Optical Engineering

Prof. Shanti Bhattacharya
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture – 30
Application of Gaussian beam equations

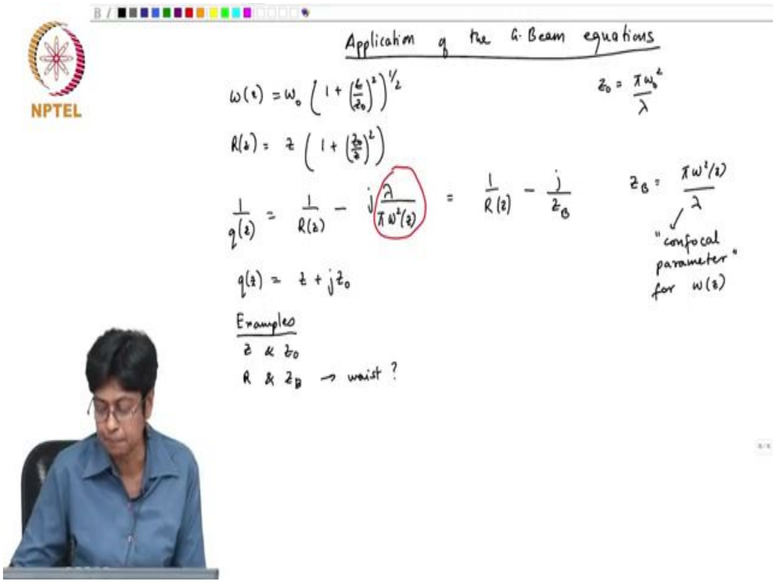
Good morning. So, we are at the last class for Gaussian beams. So, let us do a quick recap, we have started on Gaussian beams after looking at ray optics because, in certain conditions it is necessary to take the wave nature of light into account.

And we got a sense of where that is yesterday because we saw when we were focusing a Gaussian beam only under certain conditions is the imaging equation similar to that obtained when doing ray optics. We looked at Gaussian beams without going through a detailed derivation. So, I gave you a set of equations and what I want to do now is to or in today's class really look at where we can use or how we can use those equations in different applications ok.

So, let us look at applications or Application of the Gaussian beam equations. So, I want to just write out some of the important equations you had the equation for the beam diameter sorry beam radius, you had the equation for the radius of curvature right and I am not sure if you remember I had also very early on defined a q parameter.

I said this q parameter is actually a function of both R of z and $\omega(z)$ and we had given it appears in two different forms. I will give you both of them or sometimes you will see it in this form also. Now, if you remember z_0 was the Rayleigh range defined like this.

(Refer Slide Time: 01:25)



So, you can see that this term here is similar to the z naught and so sometimes this is actually given as so, sorry right where, $z(B)$ is defined by this right. So, z_0 was the Rayleigh range of the confocal parameter you can consider this to be a confocal parameter for $\omega(z)$ ok. So, its because its similar to z_0

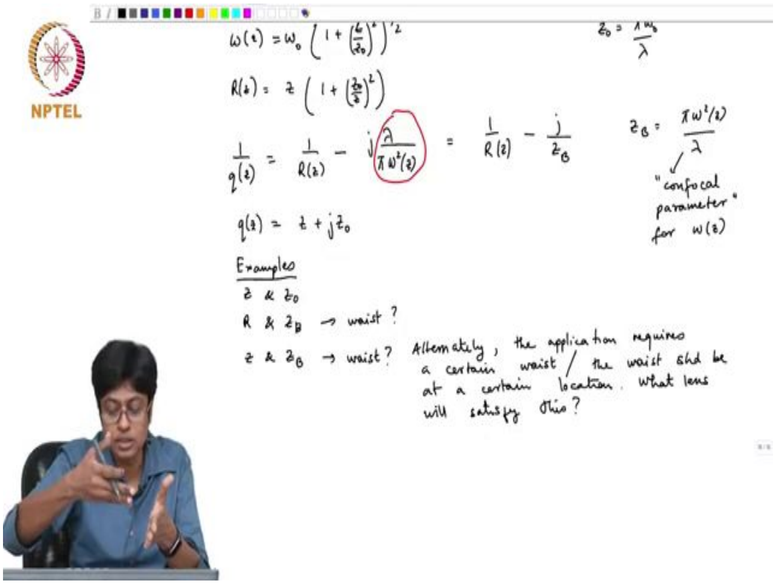
Now, depending on the application that you are looking at you may have some information. So, you can see you have an $\omega(z)R(z)q(z)$ which is a function of both of these z_o this ω_0 right. Depending on your application some of this information may be available to you and your application will require you then to calculate the remaining terms ok.

So, it could be that, so maybe you know z and z_o righ,t z is the location of the waist z_o is the Rayleigh range. So, maybe you have that information and you are being asked to find out what is the omega of z at that z value right. So, what is the beam waist? It could be that you know the radius of curvature capital $R(z)$. I am not writing that and you know the z B value.

So, that is you know at some plane the radius of curvature at some distance z you know the radius of curvature and you know the beam radius at that place ok. Now, you are being asked where is the waist, where is the waist, what size is the waist? Then we will see where these applications come.

So, as long as you know any two of these parameters you can calculate the other parameters, maybe you know z and zB and here maybe again you could say well if I know z and zB again I could ask where the waist is? But maybe my application instead of I could just ask where is the waist, what size is the waist?

(Refer Slide Time: 05:43)



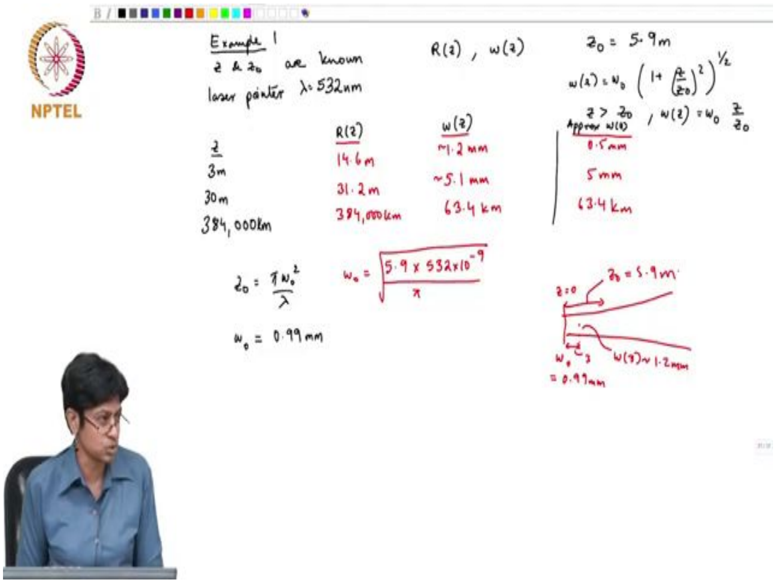
Alternately, I could say given this z and zB I that the application requires a certain waist; or requires the waist to be at a certain location and then you could ask what lens will do that? What kind of application could this be? When you could say I have light coming out of a laser and I have gone through some optics. So, I know zB right.

Now, I want to change that, such that I have a certain waist size. So, that it enters maybe I wanted to focus, sorry I want it to be at a certain location because in my system say I have an endoscopic system for example, I know that the next element is going to be at a certain location. So, I wanted to be at that location. What lens should I use to achieve that?

So,, like this I have listed out some here, but I can take any combination I could say I know z_o and z B I know, R and z_o I can take different combinations saying; if I know two of these parameters. I can use all these except for equations in order to calculate the other parameters and depending on each application you will be asking that question for.

In this application these are the measurable parameters; this is what I know. And this is what I need to know in order either to make that application do something specific or to get some information from that application. So, let us look at some examples ok. So, the first case is a fairly simple case ok. Let us say z and z naught are known example 1. So, z and z_o are known.

(Refer Slide Time: 08:44)



So, for example, let us say you have a laser pointer. It is a green laser pointer. So, let us say the wavelength is 532 nanometers. What do you want to do? z is known, what you want to calculate is what is R(z) and what is $\omega(k)$ ok. So, what is the radius of curvature? What is the spot size at that value of Z?

Now, here all I have is a laser pointer. So, when I say what is R(z) and $\omega(z)$ Basically, I am saying if I were to point that laser pointer at the light across this room for example. So, I am talking a few meters away. What is the size of the spot when the beam has traveled a couple of meters? I might say well I am going to sit in my office and I am going to point it across to the tennis court.

What is the size when it travels across or from my block to another block? So, I am talking a little longer distance or I could say let us point it up at the moon what is its size at the moon ok. So, calculate these three cases for me. Let us say we take z as 3 meters 30 meters and let us take their distance to the moon. What is the distance to the moon?

Yes so, it is something like this it is not a there are a few more 100 meters, but. So, calculate for me quickly if I just give you the numbers it's boring. Why do not you calculate and let us do it there is a number of you. So, let us have the first row calculate the 3 meter distance what do I want you to calculate , you do not need to calculate R(z) I will give you that calculate $\omega(k)$ ok.

So, you need some information from me to do that. I will tell you that z_0 is 5.9 meters and I have given you the wavelength ok. So, let us have the first row calculate $\omega(z)$ when z is 3 meters, let us have the next 4 people calculate at 30 meters. So, you are calculating at 30 meters and the last row is doing it for the distance to the moon ok. Oh sorry, yes kilometers yes.

So, you are using this expression omega of z is ω_0 I had just written it down $(1 + z/z_0)^2$. So, the first thing you need to calculate is ω_0. So, my z_0 is $\pi\omega^2/\lambda$. So, what do we get for ω_0? 0.99; 0.99 what? Ok then do one thing calculate for calculate this also for me for these three distances you can do this calculation.

Let us also calculate this approximation where, if we assume $z > z_0$ then $\omega(z)$ will so, the term $\frac{z}{z_0}$ the whole square is much larger than 1 right. So, then I am going to just ignore the 1 right. So, then my $\omega(z)$ will reduce to $\frac{z}{z_0}$. So, calculate this; right now just take this approximation and calculate it for all these three cases. So, we can compare the approximation with the actual value. I will give you the value of R I mean you can check it later, $\omega(z)$ is 1 point?

Student: 215 millimeter.

Millimeter ok. Do I have a value for 30 meters? 5 points.

Student: Mam (Refer Time: 13:38).

It is just 5 millimeter?

Student: (Refer Time: 13:43).

Let us put 5.1 millimeter approximate. It does not matter if these are not exact the idea is to get a sense of how things are changing ok. I am not worried about the accuracy to several decimal places ok. And what do you have for $\omega(z)$ for the distance to the moon? 60.

Student: 63.4.

63.4 kilometers, right. And do we have the approximate values in each case? I am going to write it down because it is taking too long again do not worry if these are not exact values you can calculate the exact values ok. So, what have we done with this exercise? I have been giving you equations endlessly over the last few days. I just wanted you to get a little familiar with those equations.

So, in this case you were given two parameters you were given z. In other words, when you are saying given z you are being told at this z find out something about the beam ok. So, that is what I mean by given z, but what you are really given as a piece of information is a Rayleigh range of this beam. The moment you gave z_0 and of course, implicit in all of this I did not mention this earlier you need to be given the wavelength.

Because, if you are going to use z_0 anywhere or ω_0 this relationship between ω_0 and z_0 of course, you need the wavelength of the light. So, this example was off a laser pointer, so we took something in the visible region with the lambda with the z_0 you can calculate omega naught and with that value and the other parameters that are known if then send for different values of z what is the beam like ok?

Now, you look at this at 3 meters. So, you started out with an omega naught of a little under 1 millimeter at. So, its leaving at the beam waist its less than a millimeter. It travels 3 meters away, because your z is always measured from the beam waist. So, it travels 3 meters away and its 1.2 millimeters.

Now, what is z naught here? z naught was 5.9 meters right. So, you have a beam that is ω_0. So, this is z=0 this is where you have omega naught at this place ω_0 is 0.9 millimeters, if you go 3 meters away ok. Your ω of 3 is 1.2 millimeters.

So, the beam has hardly diverged, but you should not be surprised because the z_0 of this beam was 5.9 meters. So, you expect that the beam diameter stays more or less in that region right, the large divergence is going to happen beyond z_0 we that is why we call z_0 the depth of focus because there is. So, little change in that region.

So, the moment you are asked to calculate, if I had asked you to calculate for 1 meter, for 2 meters, for 3 meters this is all less than the Rayleigh range of this beam you should not expect a large change from the omega naught value. Depending on your application you might say I do not even need to calculate it; depends on how fine a sensitive application you are working with. But if you just need an approximate number you just say I know $\omega(z)$ I am going to assume it is more or less constant over the Rayleigh range.

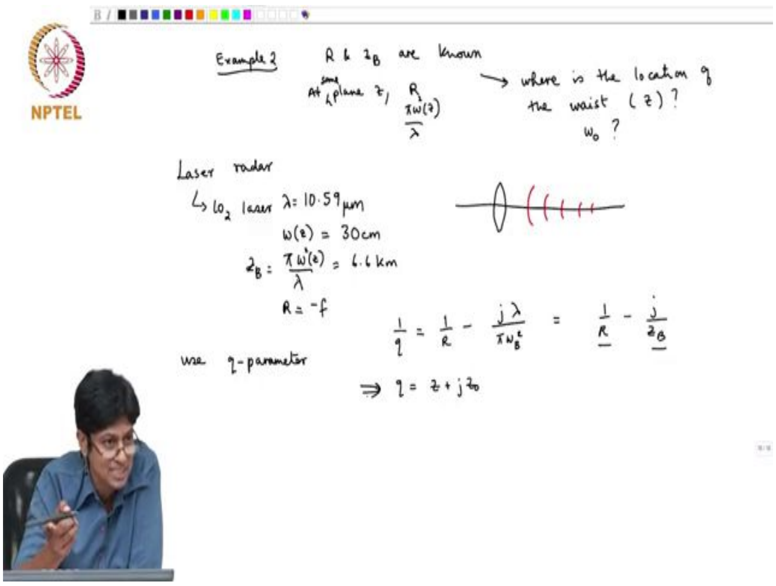
This is to say I am using the beam here and was pointing at the screen. It is a couple of meters distance and the beam diverges very much from when it leaves a laser to when it reaches the screen. If I were to send it across one building to another building now its travelling a greater distance in 10s of meters and there even there the $\omega(z)$ is still fairly small right, but you see when you go the distance to the moon right, what has happened there?

You have huge things you are talking about. I mean the units are changing. You have gone to kilometers and it is not surprising considering the distance you were talking about meters units of meters and now you are talking 384 thousand kilometers right. Of course, the beam is going to be very large. This example is to give you an idea of how the sizes change or why you might want to calculate the radius at different sizes of course, you would not be pointing a laser pointer at the moon.

Well you could point a laser pointer at the moon, what would be the problem? You would not have enough power reaching the moon right because, now the power that was in that beam of omega not size 1 millimeter that power is spread over 63.4 kilometer spot size. I mean you are not going to get enough power to have anything measurable and it would be impossible if you said that is getting reflected and coming back you really have no power ok.

So, this is just an exercise to give you an idea of the numbers or the sizes, it is in practical. The last part is not really practical in terms of the power of the source that you are using. Let us take another example, in this case let us say R and z B are known; that means, this R of z is at some plane; so at some plane z radius of curvature is known and pi\ omega B squared off.

(Refer Slide Time: 19:42)



So, I know the local size of the beam. I know the local radius of curvature and I am focusing this beam. I might ask, given that I am focusing this beam where, is the location of the waist? In other words what is Z? Right. I can also ask what is the size? No I know z B. So, what is the size of the waist?

Now, an example of where this might be used let us say you are using a Laser radar system. So, it leaves and you know the size, the radius of curvature and the size of the beam. When it leaves you have optics its focusing it you need to know where the focus is. Why? Because that is how the radar works it focuses on some object and then that focused light is reflected back and you are sensing that focus light and from that you are able to tell where the object is ok.

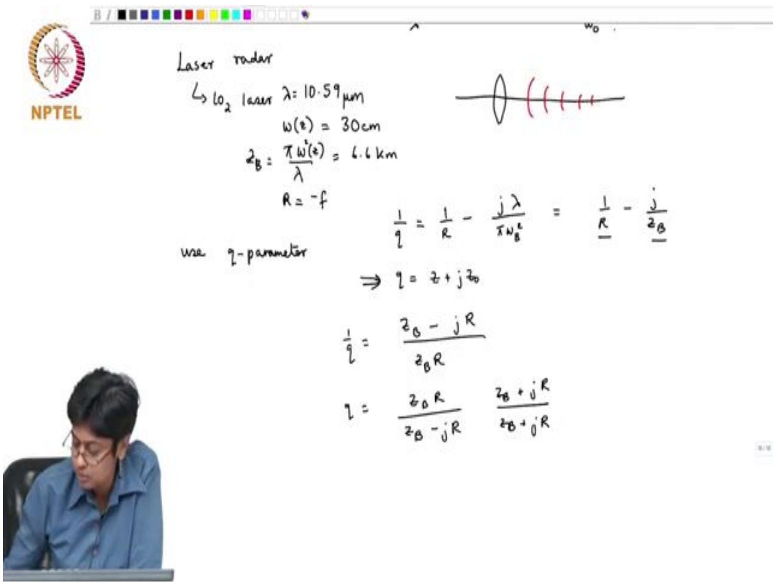
So, in such an example what kind of laser would you be using? A very common laser to use is a CO 2 laser; CO 2 lasers can laser at different wavelengths, but let us say you are using this 10.59 micron line of the laser ok. So, it's in the infrared and the beam has been expanded to this size it's quite a large beam quite different from the case of the laser pointer where, omega naught was 1 millimeter right.

You have been given what is known as z B; that means, this is given to you alright this is your z B and this is given as 6.6 kilometers, R has been given to you the radius of curvature and I am going to say it is minus f of this of the focal length of the lens that you are using. Why is it minus f? Because if you think about it I am talking about a converging sector right.

So, that my negative sign there is indicating that this is convergent. Now, in order to get useful information out of this we are going to use the q parameter. So, you remember 1 over q I will not write q of z R or z and so on please remember these are functions of Z. I had - $-j\pi\omega B^2\lambda$ right or let us for the first part let me write it just in terms of z B ok.

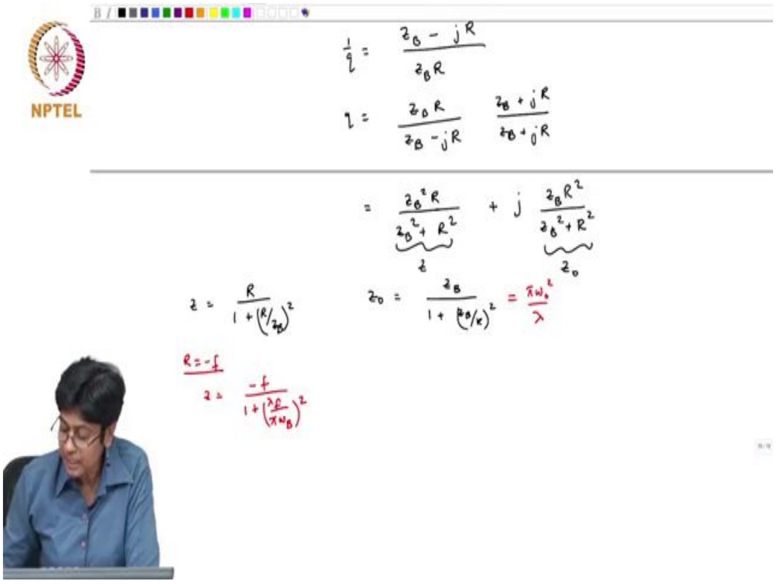
I have not directly been given ω_0, but I can calculate ω_0 Why? Because q is also $z + jz_0$. So, if I can extract, I know R I know z B. So, if I rewrite this 1 over q expression in this form I can get naught and I can find out omega naught right. And that is what I want to do. I want to find ω_0 and I want to find out Z where that waist occurs.

(Refer Slide Time: 24:47)



So, I am going to rewrite this equation $\frac{1}{q}$ So, $\frac{1}{q}$ will be zB - j; jR or q is z B R we just extract setting it up. So, that we can extract the real and imaginary parts. So, I have zB^2R/Z_B + zB squared R by z B squared plus R squared minus j no plus j z B R squared by z B squared So, this is nothing, but z and this is nothing, but z_0 t right.

(Refer Slide Time: 25:34)

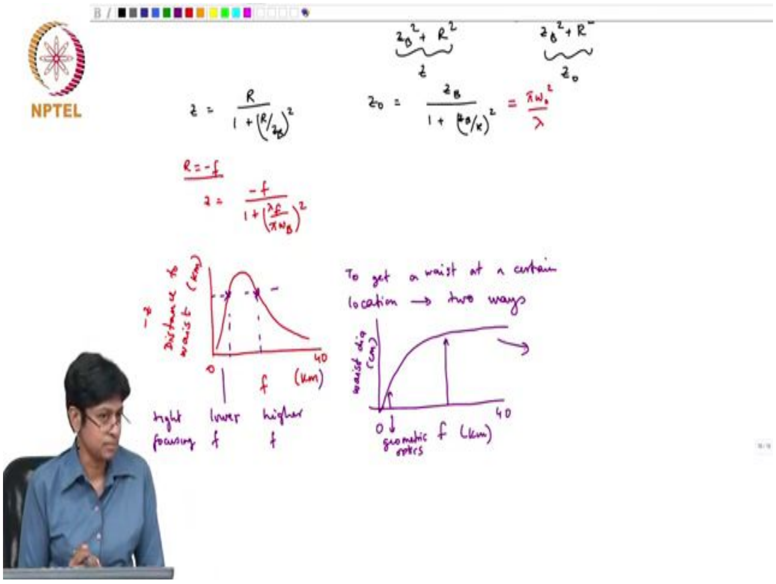


So, I can write z and I will just divide everything by $Z_B{}^2$. So, that I have it in this form and z_0 will be z B 1 plus z B by (Refer Time: 26:42) ok. Now, if you remember when I gave you the information about this problem you were given Z_B and you were given R and we have taken R as -f. So, I am going to substitute now that is a given remember the givens of this problem are R and z B.

So, given that R has this value z is going to be $(-f_1 + f\pi\omega B\lambda)^2$ and I can use this zB has been given to you also. So, I can use this, but I can. So, in principle I have got the information that I wanted I said for a given zB for a given R I need to find out ω_0 and I need to find out where, the waist occurs in other words I need to calculate z and that is what we have done here ok

But it is interesting if you look at what happens if you change the lens that you are using ok. So, we said for a particular lens we had set up the system such that R was - f. So, you can see what happens when I change the lens and if you do that and plot a set of curves using these equations, but you plot how z how. So, this is the distance to the waist that is - z and this is f.

(Refer Slide Time: 28:31)



And in fact, if I plot it for this particular example with the numbers you are going to get something like this and this is an interesting result because, what am I plotting I am saying as I change f. I am changing of course, where the waist occurs, that should not be surprising, but this tells me that to get a waist at a particular location say my goal was to get the waist at a particular location, at a particular value of z there are two different values of f that satisfy that.

Student: (Refer Time: 29:39).

Pardon.

Student: (Refer Time: 29:43).

No, I am changing when f appears in various places right. So, I am changing when I change and R is also changing. So, this is telling me to get a waist at a certain location there are two ways of doing it. And if I plot for the same range of focal lengths, ok.

The waists diameter the waist size let us see the waist diameter by the way, if you solve this for the problem that we are discussing with those numbers this distance to the waist is going to be in terms of kilometers this is in terms of kilometers it goes from 0 to above 40 is this and this waist diameter is in centimeters and of course, I am plotting it for the same set of focal distances, so, it is about ok. And it will do something like this.

So, what is this telling me; that if I wanted the original goal or the region the original way, I stated this problem was to say that given R and given z B given that you know the radius of curvature and diameter of the beam at some location. What is the location of the waist and what is the size of the waist? And we calculated that. But we are talking about in this example a lens the beam of light that is being focused with the lens of focal length f.

And now, we are saying if I change my question a little bit and say I want the waist to happen at a certain location, what lens should I use in order to get the waist at that location? It turns out I have two possible ways of doing it and if I look at these two graphs together you can see that the initial value here corresponds to a lower value of focal length and this value corresponds to a higher value of focal length. So, if I look at those corresponding values here maybe they correspond to somewhere here and somewhere here.

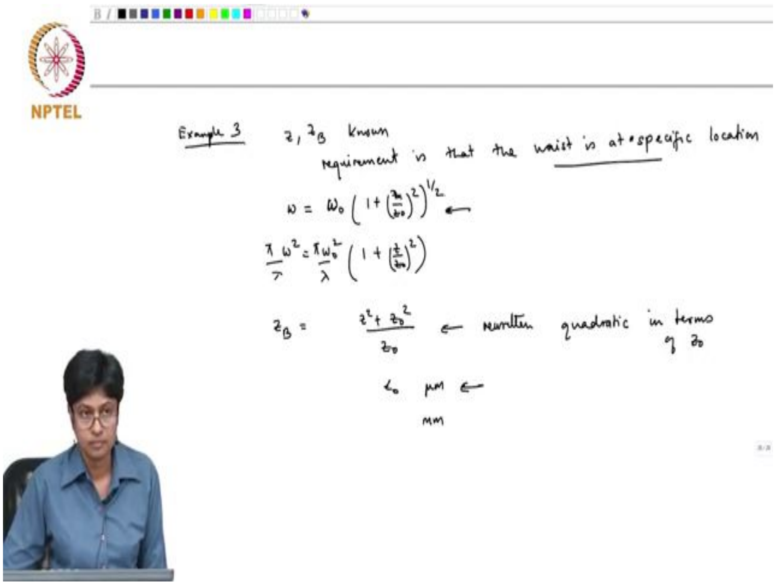
Now, what does a lower value of focal length mean? If I have an optical system and I have a lens with a small f, what does it mean? It means you are talking about tight focusing. Because if you had a collimated beam incident on a lens and the focus point is close to the lens, it means that is a lens with high power it has to very quickly bring the beam to focus right. If I have a large value of f you are saying that it comes to focus much much after the lens.

So, the power of the system is less right. I can consider this to be similar to a region of geometric optics I am focusing as I would in geometric optics and this is more like I have got a collimated beam and it is got a fixed diameter ok. But the point is if the goal, if the mandate had been found, what I should use, what lens I should use?

Such that the waste occurs at this particular distance you have an option of seeing I will either use a tightly focusing system to do it or I would use the Gaussian beam if its almost collimated form ok. So, again depending on your application you might decide to use one over the other ok.

So, I this example I wanted you to look at just to show you that its sometimes instructive to plot these kind of graphs because it tells you or gives you some more freedom that you have with your design you are not stuck only with one type or one way of solving a problem you may have more than one way of solving a problem ok. I will not do the third case in more detail, but a third example would be similar to the second example.

(Refer Slide Time: 35:01)



But let us say in this third example. So, again let us say z is known and as in the previous case ZB is known, so you know the local beam diameter. And as I had mentioned in the discussion, we have just had perhaps in this case you want the waist. So, the requirement is that the waist is at a specific location.

So, here you could look at it another way. If I go back to this equation $\omega(z) = (\omega_0 + 1z/z_0)^{1/2}$ I could rewrite this. So, let us just square it initially to get rid of that square root and multiplied by this, so that I can write it in terms of.

So, this is nothing, but my z B this is z_0 and this is $z^2 + z_0$ plus z naught squared by in fact, let us just write it right. So, this equation can be rewritten. So, that it is in the form of a quadratic in terms of z_0 right; which means, when I solve that quadratic, I will again have two solutions for it and you want the waste at a specific location.

So, you would look at those two solutions and see which one matches your requirement closely. So, the point to get take from all of this is unlike geometric optics where you mostly always have very specific solutions. Here we are saying just because of the nature of the beam the way you arrive at a solution could be quite different ok.

So, here if you were a good example as I had said is if you were trying to solve something for endoscopy you know that you want the waist to be at some location cannot be very far away. So, when you solve for the quadratic equation you have z_0 you will get two different answers and you will pick the one that suits your application more suits the distances that are relevant to.

So, for example, if its endoscopy the distances you will be talking about may be in microns in tens or hundreds of microns and when you solve for the quadratic here you may get 1 z_0 which is in microns maybe 10s of microns and another one which is in millimeters. And so, for this particular application may say this is the solution that I am going to and therefore, based on this what is the lens that I will choose that gives me the solution ok.

So, I hope with this discussion you have got a sense of how you can take all these equations that I have been throwing at you over the last few classes and play around with those based on your application based on the knowledge that you do have to calculate the other parameters which you need for whatever it is you are trying to do ok. Any questions still now? Yes previous example.

Student: (Refer Time: 39:37) second.

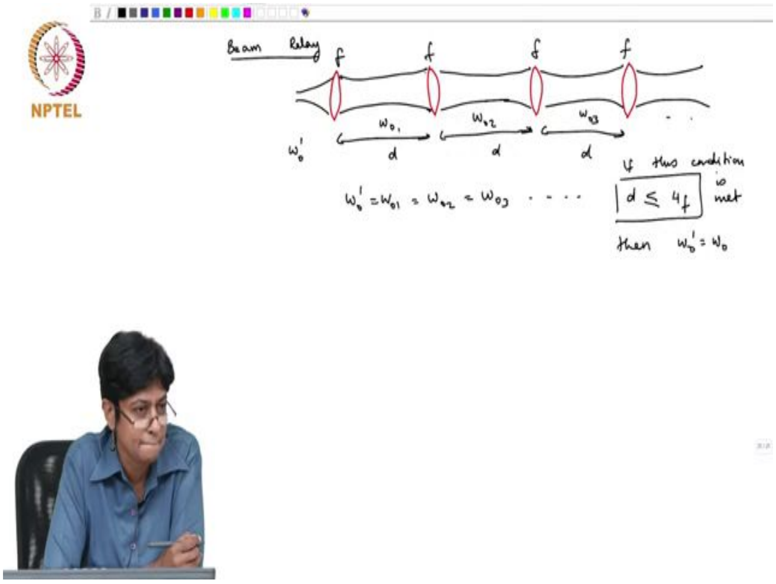
Pardon.

Student: Second (Refer Time: 39:40).

I am plotting so, once I have I you have been given in this case you were given R and zB right at some plane z from this you are able to calculate z_0, that gives you ω_0 you are also able to calculate z. So, you know where the waist is occurring you know the size of the waist once you have this then you know everything in this expression right. You know everything in this expression, don't you?

Student: (Refer Time: 40:25).

(Refer Slide Time: 40:47)



That is of omega of z yeah that is what is plotted on the Y axis ok. So, the last thing I want to do in this class is work out one more example and that is to do with what is called Beam relay system. Now, when we want to send light from one place to another place you know the problem is that it diverges.

So, we have looked at if you want to send the beam a very long distance yeah and it is a Gaussian beam what you would do is to use a beam with a fat waist, because then its divergence is going to be as less as possible. But you saw that even if you do that if the distances are fairly large, for example to the moon, you still can never compensate or completely get rid of the time you are never going to have a beam of usable dimensions right.

So, typically for example, if we say all the light that comes through for all our information that comes to the internet for the internet all of that is coming through fibers. So, what people do then to send light a long distance is to send it through a waveguide. What does the waveguide do? The beam has a natural tendency to diverge, the waveguide has some property that compensates for that and therefore, the beam is kept in as an acceptable size right.

Now, the example I am going to talk about now is again just to further your understanding. It is not that this is really used; the fiber is an example of an enclosed waveguide, but I could also use some idea concept of wave guiding in free space. So, if I had a beam that is diverging I could then use a lens and that is going to, you know compensate for that divergence, but of course, once it's focused to the beam waist it is going to diverge again.

So, I could put another lens and in fact, you can do what is called a beam relay by having a number of lenses ok. I have not really drawn this correctly. Let me correct that. Because what I am saying is, I could do this, but of interest really is to keep it the same between all these clearly lenses ok.

So, we are saying we have the same distance d between all these lenses. All of the lenses have the same focal length f and our goal is to send the beam from one place to another place and maintain its size ok. So, in a sense it is like a free space waveguide. Now, if I say this is what I want to do. In other words, we are saying that if the beam had a spot size omega naught dash before a lens, I have set up my system such that the spot size ω_0 after the lens is equal.

So, if I said this was ω_0 1 and this is ω_{02} and this is ω_{03} and so on. I am saying ω_0 ' is equal to $\omega_{01} = \omega_{02}$ is. So, on that that is what I want to do ok. And it turns out that you can do this, but only if a certain condition is met and that condition is if $d < 4f$, if this condition is met then you are able to achieve this otherwise you are not able to achieve this.

So, how would you prove that? How would you prove that? Can you tell me how we will prove it based on the lens transformation equations that you were given? I see that we have run out of time. So, this is just the last example that we have to work out. We will do that in the next class ok.