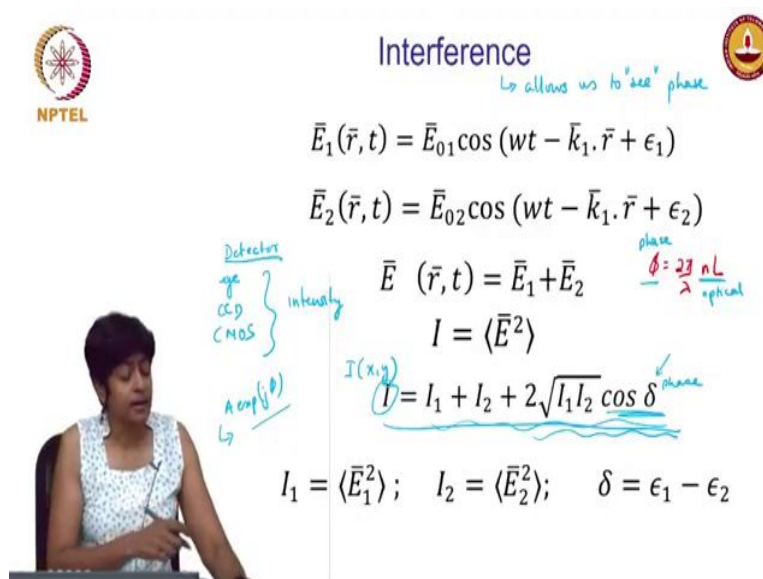


Optical Engineering
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Lecture – 36
Interferometry basics – part 2

(Refer Slide Time: 00:16)



The slide is titled "Interference" and includes the following content:

- Logos:** NPTEL logo on the top left and a circular logo with a lamp on the top right.
- Handwritten Note:** "allows us to 'see' phase" written in blue ink below the title.
- Equations:**

$$\bar{E}_1(\vec{r}, t) = \bar{E}_{01} \cos(\omega t - \vec{k}_1 \cdot \vec{r} + \epsilon_1)$$

$$\bar{E}_2(\vec{r}, t) = \bar{E}_{02} \cos(\omega t - \vec{k}_1 \cdot \vec{r} + \epsilon_2)$$

$$\bar{E}(\vec{r}, t) = \bar{E}_1 + \bar{E}_2$$

$$I = \langle \bar{E}^2 \rangle$$

$$I(x, y) = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta$$

$$I_1 = \langle \bar{E}_1^2 \rangle; \quad I_2 = \langle \bar{E}_2^2 \rangle; \quad \delta = \epsilon_1 - \epsilon_2$$
- Handwritten Notes:**
 - On the left, "Detector" is written above a bracket containing "age", "CCD", and "CMOS". Below this, "Amplifier" is written with an arrow pointing to the intensity equation.
 - Next to the intensity equation, "intensity" is written in blue ink.
 - Next to the phase difference equation, "phase" is written in blue ink, and " $\delta = \frac{2\pi}{\lambda} nL$ optical" is written in red ink.
 - Below the intensity equation, "I(x, y)" is written in blue ink.

I have given you examples with the temperature sensor, or the pressure sensor, or measuring refractive index or directly measuring physical change in length. But, you could also think of what if I wanted to do imaging? And I wanted to image a biological sample.

(Refer Slide Time: 00:34)



So, on the left here you have an image of some cells ok. So, these are cells. Now, you can see that this image is very hard to see clearly, right. And we talked about what makes a good image several classes back. I showed you this picture of the India Gate under fog, right. What was it we said that distinguishes a good image from a bad image?

Student: (Refer Time: 01:05).

The visibility, the contrast right, so you can see here you can make out that there is a cell you can even make out some of the structure within the cell. So, you know there is something there, but the contrast between the cell, so this here, and the structures within the cell and the region surrounding the cell is very poor.

So, this image is not a very good image and if you are using this image to help make a diagnosis, is there some disease present in the person responding to some medication as a cell structure change is that is this a cancerous cell or not? Is it malaria? Is it AIDS? I need to have very clear pictures.

So, this picture does not have good contrast. In the case of the India Gate picture, it did not have good contrast because of the fog which was scattering the light. Otherwise, you would have had a very strong reflection from the building and from the regions around the building there was nothing you did not have a reflection and that is what created the contrast. Here,

you do not have a contrast because the refractive index of what we want to image, that is the cell, and the refractive index of the medium around the cell are very similar. The difference between these refractive indices is small.

So, whatever the change is happening to the light, say, we this image was taken in a microscope where light passed through the sample. Whatever difference or change happened to the light passing in the medium around the cell, a very similar change is happening to the light travelling through the cell and therefore, there is not much difference and therefore, we do not see that.

So, there are many microscopic techniques which some other, now, the point is if there was 0 difference in what was happening in these 2 regions, there would be nothing I could do. But, there is a slight phase difference between the cell and the region around the cell. Its too small for our eye to make out, but there is a small phase change and if you can enhance that, or use the fact that there is some there is some difference and enhance that difference in some way, you can then make this a clearer picture.

So, there are many microscopic techniques which somehow extract that phase and here I am just showing you an image of something that has been done using what is called phase-contrast microscopy,. So, we are not going to go into the details of what has been done, but the image on the left was taken with a regular microscope. It's called a bright field image and the image on the right has been taken with the phase-contrast microscope.

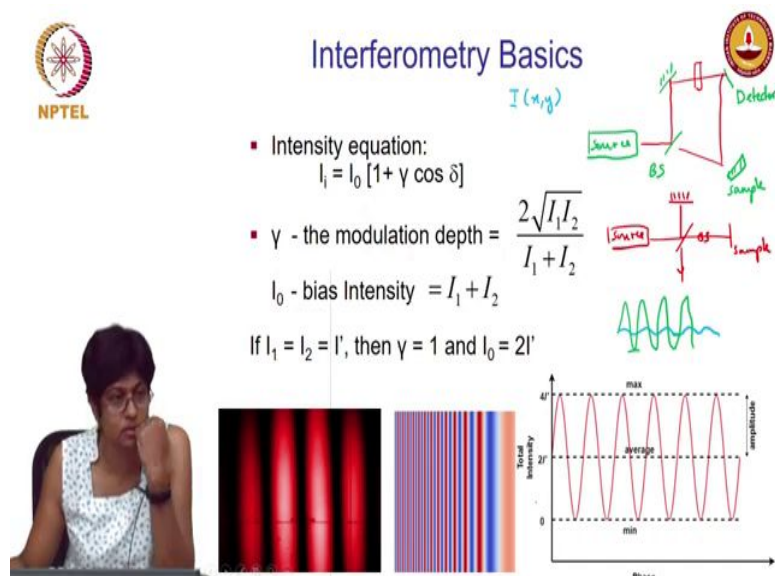
What I will tell you is that the phase-contrast microscope uses the fact that you have lights scattered because of the structure without that small phase difference, and you have light that travels through the main other surrounding medium. And if you somehow take the scattered light and introduce a phase additional phase there and then interfere with these 2 beams. So, you are using the phase-contrast microscopy using interference. And therefore, enhances the phase differences within your sample, ok.

So, the image on the right then is this enhanced image, you have taken whatever small phase differences are there and used interference to enhance those phase differences and you can see the effect of doing so. And suddenly, it becomes much more clearer and there is much

more detail and of course, you can see that the phase contrast image is going to be a much more useful image for you to extract information out,.

So, this is just one example of where you could use interference. But how do I extract the phase? We are still all our detectors are only measuring intensity, they are not measuring phase. So, if I go, I keep saying we will use the phase the phase gives us this information, but how do I get the phase? Ok. So, that is the question you should all be asking, ok.

(Refer Slide Time: 05:52)



So let us look back at this equation. So, if I assume I have 2 beams interfering, then I have the intention again, I have always written this just like I but, remember it I can always be more specific and say its $I(x,y)$; that means, that there is an intensity variation. There is a different intensity at every point on the beam, on the interference pattern and with our sensor variable to capture that variation.

We can not capture phase directly, but of course, every pixel of the sensor is going to measure the intensity lying on that pixel and it need not be all the same, it will vary, ok. All I am able to measure is this intensity $I(x,y)$. But buried in that intensity is this I_0 and this γ , where I_0 is nothing, but the bias intensity which is the sum of the intensities of the individual beams that were interfered; and γ is something called the contrast or the visibility or the modulation depth which is this function over here.

So, its again a function of the intensities of the individual beams, ok. So, I do not know I_1 and I_2 , I do not know the delta that is the phase difference between these 2 beams, all I know and I know this because I can measure this with the detector is I, this I here. So, buried in this is I_0 , γ and Δ . And I_0 and γ are functions of the intensities of the independent beams, ok.

If you were somehow able to ensure that the two beams were of equal intensity, I can reduce this equation, right. Then your gamma will reduce very simply to 1 because $I_1 = I_2$, ok. But in general, I will use this expression for the modulation depth and this expression $I_1 + I_1$ for the bias intensity, ok. What is the modulation depth? What do you think the modulation depth is giving me? What information?

Student: $\cos(\theta)$.

Sorry.

Student: $\cos(\theta)$ (Refer Time: 08:33).

Its γ is the modulation depths. So, its multiplying the cos term right. So, maybe I did not follow your answer. I want what is the maximum value that the modulation depth can have? It can have a maximum value of 1. If $I_1 = I_2$, it will have the value 1, for any other ratio, it is going to be less than 1 and in fact, if one of these intensities is 0 what happens?

Student: (Refer Time: 09:12) 0.

If one of these is 0; that means, there is no interference; that means, intensity of 1 beam is 0. I am not interfering with 2 beams, the third term in that expression, remember your interference expression was $I_1 + I_2$ plus this cosine term that comes because of the interference. So, if I remove 1 beam of course, I do not have that interference term and the intensity is just going to be the intensity of the individual beam, ok.

So, of 0 is not of interest to us. We always are looking at the case where you have 2 beams. But it can happen that the intensity of 1 beam is much lower than the other beam, right. Why could that happen? Well, we are creating 2 beams, but now we are looking at specific

applications. So, we are saying 1 beam acts as the reference beam. I need interference, I need interference because interference is the way by which I extract phase.

So, I am putting in an extra beam there just so that I can have that interference happen. 1 beam is the reference, the second beam is the beam that carries the information I am interested in. So, that beam is either traveling through a sample or getting reflected off a sample, That, in That is the way by which I get that information of the sample.

Now, when it transmits through a sample or reflects off a sample it should not be hard to imagine that the intensity of that beam is going to fall quite a bit, right. If the sample is not very highly transmissive, even when it reflects it's not a mirror like reflection necessarily. Ok, sample could have been rough surfaces. So, there is a lot of light going off in other directions.

So, some light is what you capture. So, the intensity of the object beam is usually weaker than that of the reference beam. Now, this expression he was telling me the highest value of modulation depth I can get is 1. So, and gamma is what multiplies the cosine term, right. So, that is what is giving me my fringes, right. These are my fringes over here, this is an example of the fringes between 2 beams.

In order for me to see those fringes clearly, I want this modulation depth. The difference between the maximum and the minimum here to be as large as possible, right. I could have fringes where this is the, right. Remember, these should be periodically spaced. I am right, I could have with the same period fringes like this, all right. Which one has a larger modulation depth?

Student: (Refer Time: 12:27).

The green one has the larger modulation depth, right. Maybe it has a depth close 1 or close to 1. Now, if you were looking at fringes which one would you see clearly? You would see the one with the larger modulation depth. So, you know this refers to the low frequency. So, if the low frequency is sorry this is the low intensity.

So, if the low intensity and the high intensity values are almost similar, that is when the contrast goes down. I cannot tell the difference between them, there is a difference, but we are going to, our detectors. I am not going to be able to tell the difference. In order to tell the

difference the minimum intensity and the maximum intensity should be if as wide or as far apart as possible. In other words, I want as large a modulation depth as possible, ideally, I want the modulation depth to be 1 because that is the best I can get. So, when will I get a modulation depth of 1?

Student: (Refer Time: 13:31).

If the beams have equal intensity. So, I really want the 2 interfering beams to have equal intensity and if I had one beam reflect off a sample and because in the process of reflection, the intensity dropped greatly, how would I improve my interference? What could I do? I don't really have much control on the object beam right.

It is reflecting the sample. The sample is what the sample is, right. In fact, the whole idea is to measure what the sample is. I do not want to manipulate the sample. So, if I want to improve the modulation depth in my interferometer, what other is there something I can do? I just have to live with the visibility I get.

Student: (Refer Time: 14:17).

Sorry.

Student: (Refer Time: 14:18).

Surprisingly, yes. Because you always think that I must improve things, I must increase things. If I want to increase modulation depth here, you would actually lower the intensity of the reference beam. You want it to be closer in intensity to the object beam. So, you might actually throw away power in order to increase modulation depth. Could you think of anything else you could do? Think of the setup, let us say, I had a setup like this, this is source, beam splitter.

So, mirror and I am I am drawing and my sample is sorry or this is not a mirror, let us make this the sample and detector. So, this is not a Michelson interferometer, because its not retracting paths.

So, the light is traveling like this, the beam splitter sending it here and here and the mirror has been aligned to send it like this. Its I of course, optically path length matched and so on. So

one thing you said is, lower the intensity how would you do that? Or let me ask the question differently, how would you lower the intensity of the object beam?

Student: Instead of (Refer Time: 15:49).

Exactly, you could change the beam splitting ratio. That could be one way, or I could put a filter in this arm that cuts their intensity here, but the filters are less efficient because there I am actually throwing away the power. Whereas, if I change the splitting ratio of the beam splitter, I am sending more power to the sample which is making the sample beam higher intensity and less power to the reference beam which means it might end up being equally intensity to the reflected beam, right.

So, having a beam splitter of a different splitting ratio is good. There is a reason why I did not draw the Michelson because, in the Michelson, what is the difference in the Michelson? You have the beam splitter and then you would have this and you would have your sample here, mirror here and then they come. Why did not I draw those Michelson? Why does not, why is changing the beam splitting ratio and the Michelson not really going to serve your purpose?

Student: (Refer Time: 17:08) because we could.

You go through the beam splitter twice. So, if you say 40 60, this beam goes 40 and this goes 60. On the return, this will go 40 and the will right. So, it does not help you, right. So, the Michelson is not necessarily the best to use if a beam splitter of a variable splitting ratio is not very good with the Michelson. But if you are not retracing the path then you can use that, ok.

So, here what the images I have shown over here are some interference patterns. So, the image over here may be taken with 2 beams generated from a Helium-neon laser. So, that is why you have this red colour. Now they are both interference patterns, but they are not exactly the same, right.

The one on the left seems to have a constant period, fringes are slightly thicker in the middle and there is some difference, but the period that is the distance from the centre of this fringe to the centre of this bright fringe to the centre of this and to the centre of this is constant.


Whereas, if you look at the fringes over here, there is a variation, ok. So, this is an experimental image, this is a simulated image.

So, the you know you do not have the absolute dark here, but basically they are just same, you have regions of constructive interference and regions of destructive interference, right and from 1 constructive peak to another constructive peak is what I call the fringe period and the image on the left as a constant period whereas, the image on the right does not have a constant period. What do you think?.

Could be the cause for this, what could be interfering here and what could be interfering here? Remember, we yesterday what did I say? Can think of your interference fringes as tracing the paths of constant phase difference. Just like an isobar or isotherm is a graph that gives you constant temperature or constant pressure, right and interference pattern, the fringes are indicating regions of constant optical path length right.


So, what could have caused this image and what could have caused this image? You do not have to think about whether you can not go into the type of beam, but if you were to think of the 2 wavefronts that are interfering, what would you do differently between those 2 wave fronts? Maybe, make it easier for you, look at this Michelson interferometer that we have drawn here, ok.

(Refer Slide Time: 20:15)



NPTEL

Interferometry Basics




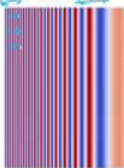
$I(x, y)$

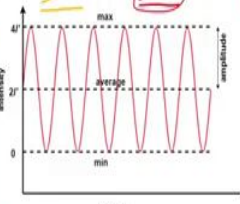
- Intensity equation:

$$I_i = I_0 [1 + \gamma \cos \delta]$$
- γ - the modulation depth = $\frac{2\sqrt{I_1 I_2}}{I_1 + I_2}$
- I_0 - bias Intensity = $I_1 + I_2$
- If $I_1 = I_2 = I'$, then $\gamma = 1$ and $I_0 = 2I'$

Interference of two tilted plane waves







If I consider that I have a beam of a certain width, ok.

So, I have a beam of a certain width travelling, this reflection of the beam splitter goes to the top mirror and comes back. So, the returning beam from this mirror is this, right. Its travelling like this is after the beam splitter, it's a plane wave let us say I have plane waves. So, its wavefront is going to be this plane surface here. So, if I draw the wavefront of the beam that is coming back from the mirror M, it is the wavefront of this beam.

Now, let us say I have this is also a mirror, part of that beam gets split at the beams splitter and travels straight on to this mirror, it gets reflected it comes back and it also is a plane wave. So, its wave front also looks like this. Do you agree? Ok, yes. What do you think the fringes will look like now? What is the path length difference between these 2 beams?

Student: 12.

Pardon?

Student: P w path length (Refer Time: 21:25).

Pathlength yes, you are right. In the sense, if I trace one beam, it travels this distance, it travels this distance, it travels back this distance and then it travels this, ok. I have said let us assume all of that has happened, ok. So, let us say beam one has travelled a path length optical path length OPL 1. That takes into account all this double distance.

And beam 2 has travelled up optical path length 2. And having each travelled this path length, they have arrived at the detector here with wavefronts as I have shown over here, ok. So, it takes into account the entire path length that is travelled. I am drawing the beam splitter as if it does not affect. It also has a thickness, it also has a refractive index.

So, I have taken all of that into account, ok. And we are taking a very simplistic picture, we said I have a plane wave incident and the plane wave stays a plane wave through all of this, And now I have these 2 plane waves that come and hit the detector. And I have drawn them the way I have drawn them, in this image here, right. What do you think the interference pattern is going to look like? What will you see on the detective, let us say I have a camera, what will you see?

Student: Same as source low interference

You will not see anything, I mean oh that is not correct, you will see something you will see as he said he said the same as source. So, you will see a uniform intensity. Why? you do not answer yet. Why would you see a uniform intensity? When do you see interference happen? I am also assuming that $OPL1 - OPL2$ is less than the coherence length of your source.

So, the conditions for interference are set. We are not violating that, but as he says, if I assume everything is perfectly aligned and the wavefronts are as I have drawn them, you will just see a uniform intensity across that camera, as if one beam was incident on that.

Student: No, I am going to (Refer Time: 24:00).

There is no path length difference across laterally. When do I get fringes? The fringes here, you are saying this region constructive interference happened, here destructive interference happened. That means, a path length difference between the 2 interference beams is not the same everywhere. If you have the same path length difference everywhere, then over here, you will just see the same intensity, right. In order to see a path length difference, I need to have a spatially varying path length difference.

So, if I in this case what you will see is a uniform intensity. So, the result will be a uniform intensity. If I could somehow get the plane waves to be like this, then there is a spatially varying path length difference. That means, in some places the difference is going to result in constructive interference, in another place it may result in destructive interference, in another place constructive, and then I get fringes. I can not get fringes if the path length difference is everywhere the same, right.

How would I achieve this? What would I have to do to my interferometer to achieve that?

Student: change the sample (Refer Time: 25:45).

How? Change the sample arm? How do I change the sample arm? I am saying the wavefront, that is if everything is perfectly aligned, the wavefronts coming from these 2 mirrors are parallel to each other. And now I am saying have 1 wavefront have a tilt with respect to the

other. The wavefront is nothing but the beam that has come off those mirrors. So, if I want to change something about the wavefront, what is my control to do that?

Student: Refractive index.

Sorry?

Student: Change the refractive (Refer Time: 26:22).

Why will changing the refractive index change the tilt of the wavefront? Or change the refractive index of what?

Student: The medium from the beam splitted to the detector.

If I change the refractive index here, I affect both beams. Both beams travel through that region. So, the overall result is no change at all, right. Both, the beam coming from the reference arm and the sample arm are travelling in this region. Any, sorry any change I do here, affects both the beams equally.

Student: Uniformly, mam, it is slowly varying from the same tool.

Whatever I do, even if I do a slow variation, a gradual variation, spherical variation, a paraboloidal variation, it affects both beams. If I want to have a difference, I have to affect 1 beam and not the other. So, I can not ever work in this space.

Student: Change the calculations or on a (Refer Time: 27:26).

Change the.

Student: A research into that important what are like a del one the first mirror and the (Refer Time: 27:33)

Ok, but I'm asking if you are getting the point? You have to change 1 arm, but why you refractive index? I have a wavefront. So, you go back to your beam theory, we are not doing anything different from what we are doing here. These could be Gaussian beams travelling

through, if I wanted to change a Gaussian beam is travelling like this, and I wanted to travel like this is your first response change refractive index.

Student: Distance, yeah.

What would I change? I have a beam you are getting confused with, I think you are just thinking of your Gaussian beams. I want to change the direction of travel of a Gaussian beam, of any beam if I want to change direction of travel.

Student: Change the angle.

Tilt the mirror. So, all you will do is in the original set of these 2 mirrors are 90° to each other. I will introduce a small tilt. So when I say perfectly aligned in a Michelson, if I want to see fringes, it does not mean the mirrors are at 90° to each other. There must be a tilt, it has to be a very small tilt. But there has to be a tilt and usually, it is quite impossible to set the mirrors at 90° So if you are assembling it, you are going to invariably have a tilt right.

I will ask you another question later. What is the problem if the tilt is too large? Ok. But right, so think about that. But right now, in order to get a tilted beam, basically, if I say the wavefront is like this and I want the wavefront like this, what I am asking you is, how do I tilt the beam? How do you tilt the beam? You tilt a mirror. So, instead of having the mirror as shown, if I had one of these mirrors slightly tilted like this, then the beam that is reflected is going to be I if I tilt it too much it does not retract it has to still come through that beam split and come.

So, it is a small tilt right and then I will have a so now, you could consider you have one beam this is a untilted beam and then you have the tilted beam and its only in this region you will see fringes. Otherwise, you just have uniform intensity, ok. So, let us go back to my original question. What could cause this fringe pattern now? We have already answered it. If I have one wavefront like this and one wavefront like this, how is the path length difference varying?

Student: (Refer Time: 30:21).

It is a linear variation. So, I would expect that if constructive interference happened at some point or some line, at some distance away. the next constructive interference is going to happen. So, that is the period and that is just going to repeat because the optical path length is varying in a linear way, right. So, periodically the fringes or the locations of constructive interference would happen.

So you can imagine that this is nothing, but the interference of 2 tilted plane waves. Now, in the second image the fringes do not have a constant period. The period itself is changing, right. So, let us say my reference wavefront is this. What do you think the second wavefront could be that could cause this interference?

Again, when I ask that question what you should be asking yourself is, how is the optical path length changing? And if the optical path length is changing in this way, to cause this picture, what beam would give me that kind of optical? I have to draw something here. So that when you look at the interference between what I have drawn here and what I have drawn here it gives me in this pattern.

Now, what I have drawn now is some random squiggle. Do the fringes below look like they randomly place this constructive interference and is it random? Its not. So, it's unlikely the beam interfering with the plane wave is some randomly carrying phase, right. It has some specific behaviour. Gaussian is if I am you have to be very careful when you say Gaussian, all of these could be Gaussian beams.

Student: In the sense Gaussian next to them.

Gaussian is intensity, we are a little more specific when you say Gaussian. Because, Gaussian, if you have to be specific why? Because we are interested in what is the phase? When I ask you what is the wave that is interfering with the plane wave, I am really asking you what is the phase of that wave? And the Gaussian beam, I can not see the phase, right. If you say Gaussian, you have to say do you mean at the waist far away from the waist, right.

Because the phase is different everywhere, if I say plane wave, I do not have to say that because the plane wave has the same phase behaviour everywhere, right. But if you say Gaussian, you have to tell me where that? Which part of the Gaussian?

Student: (Refer Time: 33:14) they should be neither to the (Refer Time: 33:16) Gaussian beam and then the path (Refer Time: 33:18) the path difference should be is that vary from (Refer Time: 33:26) should be spherical right.

Ok, so let us be, but we typically do not generate wavefronts with Gaussian phase right. So, I want you to use all the optical terms you have used. We have wavefronts, with you are talking about a variation like this. What can give us a variation like that? Do I mean think of a more general shape.

Student: Spherical.

Spherical, that is why when I said you said Gaussian, you could have said Gaussian far away from the waist is kind of spherical, right. So, its a spherical wave. So, if my wavefront was something like this, then this is not a linear variation in optical path length anymore. The optical path length is increasing faster as you go further away from the center, right and therefore, the fringes are coming the places where the constructive interference is happening is happening closer each time.

And so, the fringes come closer and closer, right. Now I will ask you a question, I asked you some time back. With a Michelson, in order to see fringes you should not have your mirrors perfectly at 90 degrees to each other. Because then, you just have 2 parallel plane waves coming and you have a uniform intensity. It is $I_1 + I_2$. So, there is more than the intensity of a single beam, but there is no you do not see that interference, ok.

In order to tilt one plane wave with the other we said we introduced a small angle to the mirror. Now we will see that, when I have a varying path length and you have the variation is not constant, it is not linear, you can see the fringes start coming closer and closer. Keeping all of this in mind, I am asking you, if I tilt the mirror too much in a Michelson, what I might not see fringes. When I say too much, I am not talking a large angle, I am not talking such a large angle that the returning beam does this and does not even come back the right way, I am not talking about that.

Let us assume that it still hits the beam splitter and still comes and there is still an overlap between beams, ok. I will only have interference if I have an overlap between beams, right.

So, I am not talking about the case where I have tilted the mirror so much that the object and reference beams do not even overlap. There is still overlap, but if I have a large angle, I may not see interference again, why?

Student: The (Refer Time: 36:07) will be.

The fringes will be too close, my detector may not have the resolution to see them, ok. You can start to see that these fringes are easier to see than these over here. This is a simulation mind. So, if you were doing an actual experiment there is going to be some noise in your system and you may not see the fringes in this region at all.