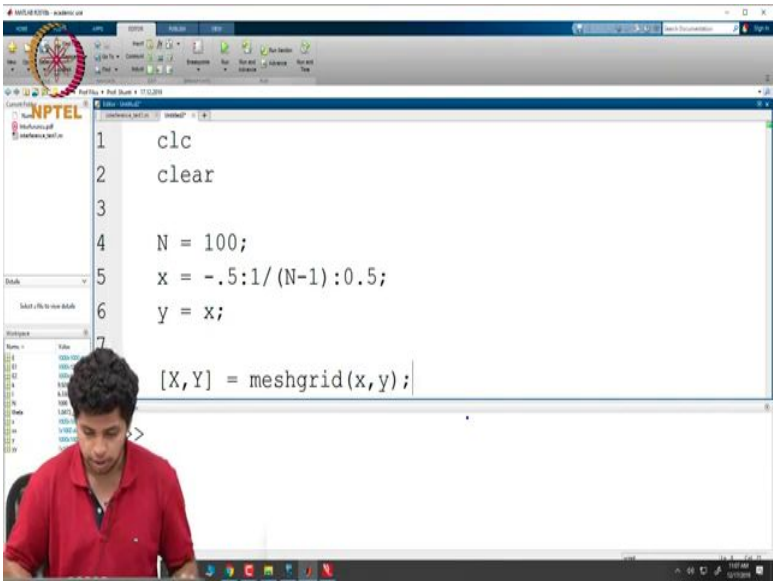

What we do now is use the MATLAB editor to write a program for this. Use these commands to clear any data that was already present in MATLAB. Now, we create two variables to denote the limits of x and y direction because what we essentially want to look at is a 2 dimensional view that is a cross section of a beam.

So, it will look like an image that is why we have to give a 2D representation. So, say x goes from -0.5 to 0.5 with a step of say we want a 100 steps. So, 1 by N -1 we will define N, we have to do it before x + 0.5. So, say we want 100 points by 100 points. So, we give values like this and the reason we give x as this value 0.5 to 0.5 with a step of 1 by N -1 is because the first element is going to be - 0.5. And the last element is going to be 0.5 and we want 98 elements between them.

So, that is why and the last element since it is 0.5 we have to give such that after 0.5 there have to be 99 elements and that is why. And then we want the y to be the same as x, so instead of putting the same statement again we just put y equal to x. So, now, it represents y in the same way.

Now, what we do is we use a command called mesh grid and the syntax for this command is we give the variables which we want equal to meshgrid x and y. Now, what this mesh grid command does is it creates ordered pairs of x and y and we can represent any function we want using these capital X and Y variables.

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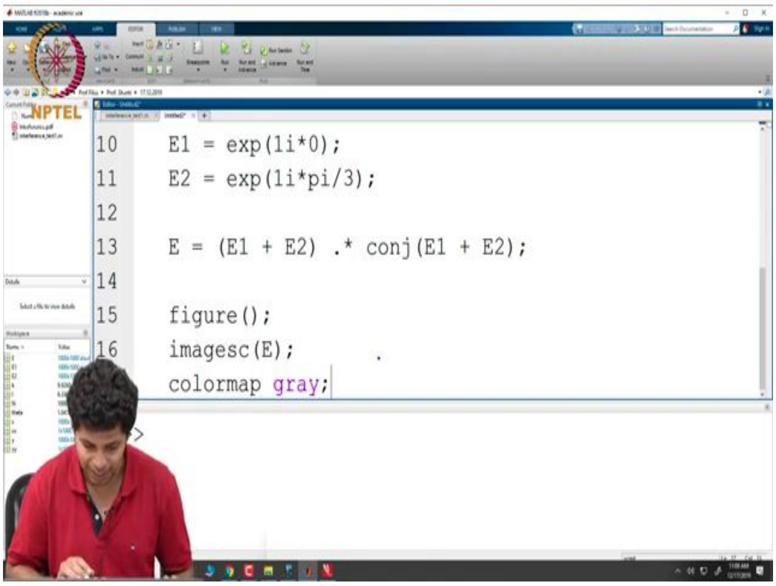

Now, what we do is we define E_1 as equal to exp which is the syntax for exponential function. And as we discussed here the $\omega t - k.r$ k . r values are going to be the same for both the equations. So, we neglect both of them and we just give a ϕ_1 value to the first field and a ϕ_2 value to the second field and both of these are phase terms. So, what we do in our MATLAB program is we give it the amplitude as 1 for both of them which we want the same.

And we give an exponent of 1i which is a which denotes a complex exponent which we want and then we give it first one as a value of 0 just considered a 0 as a reference and for the second wave E_2 what we do is we give it a different phase value, so we give 1i multiplied by. So, now, because we have discussed the constant phase, but a different value I will give it a constant phase of say $\pi/3$

Now, in order to implement this interference equation in MATLAB, we can use the following syntax like I will explain now. Let us consider E is the field total field, it will be $E_1 + E_2$. multiplied by we use this dot multiplied by because we want term wise or element wise multiplication of 2 matrices. And the second matrix will be conjugate or complex conjugate of this sum $E_1 + E_2$. Once we take this multiplication we will have our interference diagram ready.

Now, what we want to do is we want to plot this E function, but since it is an image we do not use this command plot, we use image scan and the way it is done is in this way. So, first we give the command to create a blank figure; the command is given a figure, then we say image scan and in the argument we put E and then because we want the plot to be in black and white scale.

So, we do a colormap and we specify the colormap that we want its gray and because we also might want to look at what value a point has we also want a reference or a colorbar to be on the side. So, we use this command colorbar.

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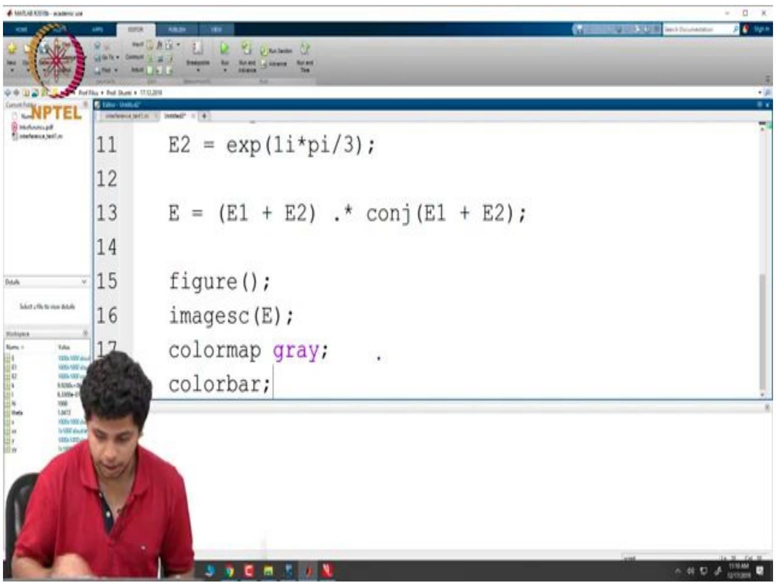

Now, we run the entire program.

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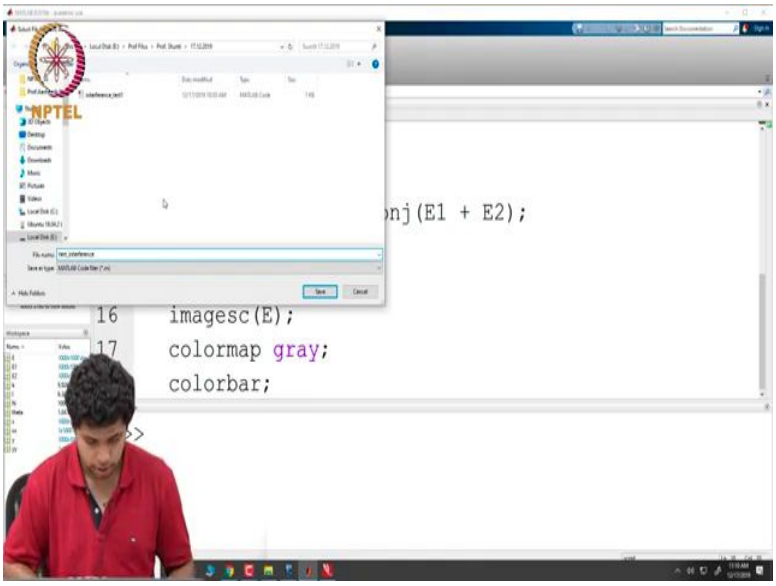

I will just save as test interference.

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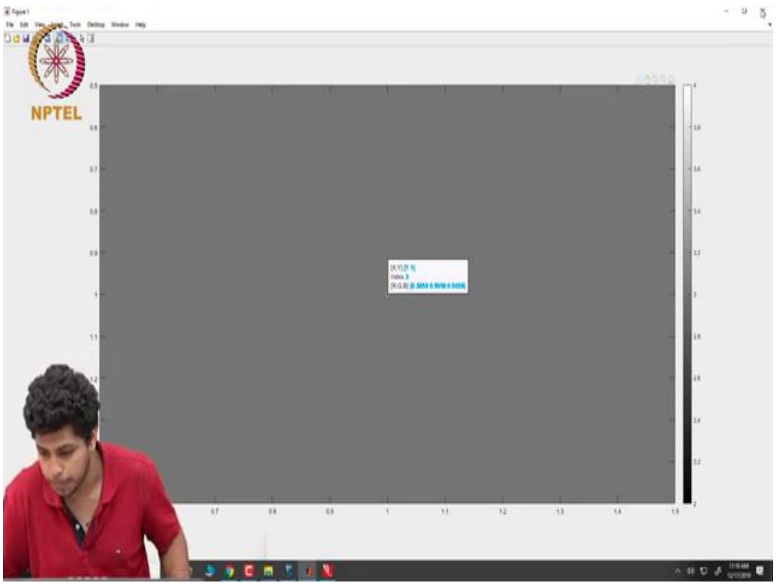

And now we have a field with constant phase or constant intensity all over it. And if you want to have a look what we can do using MATLAB is we can use tools, a data cursor and we can put a data cursor anywhere. So, it is showing RGB equal to 0.5 index is 3 which means this is the value that it is using, it is showing 3 all over the area. So, which basically means that the entire area of the interference pattern has the same value and the value is 3.

Now, this was a fairly simplistic example and it did not really show any special variance in the phase values. So, the next example that we will look at it will be slightly different.

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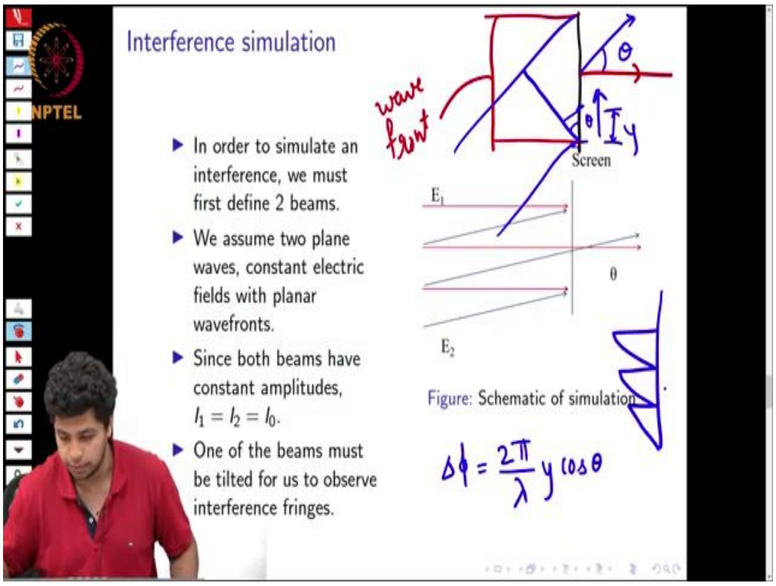

So, an interference simulation again will define 2 beams; both beams will again be plane waves; that means, constant electric fields with planar wave fronts, but there will be one key difference here again both the amplitudes will be the same.

But one beam will be tilted with respect to the other now what; that means, is explained in this schematic. Now, the red beam is the first beam which is coming normal to the screen and the blue beam is the second beam which is coming at a certain angle with respect to the normal and that angle is given as theta. So, I will just represent it again.

So, say we have a screen here and we have one beam coming normal on to the screen and the vector is like this, the wavefront is going to be perpendicular to the direction of propagation. So, this is the wavefront and the second beam that we have is coming at a certain angle and again the vector I will show like this; this is the angle theta. And the wavefront for this beam is going to look like this perpendicular to the direction of propagation this angle is again theta.

Now, what if we choose a point here where this point is y and what we see here now is there at this particular point which is at y distance from the bottom point, the beam the blue beam has travelled a little extra distance which means it has gathered extra phase. How much extra phase it has gathered can be given by this particular equation that I will just write down its 2 pi by lambda into $y\cos(\theta)$

So, as it happens as you go up from the bottom point the phase keeps accumulating and as we know after the value of 2π, the phase can be approximated can be considered to be again jumping back to 0. So, what happens is the way we see the phase varying over this plane is something like this: it goes from 0 to 2 π goes back to 0 again it goes to 0 to 2 π goes 0 again and goes 0 to 2 π and this keeps happening.

So, this is something which is called in the language of MATLAB as wrapping of the phase function, it is being wrapped around 2π So, what we want to do now is simulate this particular interference, now the way we do it is I will show it to you on the MATLAB editor itself. So, again what we do is we consider this very similar syntax as previously. And since we want one of our beams to be normal to the screen we let the E_1 the equation stays the same. For the E_2 expression we use a different formula.

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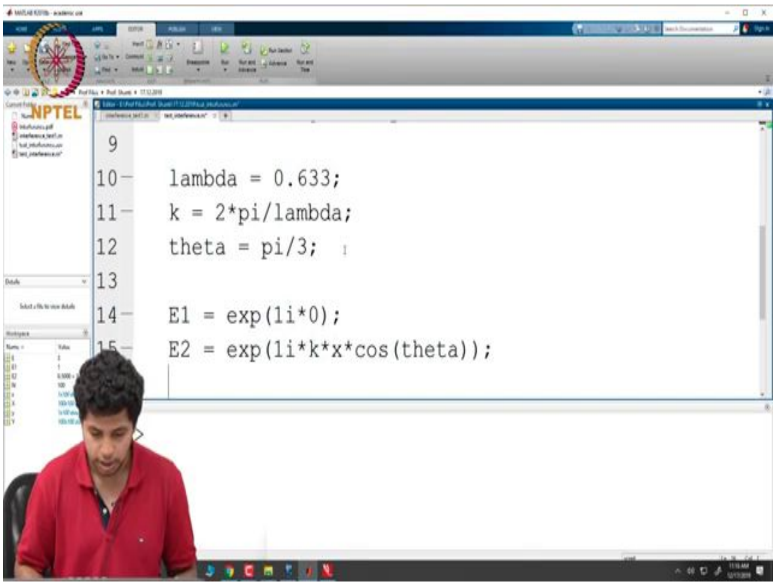

And for that we will need to specify a few things first. First we specify the value of lambda to be say 0.633 which is the wavelength in micrometers for a red laser, then we specify the value of k which is equal to $2\pi/\lambda$

Now, we also want one more thing which is the tilt angle and θ. For our purposes we will use theta as $\frac{\pi}{3}$, we could use any angle, but $\pi/3$ is a reasonable guess or a reasonable value to consider at the start. Now, what we will do is for E2 we will change the expression to exponent of 1i*k*x because we want the variation to happen in one direction multiplied by $\cos(\theta)$

Now, what we should see is an interference pattern which will contain fringes; that means there will be black and white fringes along one direction which is the x direction. Now, we will plot the entire variation or plot the entire interferogram.

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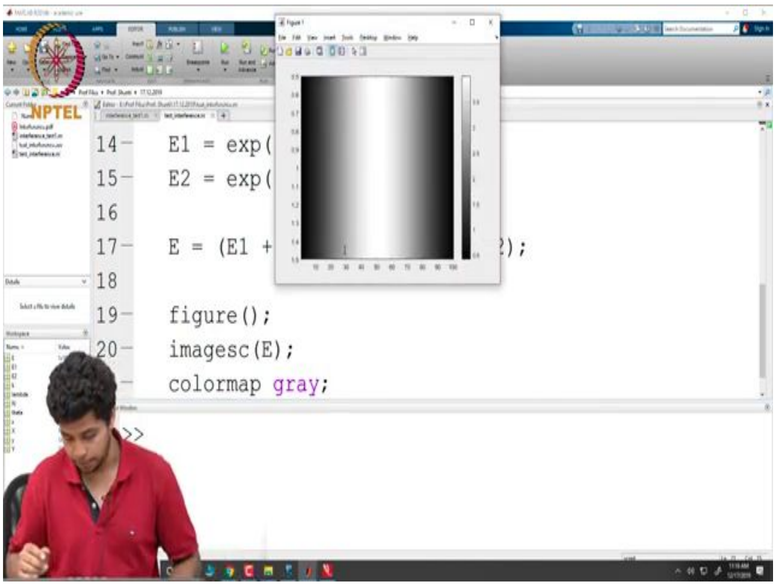

So, this is how it looks. Now, there is only one key difference here that there is a region with a black region on both sides and then there is a white region in the middle. Well there is one more way in which we can show you the fringes in a better way. And that is I will just write another program in case it is not very clear.

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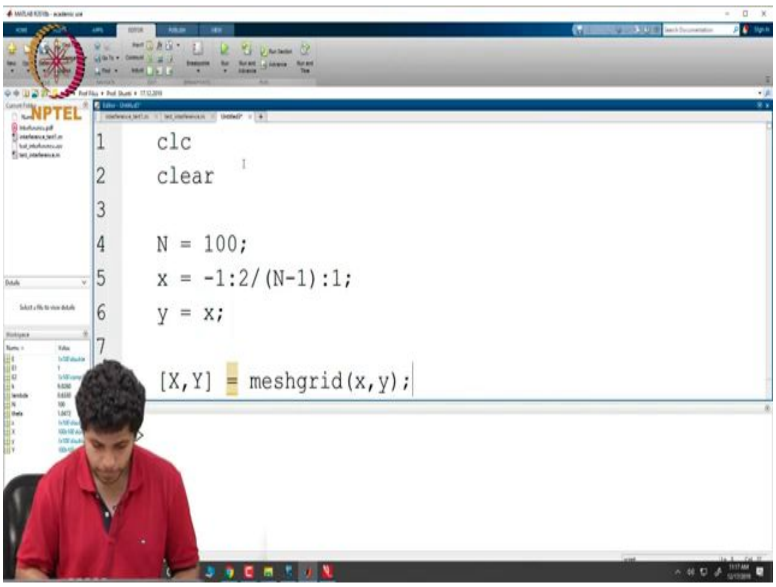

So, we use again the same syntaxes x is equal to but now I will just change the few values here. We will consider N equal to 100 like we mentioned 100 pixels by 100 points by 100 points image. x=-1 t0 2/N or N - 1 in our case actually N - 1 to +1 y=to again x which we want the same values. Then we use again the same command X, Y is equal to meshgrid of x comma y.

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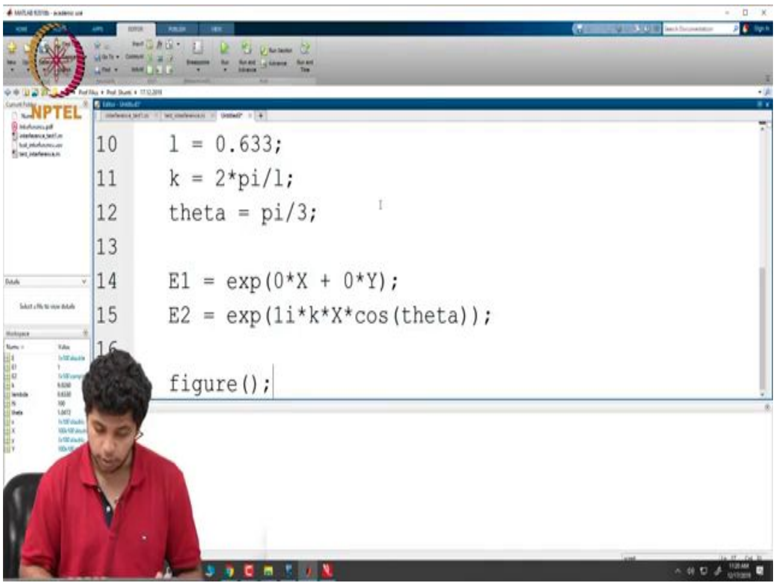

And now what do we do? Now what we do is we define l again as our wavelength as 0.633, then we define k is equal to 2π, then we define our θ is equal to say again $\frac{\pi}{3}$. Now, there will be one key difference here is that E1 is again going to be an exponent of 0 into X plus 0 into Y for our program.

And then the second beam where we want a variation in phase along one direction what we do is we use an exponent of 1i*k*x into $\cos\theta$ Now, we again have the equation ready, now what we do is we generate a figure, use the figure command to create a blank figure.

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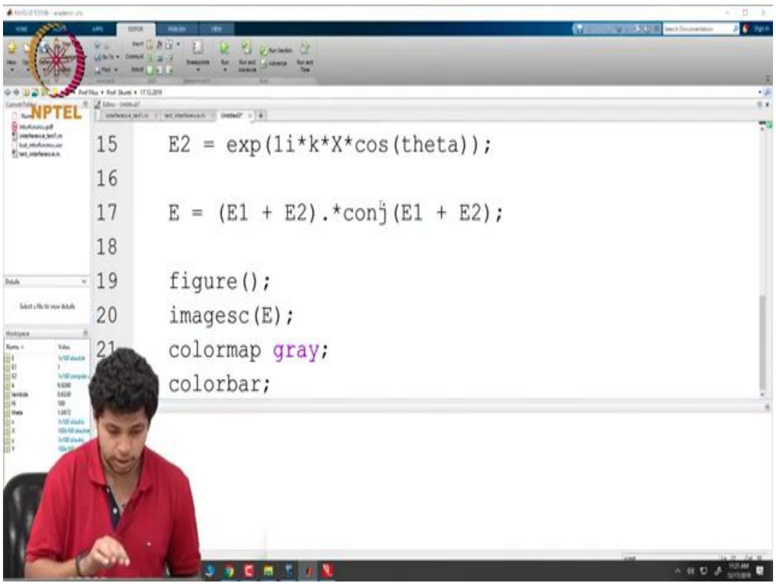

Then we put an image scan of E, I have not defined a function E, I just realized. E should be equal to $E_1 + E_2$ .multiplied by which signifies the element wise multiplication of conjugate of E1 plus E2.

And now, we have the equation e ready when we use the image scan of E, then we use the color map as gray, then we use the command color bar to give a color bar on the side, so that we can refer to the values.

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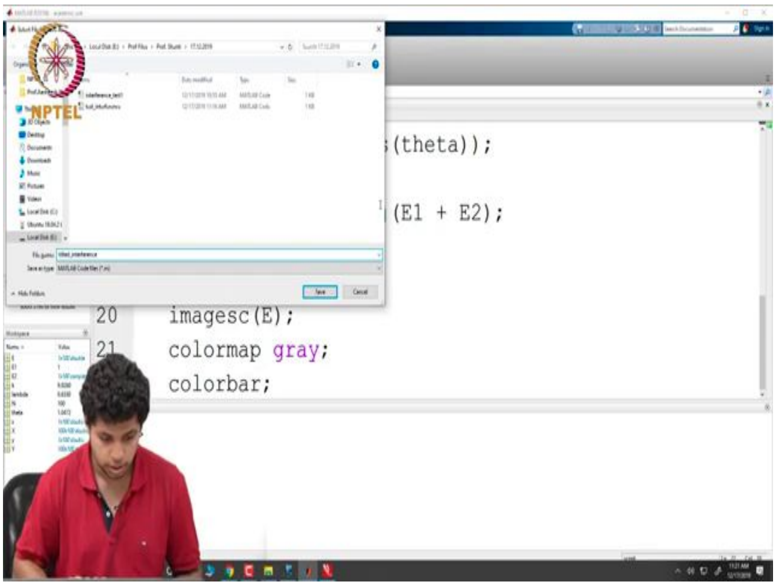

And we will run this program, we will say tilted interference.

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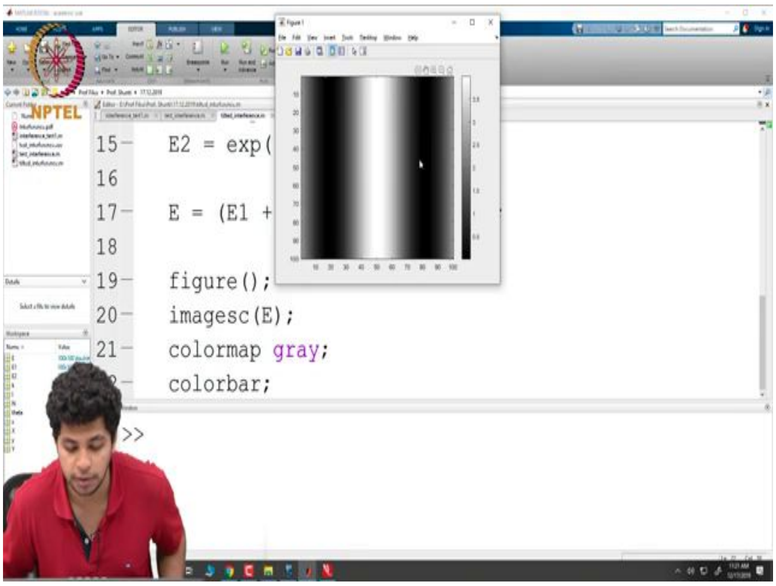

Now, it shows us this interference pattern. Now, what we can do here is play with the tilt angle that is theta, I will just make this angle as $\pi/6$.

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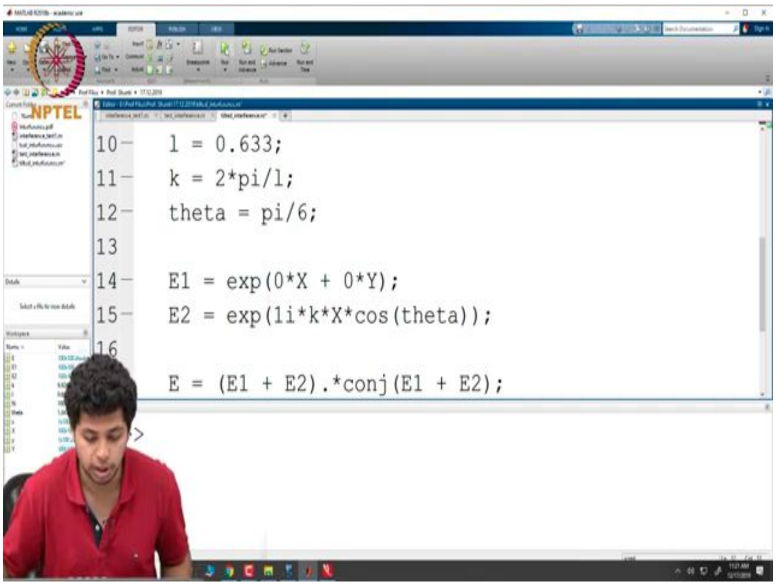

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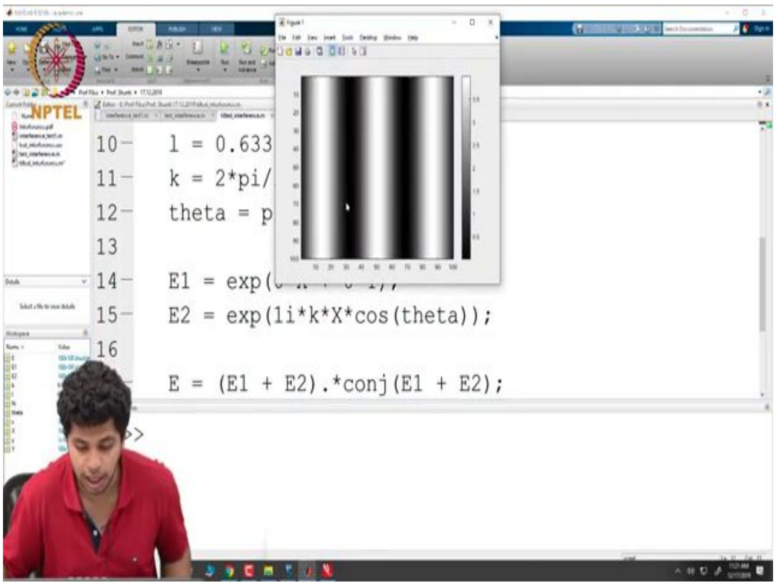

And run the program once again and as you can see as the tilt changes the number of fringes that appear changes.

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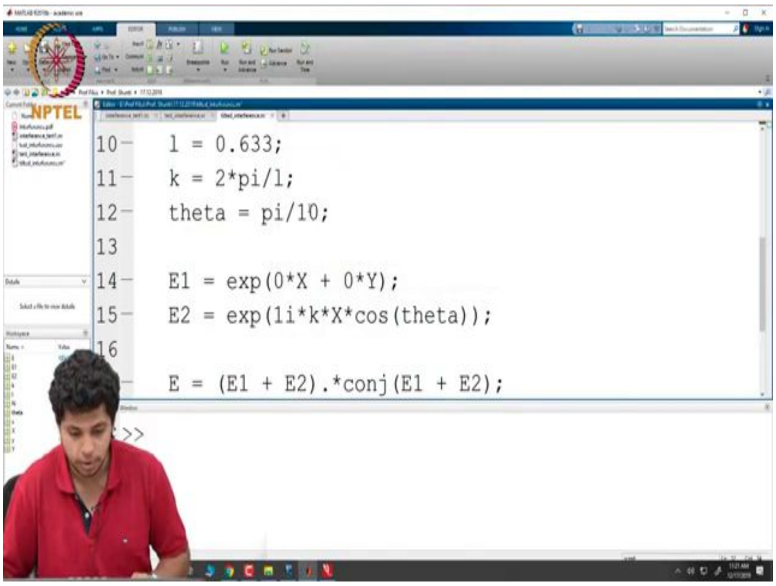

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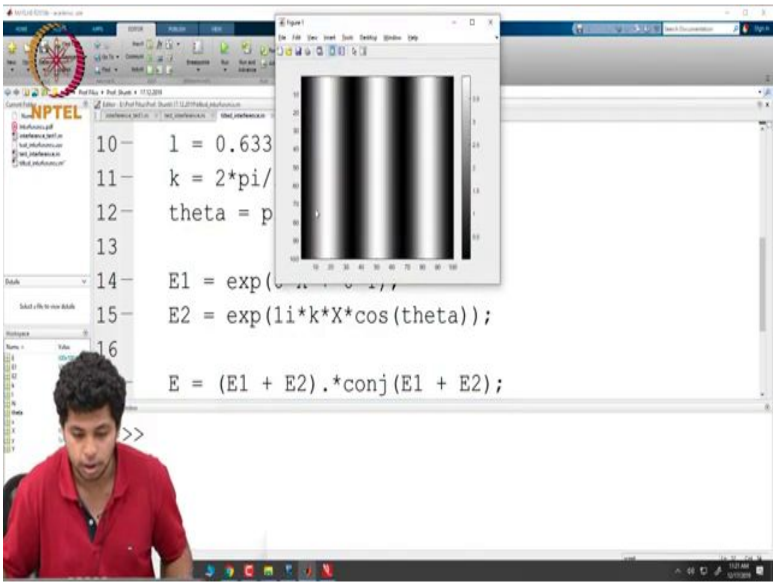

Now, what if I change it again to say $\pi/10$? The tilt angle is again changing, so the number of fringes that we see will change. The next thing that I would like to do now is to show you what happens if we take this plot and we select one particular point. And we see what variation is happening along the direction of change in the fringe pattern. So, for that we again have to use a plot command since we are taking one particular line and seeing what is happening along that line.

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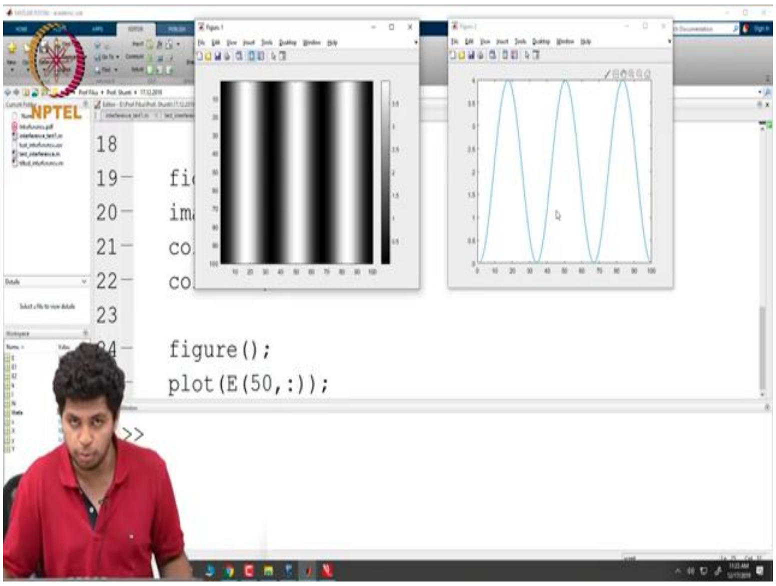

So, what we will do now is generate another figure and we will now say plot the 50th line of the E. And since we want to plot the entire line the second variable, we do not want to do anything to it because we want to plot all points along the second variable we just put a colon in there which means the 50th row in E all points will be plotted. Now, we will just run the program once again and this is what the variation looks like.

So, along the 50th line; along the 50th row we travel in the x direction and this is the variation that we see from 0 to 4 to again 0 to 4 and 0. So, this is like I mentioned for an interference of two beams with equal intensity this is what the variation will look like 0 to 4. These are the basics of interference and what you can do now is use slightly more complicated beams in order to simulate some different kinds of interferences.