Optical Engineering
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Lecture – 37
Python - part 3

Good morning, in today's lecture I will briefly explain about interference between 2 plane waves and after that I will show you how to model this simulation in python.

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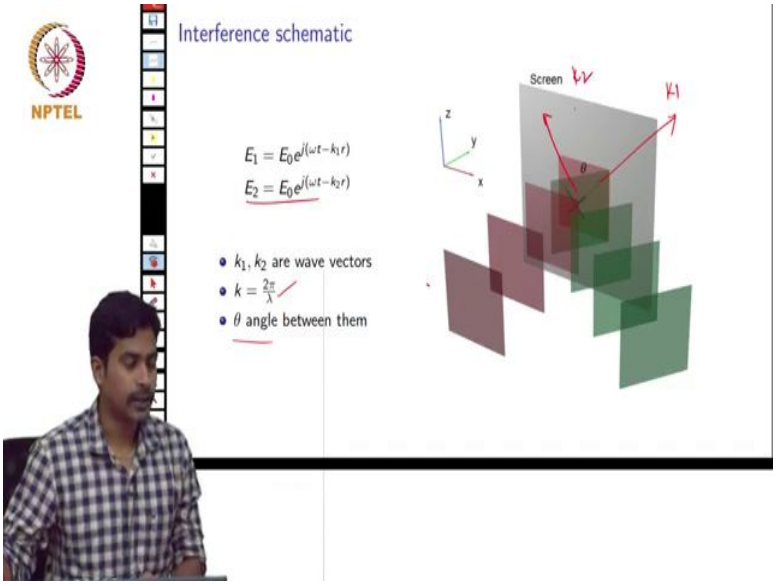

So, before that let's look at the interference between 2 plane waves, so in this figure there are 2 plane waves travelling in two different directions. So, the red one is travelling in this direction and its wave vector is given by k_1 and the other plane wave shown in green color, it is travelling in some in this direction and its wave vector is given by k_1.

So, these are the expressions for the electric fields of these 2 plane waves, so where $E_1 = E_0 e^{j\omega t - k_1 r}$ Similarly, $E_2 = E_0 e^{j\omega t - k_1 r}$ ok. And the value of the magnitude of the wave vector is given by $2\pi/\lambda$ and we are assuming the angle between these 2 plane waves is some random angle θ. So, given these parameters we want to know what is the interference pattern that is going to look like on this screen ok.

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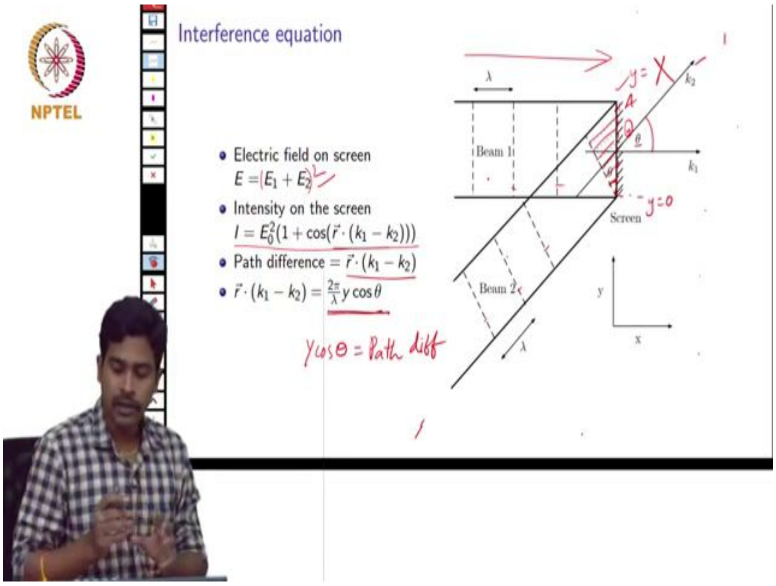

So, to understand that, this is just the top view of the beams of the figure that we have seen in the previous slide. So, where this beam is travelling or parallel to the screen and this is going at an angle theta or if you see these are the these two are the wave vectors. And the angle between these 2 beams is given by this angle theta and if you use little bit geometry you will also find out this angle is also theta. And so we said E_1 and E_2 are the electric fields on the screen. Our aim is to find the interference pattern on this screen ok.

So, on the screen when the 2 beams fall on the screen, the resultant electric field is going to be the sum of both electric fields E_1 and E_2 ok. So, in order to find the intensity, intensity is nothing, but the square of 2 is the sum of the electric fields right. So, on the screen we just have to find the value of $E_1 + E_1$ whole square, so after substituting the electric field from the previous slide and doing this algebra.

So, we will find the equation for the intensity on the screen is given by $E_0{}^2$ (1+cosr.($k_1 - k_2$) ok. And here $r.(k_1 - k_2)$ is the path difference between these 2 beams, so understand what is this path difference. So, if you look at these 2 plane waves in this figure, so the planes that are separated by lambda distance they all have equal phase ok. So, this plane, this plane and this plane they have equal phase and similarly for the second plane wave the planes that are marked in dash line they all have constant phase shifter 2π phase shifter ok.

But on the screen, there is a phase shift between the plane 1 and sorry between the plane wave plane wave 1 and 2 and that path that phase difference can be calculated by calculating the path difference between the 2 plane waves. So, which I am highlighting this red color so if you need a little bit of trigonometry. So, we can find out the path difference between these 2 plane waves at each of this point for example, let us say this is point A. So, at point A the path difference between plane wave 1 and plane wave 2 is given by $2\pi/\lambda * y\cos(\theta)$ ok.

So, θ is the angle between the plane waves and y is the position, so let us say if this is y equals to 0 and this is some y is equals to capital Y or some, some value X. So, if you want to find the path difference along this line on the screen at a point y some y value. So, you just have to substitute that y value in this expression you will get the path difference similar to the sorry phase difference.

So, path difference is Y cos θ and if you know the path difference we can find out the phase difference by multiplying the value of the wave vector which is given by this expression ok. So, now we have seen so what is the expression to use in order to find the interference pattern? So, we will see how we will model this same problem in python programming language ok.

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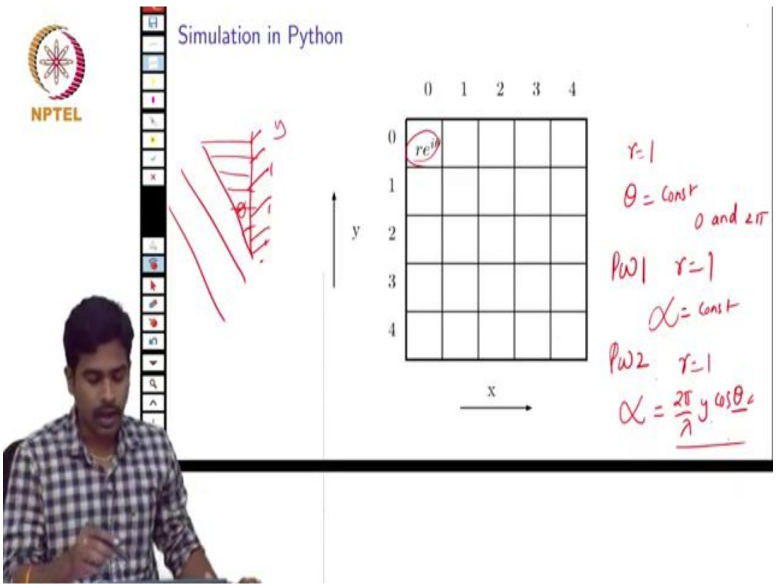

So, first of all there are 2 plane waves and we have to represent each of these plane waves as a matrix in python or an numpy array which we have seen in the previous lectures ok. So, if you look at this picture for the first plane wave the phase along this screen is constant because it is going parallel to the screen.

So, there is no phase variation for plane wave 1 on the screen, there is a phase variation for there is phase variation only for the second plane wave if you look at the projection on the screen ok. So, what we have to do to simulate is we just have to create two matrices in python one for the plane wave 1 and the second one is for the plane wave 2. And we have to represent the matrix with the phase values at each of these points on the plane wave ok. So, as I have said before the plane wave has a constant phase on the screen and the matrix is going to be constant value all constant phase values.

So, it is going to be of the form of $E^{j\theta.r}$, so in our case we are assuming plane waves. So, the amplitude of the plane wave is 1 so r is going to be 1 and θ is some constant between 0 and 2 π ok. So, for plane wave 1r = 1 and θ is going to be constant, so for plane wave 2 we again have to take another matrix of the same dimension m by m or n by n matrix. But in the case of a second plane wave as I have shown in the previous figure if this is the screen then the plane wave 2 is coming at an angle like this and this is the theta angle.

So, from mathematics I have shown you already that the path difference along this y direction is given by or the phase value this θ and the previous theta are different that the θ is the angle between the plane waves while just giving some other name. So, let us call the phase angle alpha so for the plane wave 2r=s 1 because it is a plane wave of constant amplitude.

And alpha is given by $2\pi/\lambda 2$ into y into $\cos\theta$, is the angle between plane waves and for example, if you if the plane waves are interfering in some other angle. So, this is the side view if the plane waves are interfering in another angle for example, in this figure the plane wave 1 is coming like this and the second plane wave is going at some other angle.

So, it could also be from top in that case it's going to be x or if the plane wave is coming either in some other plane instead of xy plane so then the phase variation is going to be both as a function of both x and y. So for simplicity we are just dealing with the plane wave and the plane wave were in same xy plane ok. So, in the second for the second plane wave the phase on the screen is going to be $\frac{2\pi}{\lambda}$ *y cos θ .

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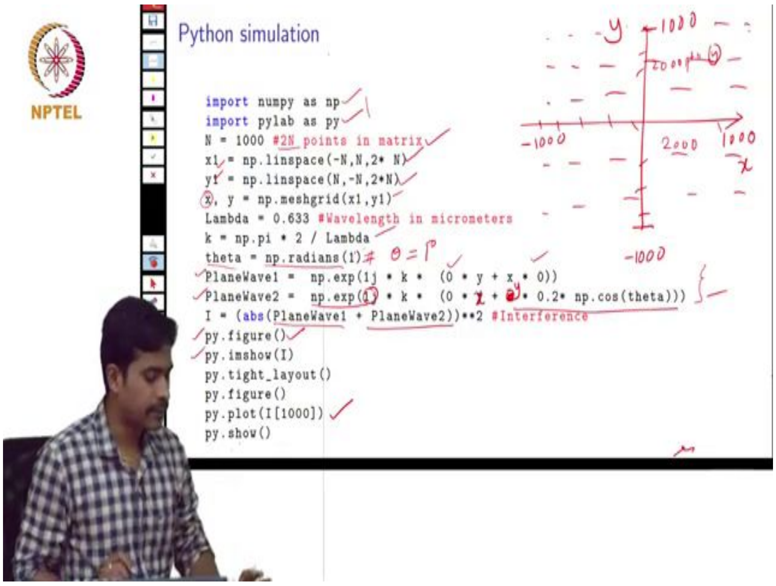

So, now I will give you the code for this python simulation. So, let me just explain what each of these lines are doing and after that I will show you what happens when we run the code and what are the different parameters that we can change. And what happens when we change some of these parameters on the interference pattern.

So, here I am using numpy arrays for the simulation rather than list, so that is why first we have to import the numpy package and I am using pylab to show the plots or the images that is why we just have to import these two packages first.

I am using a matrix of 2000 by 2000 matrix points here, so for that I just took 1000 and there will be going to be 2 n number of points in the matrix and x_1 . So, in this simulation what I am doing is I am creating a mesh grid with coordinates varying from minus 1000 to 1000 and in my direction to 1000. And this is x axis and this is y axis, so between these two points along y I am sampling 2000 points similarly I am sampling 2000 points on y axis ok.

So, that is why I have x1 varying from minus 1000 to 1000 and 2000 points and similarly for y 1. So, with these two values I am going to create the mesh grid of the coordinates system using this command mesh grid of x1, y1. So, the values x is going to have all the values in this coordinate system, all the x coordinates of the points 2000 by 2000 points and y is going to have all the y coordinates of all these points in the coordinate system ok.

And here I am choosing the wave length to be 633 nanometers and from these we can calculate the wavenumber to be $\frac{2\pi}{\lambda}$ 2π with this equation and for this simulation I am taking the angle between the plane waves to be 1 degree. So, in order to calculate the $\cos\theta$, so cos only in the argument for a cos function is always a radian. So, we cannot give degrees directly to the cos function. I am converting the 1 degree angle to radians using this command and theta is stored as a radian angle ok.

So, the plane wave 1 and plane wave 2 are calculated here, so as I said before the plane wave is going the first plane wave is going to have a constant phase on the screen. So, here 1 jk into so we are not giving any value we just it is just a constant plane, constant phase values. For the second plane wave as I have said before I am sorry there is a mistake here. So, it should have been 0 here sorry it should have been y here it should have been x here, so as I have said since the plane wave is going at an angle theta in xy plane.

So, the extra path difference is written in this expression and to calculate the phase values. And in order to say it is a phase value we are using e to the power 1j and 1j represents the ji or j complex number and k is the wave number and this is the expression. So, after we have now the expressions for plane wave 1 and plane wave 2 so we just have to use the interference equation that I have shown in the first slide.

So, which is nothing, but adding the 2 electric fields and taking the square and absolute value square absolute value of the square, square of the plane wave 1 plane wave 2 ok. So, after we got these interference values so now, what I am going to show now using this command I am showing the interference pattern and after that I am using another command. So, plot to plot intensity along a particular line in the interference pattern so after I run this call the output is going to look like the next slide.

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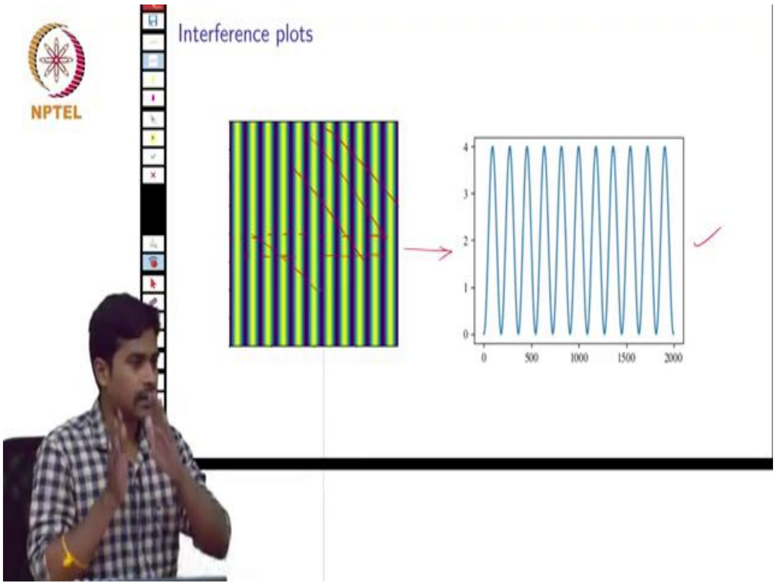

So, this is the interference pattern image, so which has a sinusoidal variation. So, what I have done in this adjacent figure is I plotted the intensity along one of at the midpoint of some, some one of the rows I have taken from this figure and then I have plotted in this graph.

So, we can see the sinusoidal variation so this is how we model interference. So, the same procedure can be used to do interference simulation for any different kinds of waves. For example, if you have a plane wave and you have another plane wave coming in some other direction rather than its coming in some y z plane or x z plane. So, in that case what is going to happen is the path difference is going to be a function of both x and y ok.

So, in that case what can happen is the fringes may be in this direction rather than y x direction. Because the wave is hitting in some other random angle or you can use the same program to simulate the interference between a plane wave and a cylindrical wave or a spherical wave you know a spherical wave has a spherical wave front and a plane wave has a constant phase.

So, you sum that two using the same method and you take the square of it and the obsolete value you can see the interference of any two kinds of waves ok.

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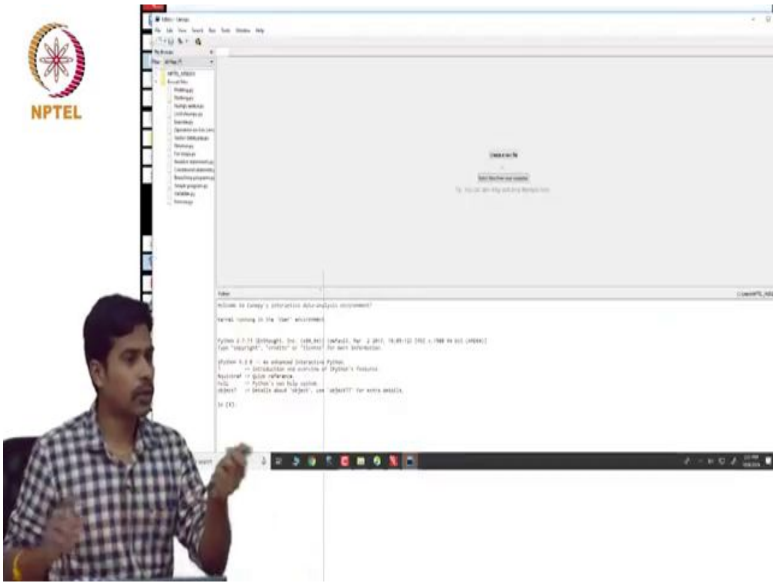

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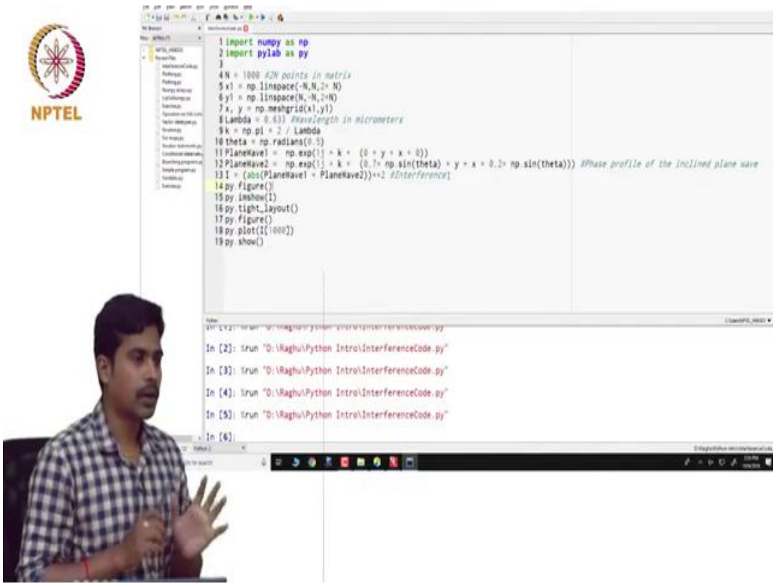

So, now I want your guides if you have the code you can open ok, so this is the same code so there are two parameters that we can vary in this interference simulation. So, the first parameter is the angle between these 2 plane waves and the second one is the direction of the second plane wave ok. So, by varying these two parameters we can see what is happening to the interference pattern. So, now let us go back to the python code window so let us say first we will just run the program as it is and.

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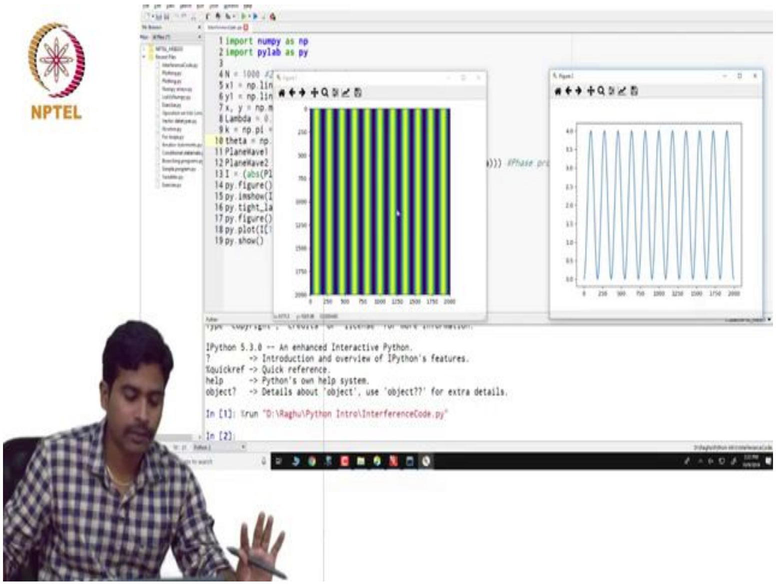

So, we got two figures so the first figure is actually an intensive interference pattern and the second one is the one of the rows I have taken from this figure and then I have plotted it to show the sinusoidal variation. So, this is for the case where the plane waves are going at 1 degree with respect to each other. So now, can you guess what happens if we increase the angle? Can somebody answer?

So, what do you think happens when the angle between these 2 plane waves is increasing so what happens to the fringes are they going to increase or the or they going to decrease ok. We will just try out here so I am giving maybe 5 between the plane waves and I will run this problem.

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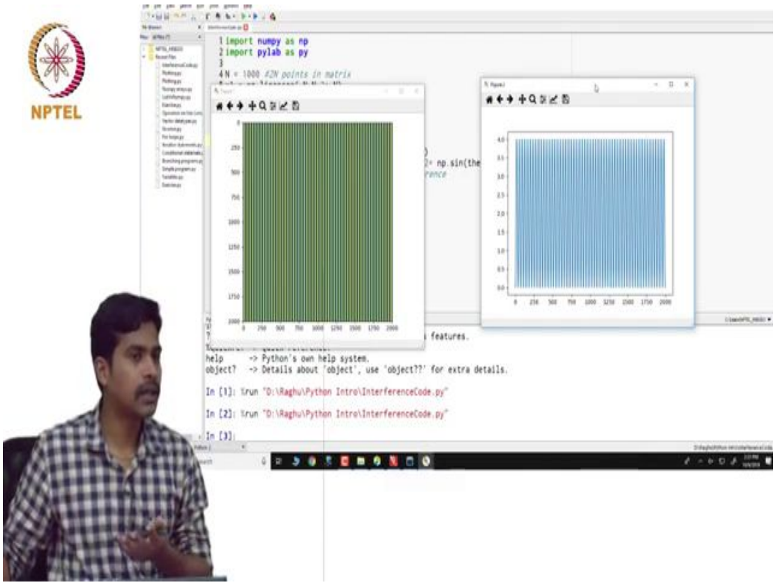

So, as you can see from this, the number of fringes have increased a lot compared to the previous case. So, one observation we can make from this simulation if the angle between the plane waves increases. So, the number of plane waves also sorry the number of fringes in interference will increase.

So, we can just stop here and then think for a moment why the fringes are increasing. It's very easy because when the angle is increasing the path difference is also increasing. So, since phase can only have values between 0 and 2π so if the path difference is increasing it does not matter the number of 2π is going to increase in that path difference so that is the reason for this large number of fringes.

So, similarly you can give a smaller angle say 0.5 and; obviously, you will now see a lesser number of fringes compared to the previous case ok. And the second thing I said was so what if what happens if we give the path difference that is a function of both x and y which means the plane waves are no longer in the xy plane. So, the second wave is coming from some other arbitrary direction, so let us just actually this should have been y, but anyway we will just use the same path difference term for the y variation.

So, now what is happening is the plane wave 1 is coming like that and the second wave instead of going in this plane it is coming from some oblique kind of plane.

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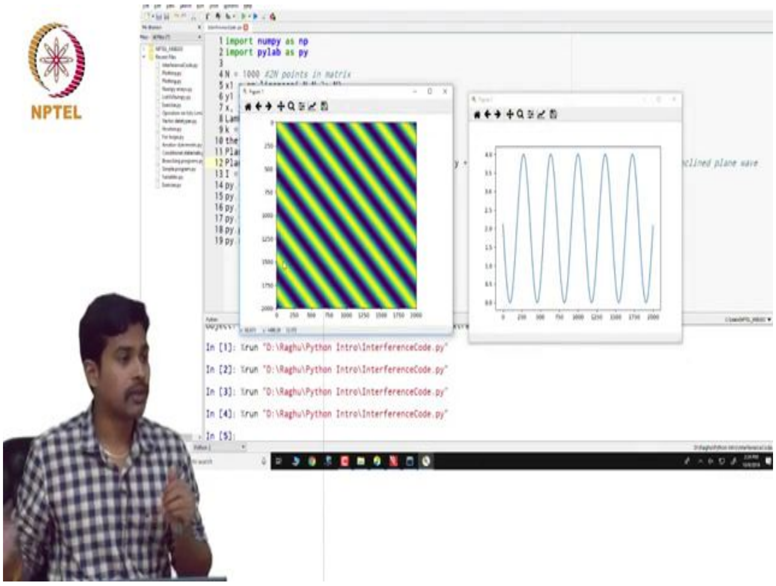

So, if we run this program now so if you can see, the fringes have shifted in xy direction and if you give more weight on one of these.

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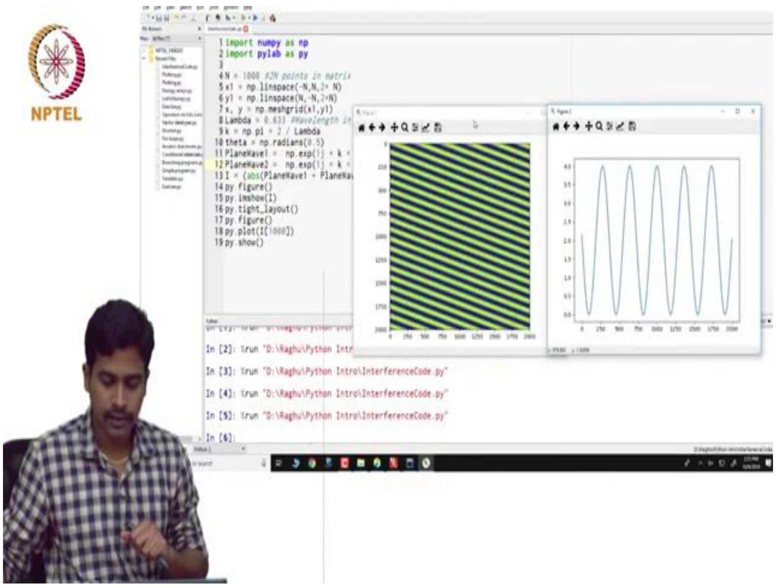

So, you will start to notice the fringes also varying so now, there are more like y direction sorry x direction. So, these are the parameters that you have in your hand to control interference. I hope you all understood the simulating interference. So, if you have any doubts you can ask me or you can start simulating the interference between a spherical wave and plane wave and spherical wave and a or a cylindrical and plane wave. So, using the same it is just the same procedure so you have a plane which is constant.

And the second plane wave we just have to model using some mathematical expression that describes either a cylindrical or a spherical or helical whatever. So, you just have to represent it mathematically and then this equation is the same. So, with this I want to conclude the interference simulation part so now if you have any doubts you can ask me otherwise that is it.