Optical Engineering
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Lecture – 03
Refraction at a single surface

We will continue with a course on Optical Engineering. In the last class; we looked at how we can start designing systems and we took material with a certain refractive index and said what should it shaped be such that we get a perfectly focused pointer image given a point object.

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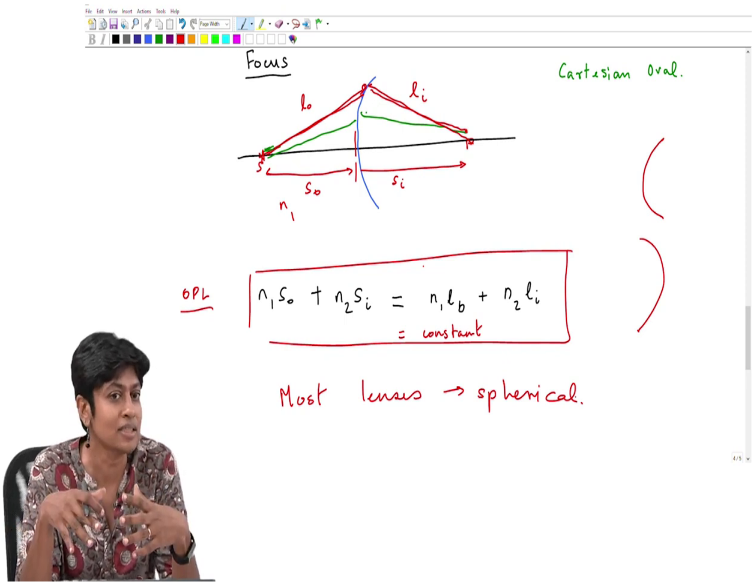

So, we said in order to do that in order to achieve that; we need in this image we need every ray of light that is leaving this source S over here to get imaged at this point P over here. And the equation that satisfies or should be satisfied in order for that to happen is what we have written over here. This is nothing but the optical path length for every ray traveling through the system.

So, we have written it for two rays over here one is the on axis ray. So, that is the ray traveling the distances S_0 in mediumn_1 andS_1 in medium n_2 And another way from the same point that is this point. It will travel through some point on the interface and then it too should come to the same point P over here. However, it has travelled a distance $l_0$in medium n_1 and a distance l_i in medium n_2.

In other words we are saying this should be a constant. And we went on further to say that; there is such an interface there is such a shape that will satisfy this equation that shape is the Cartesian over ok. Now I can see a number of you wearing glasses. I am wearing reading glasses. Do you think, the curvature of those glasses are the Cartesian ovals?

Student: No.

No. So, somebody says no that is right they are not Cartesian arm. Do you know what shape most lenses have?

Student: Convex.

Convex concave is the nature of. So, convex I will say this is convex this is concave. But if I had to describe this shape, is it a Cartesian oval, is it cylindrical, is it spherical does it have some other shape. So, most lenses are not correct to say all lenses, but most lenses will be spherical ok. So, we have just said; the best shape the shape that ensures any ray of light coming from a point object point will get image to the same image point is a Cartesian oval. And now I tell you nearly every lens that we use does not have a Cartesian oval shape, but has a spherical shape. Why do you think that is?

Student: (Refer Time: 03:26).

Sorry.

Student: (Refer Time: 03:29).

Absolutely, it is easier to build, it is easier to manufacture spherical then it is any other shape. Today we have very advanced technology where you can input a complicated shape and 3 D print almost any shape but the costs in mass production 3D printing is not very good for mass production. but, the cost even today to mass produce something other than spherical would be very high and of course, lenses and mirrors have been used in optical systems for centuries now.

And we are talking about manufacturing requirements of the ability to manufacture these shapes hundreds of years ago spherical has always been relatively easy to manufacture. So, that might have been the reason historical people started with spherical shape, but even today is for mass production spherical shape would give you a cheaper to mass produce and therefore, spherical shape is used.

Now, you have to think about this. Because, we are saying the shape that gives me the best optical performance is something and because it is not easy to manufacture that I am doing something else. Clearly the optical performance we get from this other shape has to be adequate or better than adequate. We are not sacrificing quality for ease of fabrication. So, we are going to look at under what conditions does the spherical shape satisfy this equation over here ok.

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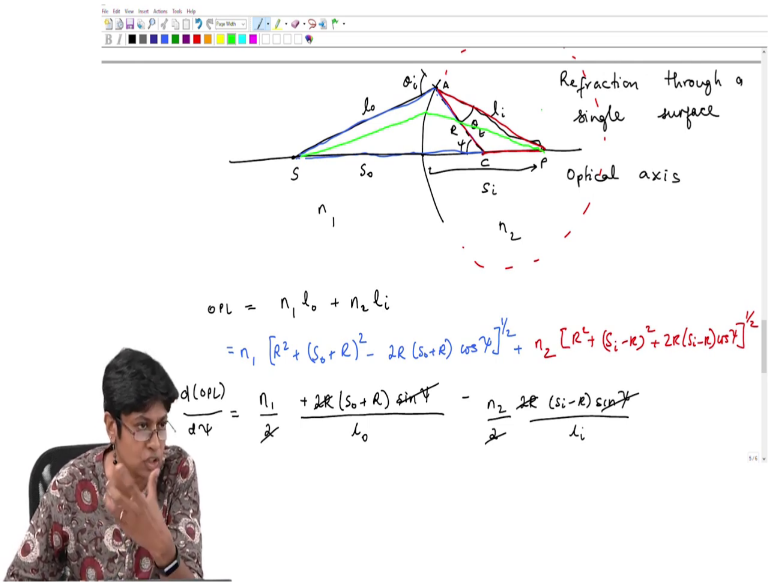

Again I am not yet starting with the lens I am just taking a single interface ok. So, let us say I am right now going to start off by looking at refraction through a single surface ok. So, that is my single surface that is the optical axis. The difference is; I am now specifying that this curvature is of a sphere ok. So, if I were to continue this I would actually draw a circle over here right ok.

This is medium n_1, this is medium n_2 The object I want to image is a point on the axis let us say it is at point s and of course, there will be a ray that travels along the axis there may be millions of rays. I am going to look at 2 rays: the one that travels along the axis and the one that travels at some other angle. We will call this distance $l_0$and this distance S_0: $S_0$is from S to the vertex here.

Let us say the image is at point P. So, this ray goes to point P sorry and this distance is S_i. Because, this is a spherical surface; there is a center of curvature c and this line can be considered normal to the surface. Because, this is nothing, but the radius of this surface right. So, in yesterday's class; we said we define angles of incidence as the angle between the incident ray and the normal to the surface.

So, this is now the angle of incidence θ_i this is the angle of refraction θ_t. And we are going to need this you will see shortly why I define this angle ψ ok. So, let me see do I have everything I need this point let us call this point on the interface as A. So, OPL=$n_1 l_0 + n_2 l_i$, but I can write l naught in terms of this triangle right.

So, I will have

$OPL = n_1\left(R^2+(S_0+R)^2-2R(S_0+R)\cos\psi\right)^{1/2}+n_2\left(R^2+(S_i-R)^2+2R(S_i-R)\cos\psi\right)^{1/2}$ We are going to use Fermat's principle; Fermat's principle states that the path that light takes is the shortest path. So, I am going to minimize this path length, shortest path here we are deciding how this ray travels and reaches this point. If I were to change how it reach this point it would basically be moving along this surface in other words this angle ψ would be changing.

So, the variable I am minimizing it with is $d\psi$ ok. They using Fermat's principle ok. I am minimizing optical path length here. So, I keep the nl unlike yesterday's example where we were minimizing the time. Then I remove the refractive index ok. So, if I carry out this differentiation; I am going to have

$$\frac{d(OPL)}{d\psi} = \frac{n_1}{2}\frac{-2R(S_0+R)\sin\psi}{l_0} - \frac{n_2}{2}\frac{2R(S_0-R)\sin\psi}{l_i} = 0$$

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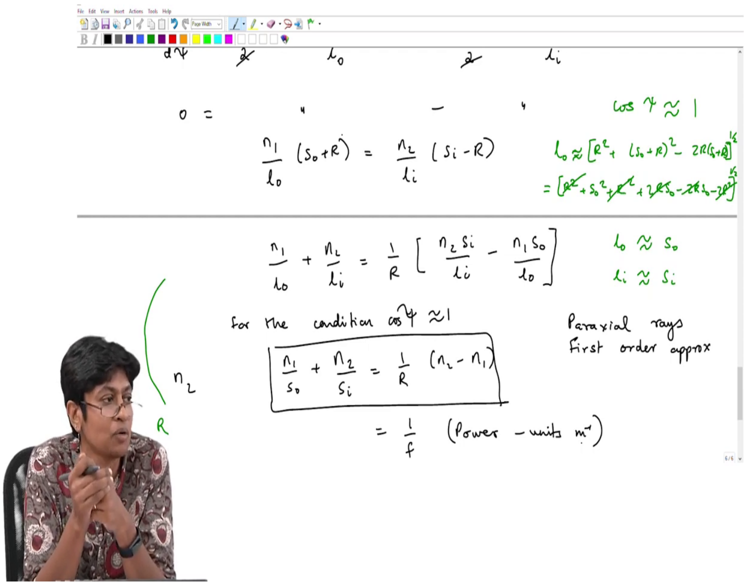

The equation in this form $\frac{d(OPL)}{d\psi}¿\frac{n_1}{l_0}(s_o+R)=\frac{n_2}{l_i}(s_i-R)$

$$\frac{d(OPL)}{d\psi}=\frac{n_1}{l_0}+¿\frac{n_2}{l_i}=\frac{1}{R}\left(\frac{n_2 s_i}{l_i}-\frac{n_1 s_o}{l_o}\right)$$

you are not going to look at this and say that really makes a lot of sense. You have applied Fermat's principle; you got an equation. This equation is telling you something, but it is not at this moment telling you anything very useful, why? Because, the goal was to say every time a ray leaves the point S. I want an expression that tells me how that ray gets to p.

And if I look back at this figure; I can see well before I go back to the figure, if you look at this equation. This equation tells me; every time I change the ray i trace. Now, when I change the ray I trace what is happening. I am changing basically if I change $l_o$here. If I change the ray i trace; I am changing l_o which means l_i is changing. But, that means; that equation the right hand side is also going to change.

In order for it to be equal. So, if I trace another ray, if I put another ray, so I have a new l_o right. S_o and S_i have not changed, because those are the points that S_o is the point I want to image. So, it should not change, but if I look at this equation if I change l_o and l_i. The only way this equation is going to stay constant is if I change S_o and S_i So, it does not seem useful at all to me and we should not be too surprised. Because, we have said the spherical interface is not the interface that satisfies the equation.

So, it should not be surprising, but I started out this derivation by saying under what conditions can we use a spherical interface in order to give us imaging. So, in its present form it is not carrying out the imaging that we want. Is that some condition we can apply that will make this equation work for us. And it turns out if you go back up to this part of the equation you see you have a $\cos\psi$ here.

Now, what if I said I can make $\cos\psi$ or let us take the condition when $\cos\psi=1$ When would that be what constraint am I now putting on the optical system if I say $\cos\psi=1$?

Student: (Refer Time: 15:17).

I am now saying; let us assume, only rays making a very small angle. So, strictly speaking if i say $\cos\psi=1$; that means, I say only on axis rays, but I will be a little generous I would not say only on axis rays. I will say rays making a very small angle with the axis. Let us look at what happens to this equation; if we only look at rays that make a very small angle with axis does that help us.

Now, the moment I say $\cos\psi=1$. What will happen to my definition of l_o

$l_o \neq \left(R^2+\left(S_0+R\right)^2-2R^{\square}\left(S_0+R\right)^{1/2}\right), l_o \approx S_o$ and $l_i \approx S_i$ if I now apply this condition

Then for the condition $\cos\psi \approx 1$

$\frac{n_1}{S_0}+\frac{n_2}{S_i}=\frac{1}{R}\left(n_2-n_1\right)$ In other words; I have removed the l naught and l i and I am saying any ray coming from the point S will reach the point P as long as the rays make a very small angle with the optical axis ok. We call such rays paraxial rays. This is also sometimes called first order optics first order approximation, because, we have made $\cos\psi$ and not to the 1 minus the next term right. You can also do third order higher order approximations right.

This is why the spherical surface is used, yes it is easy to manufacture, but it also gives us good optical images under certain conditions ok. What is the condition as long as the rays are paraxial? This is now a very useful equation. And the right hand side of this is sometimes termed as relating to the focal length. So, $\frac{n_2 - n_1}{R}$. So, remember your surface had radius of curvature R and it had the medium n 1 on one side and then 2 on the other.

And we are now saying; this surface has in fact, this 1 over f is the power of the surface right. Power and units are always given in meter inverse. The power of this surface is now determined by the refractive indices on either side of the surface and the curvature of the surface.

So, if I want to change the power of the surface; these are the variables I have I can change refractive index or I can change curvature. Now in a lot of cases of course, one medium will probably be air and you do not have an infinite basket of optical materials. So, you can say I want a material with this refractive index and I will go and pick any refractive index. You have a limited set of optical materials.

So, you are there not that many refractive indices you can just pick up. So, very often the variable you will be playing with is the radius of curvature. We want to change the power of the focal length of an optical lens or mirror you will do that by playing around with its curvature.

Now, this is still not that useful. Why? Because, my a lens it does not consist of air on one side and then glass all the way right. It has one surface, it has a second medium and then that medium ends in another interface and then I go back into air or I go back into some other medium right.

So, I now use this formula as a base. The image created by this interface will become the object for the next interface; that interface will see a certain radius of curvature. And it will have a medium of some other refractive index after it. And it will create its image and I can now do this forever the image it creates can become the image for the next surface and so on ok.

So, let us do that now and arrive at an equation for a lens. Keeping in mind all the time that we are talking about paraxial optics; that assumption is not going to be explicitly mentioned again and again, but all of this works on the basis that you are saying $\cos\Psi$ is almost equal to 1.

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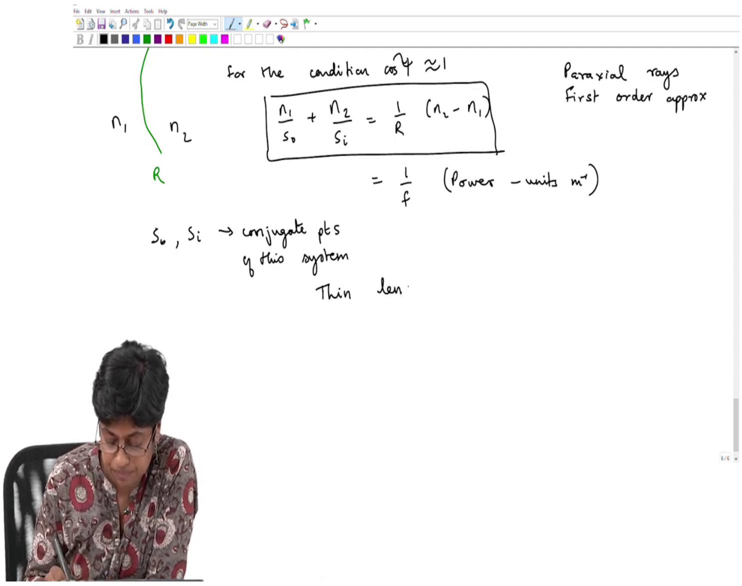

So, in order one more point we talked about conjugate points also and here we are saying S_o and $S_i$these are the conjugate points of this system. That means, in this particular example I said let us assume; we want to get an image of the source at S and we arrive at the image at P I could interchange I could put the source at P and I will get the image at S ok. So, these are the conjugate points of this system.

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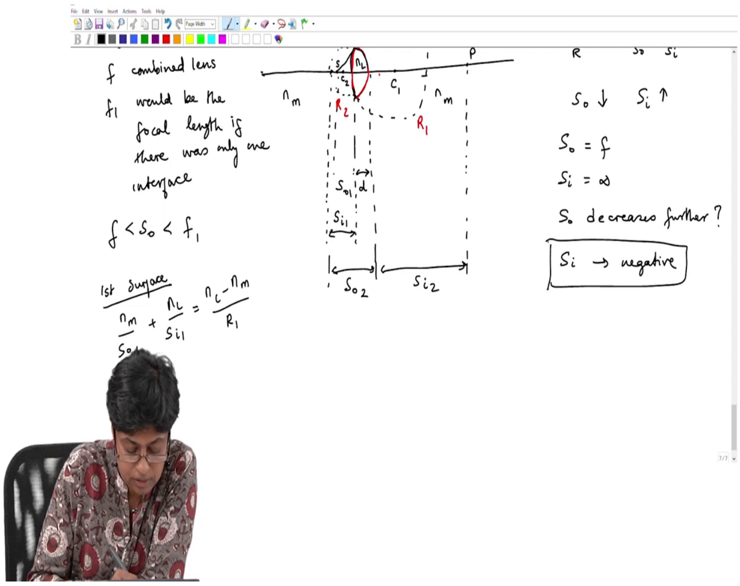

So, that was the equation for a thin for a single refracting surface. Let us get the equation of a thin lens. As usual I start with an optical axis; it is a lens there are two interfaces. So, I am going to say this is one interface remember its part of a sphere or circle and this is another. So, this is the lens this is all that we are going to use ok. One side has radius of curvature R_1, the other has radius of curvature R_2.

So, I am not drawing this properly. Please these circles should be going the optical axis is going through the center of the circles. This I call C_2 the center of curvature of this circle and this is C_1 center of curvature of the first interface light will see. This has rad R_2 this has radius of curvature R_1. And let us say our object is sitting somewhere over here, so this is point S.

So, the distance of S from the first interface; we shall call it S_{o1} it is the first object point right. The thickness of this lens let us call it d right and finally, let us say that the image is forming at P, but before it forms at P; there is an image formed by the first interface. And I am going to put that image over here and I will explain why I am putting that image over here ok.

So, light traveling from point S hits the first interface; if the second interface did not exist in the case that I am setting up now its image point would actually be at this point over here ok. And I will explain to you why I have done that ok. So, if this is the image point, whenever we are dealing with lenses; we always measure distances from the vertex of the lens ok. So, this is my first image, so I call it $Si1$. This image acts as the object for the second interface.

So, the second interface; this is now object distance 2 and this is image distance 2. P is where the final image is formed. Remember, we have just arrived at this expression right. For a single interface, this was refraction at a single interface. Now, what happens in this expression; the left hand side is constant for a single interface. You are not changing the refractive indices, you are not changing the radius of curvature it is a constant.

Now, let us say I go on decreasing the distance S_o. What does that mean? I go on bringing the object I want an image of closer and closer to the interface, because the left hand side is a constant. What does that mean for S_i? If I go on decreasing S_o; it means, S_i is going to go on continuously increasing.

At some point S naught will equal n 2 minus n 1 by R, in other words it is equal to this or sorry $s_o = f$. What does that mean forS_i? Where does the image happen if $s_o \neq f$? That means, S_i is at infinity what happens ifS_o decreases further? What happens to S_i ?

How will this equation get satisfied? It means S_i is negative or as he said it will be on the same side as the object. And that is the reason why in this drawing I have put the first image formed on the same side as the object. Because, if you think about it by doing that; what am I saying. I have my object distance and let us sayS_o is my objective distance.

I have the focal length f which is of the combined lens ok. Same combined lens I mean both the interfaces, but I have f_1 which would be the focal length; if there was only one interface. And why we do this is this condition is almost always going to be satisfied. Two interfaces together have more power than a single interface right.

So, this condition is what you would expect or is not surprising that this condition arises when you compare what is happening at one interface to what happens when you have two interfaces ok. So, that is why, the first image I have kept it on the same side as the object ok. Is that clear that point?. So, now, I want to use the same equation that we arrived at for a single interface, but apply it iteratively; first for the first interface and then the image that is formed will be the object for the second interface and finally, arrive at an equation for the full lengths.

So, let us do that. So, I hope you all have this figure; because, I am going to be using the variables from this figure. For the first surface right. Keep the equation in mind right. So, I am going to have and let us let me define this as the refractive index surrounding the lens as n_m. So, that is true here as well that is the medium surrounding the lens and the medium of the lens I will call n_1.

In the first case; this is equal to n_1 by my earlier definition and $n_l = ¿ n_2$ ok, for the first interface ok. So, I have $\frac{n_m}{❑_{□} s_0 + 1} = \frac{n_i}{S_{i1}} = \frac{n_i - n_m}{R_1}$ That is the equation for the first surface.

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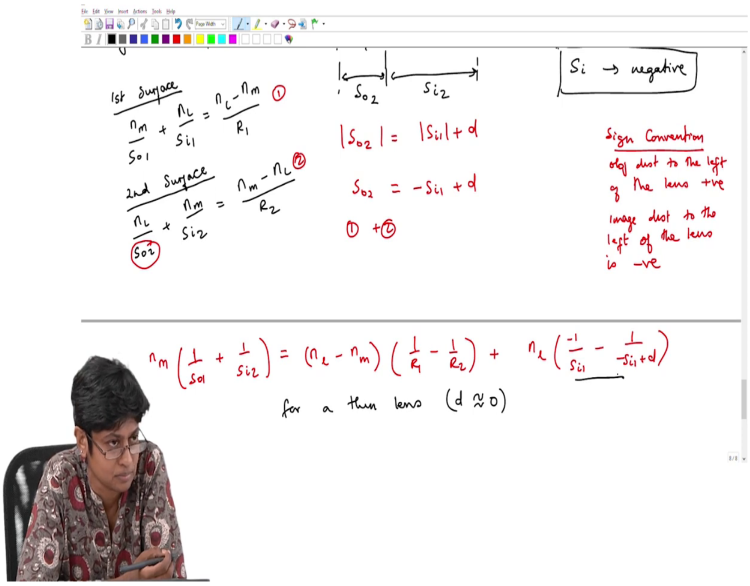

And the equation for the second surface $\frac{n_i}{S_{o2}}+\frac{n_m}{S_{i2}}=\frac{n_m-n_l}{R}$. At this point; I have to be very careful, because I need to know what signs I am going to attribute or assign to each of these variables. There is a sign convention that we need to take into account ok.

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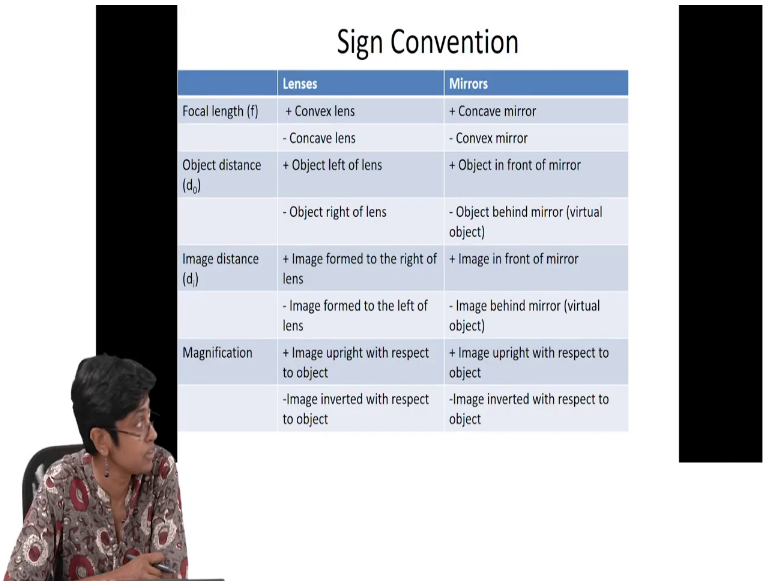

And the sign convention and let me just go to this other. So, the sign convention I have given it to you and I will share this with you on moodle. So, do not worry about it too much now, but the first column is the sign convention that we are using with lenses. So, concentrate on this for the moment. What are the signs we associate with different parameters, what are those parameters

So, the parameters of interest are the focal length of the system; the object distance, the image distance and the magnification. And you can see that for a convex or a converging lens we will use the focal length as positive for a concave lens it will be negative. And in fact, we will keep coming back to this because I add a few riders as we go along, but I think at this point it is enough to just give you this information.

The object distance and this is what you need to note because we need to use this information now. If the object is towards the left of the lens; we say that the distance is positive. So, typically, historically, optics has always been designed by tracing light traveling from left to right. And that is why; here if the object is towards the left or is left of the lens we say the distance is positive if the object lies to the right of the lens we say its distance is negative.

The reverse is true for the image; if the image forms to the right of the lens it is positive. If the image forms to the left of the lens it is negative ok. That is really what is important for you right now. We will come back to this, but keep this convention in mind. So, I have to be now careful with these distances I have written out two equations, but this equation this term S_{o2}. It actually is a sum of this term and this term.

And I need to take the convention the sign convention into account when I add up these two terms. So, S_{o2} is actually going to be S_{i1} + d. And what was the convention for object if an object was to the left of the lens it was positive. So, S_{o2} can remain like this, but S_{i2} is an image distance and in this particular case it also lies to the left of the lens. So, by the sign convention this is going to be negative.

So, I will just write that maybe the object distance to the left of the lens is positive image distance to the left of the lens is negative. So, this is what I am going to get, following the sign convention. There are other conventions and there is no problem with you using any convention.

You just have to ensure that; if you are solving or working out a problem you stick to the same convention from beginning to end and it is also good practice to state; what is the convention you are using especially, if it is going to be different from this one just. So, that it is clear ok. So, if I now take these two equations and I add them up keeping in mind this sign convention let us see what we get.

So, let us take all the n m terms $\left(\frac{1}{s_m}+\frac{1}{s_{i2}}\right)+\left(n_i-n_m\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)+n_l\left(\frac{1}{S_{i1}}-\frac{1}{-s_{i1}+d}\right)$I have done is add up these two the equations for each of the interfaces. Now to make this again a little more useful immediately; let us make an assumption that the lens we are dealing with is a very thin lens. So, for a thin lens in other words the thickness d = 0.

Why is it valid to make this even in the morning class I talked about geometric optics as being the regime where we consider λ to be 0 and that is a valid assumption to make, because we are talking about imaging objects much bigger and dimension to λ.

So, here again; if I say thin lens how far away is the object from the lens if we are talking about distances much larger than the thickness of the lens with respect t to the thickness I can consider that distance so much larger the thickness can be considered negligible ok, but the advantage of doing this is. If I look at the equation above now, if d~ 0, this term here will disappear.

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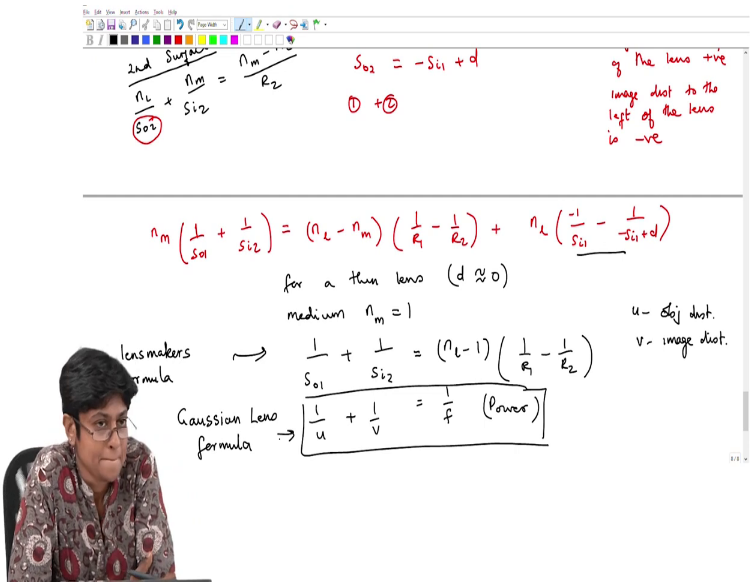

So, with that condition; I can now say and let us make some more assumptions let us say the medium surrounding the whole system, medium $n_m=1$. So, let us say it is air I will have this equation. I have an equation which is in terms of the object distance in terms of the final image distance the object distance S_{o1} is the location of the object on the optical axis from the vertex of the first surface.

The image is the location of the image point on the optical axis from the vertex of the second surface right. And the medium surrounding this lens is air. So, I am not bothered about its refractive index and the medium off the lenses n_1 that appears in the equation both the radii of curvature appear in the question. It is not very different from the equation we got for the interface that also the power was a function of radii of curvature radius of curvature.

Here it is a function of both the radii of curvature again this is called the power of the system and often you will see it written in this form where u is used as object distance and v is used as image distance. This, if you write this in terms of u and v it is also called the lens makers formula. Sometimes this is called the Gaussian lens formula ok. Pretty straightforward.

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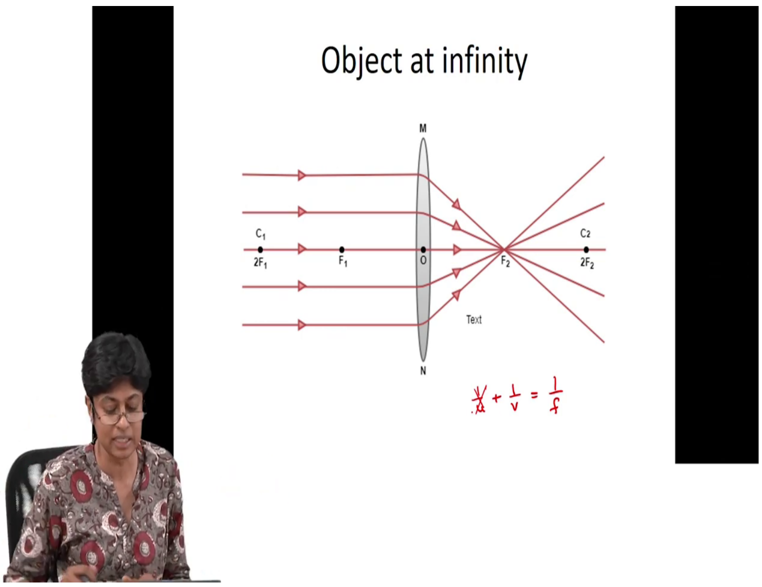

So, let us just run through some possible scenarios here, but what happens now, when the object is located at different places ok. This is again high school optics, but now you should have a little better understanding of where it is coming from right. If you go back to your equation; $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ If the object is at infinity this term is disappearing the image is happening at the focal length itself right. And you can move your object from infinity to a position to f away from the lens to something in between two f to less than f and you can see how the image moves accordingly ok.

And that is one of the first exercises you will do in the lab, the simulation lab is to simulate by changing object position where the image is forming and also looking at the nature of the image, because mostly you may get a real image, but at some point the image will appear to be on the same side of the object and then you get a virtual image ok.

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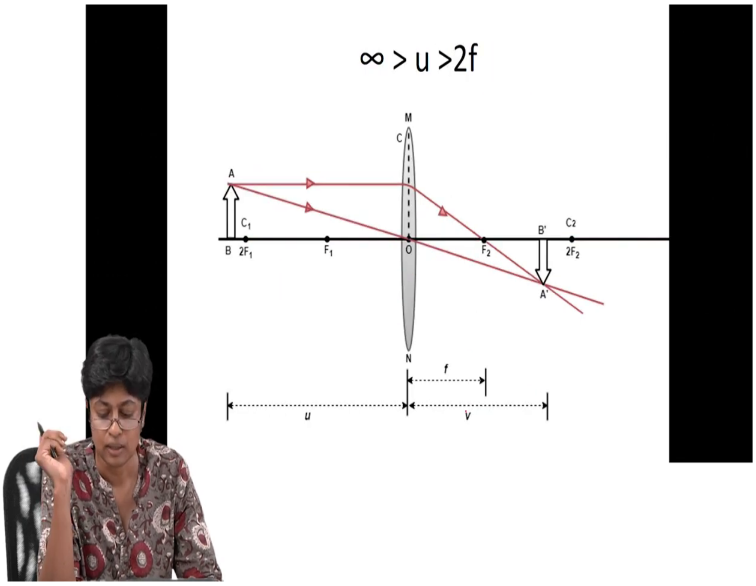

So, I will just scroll through these, but this is actually what you need one of the exercises you need to do in the lab today is actually send light through a lens and change the object position and then monitor where the image is forming.

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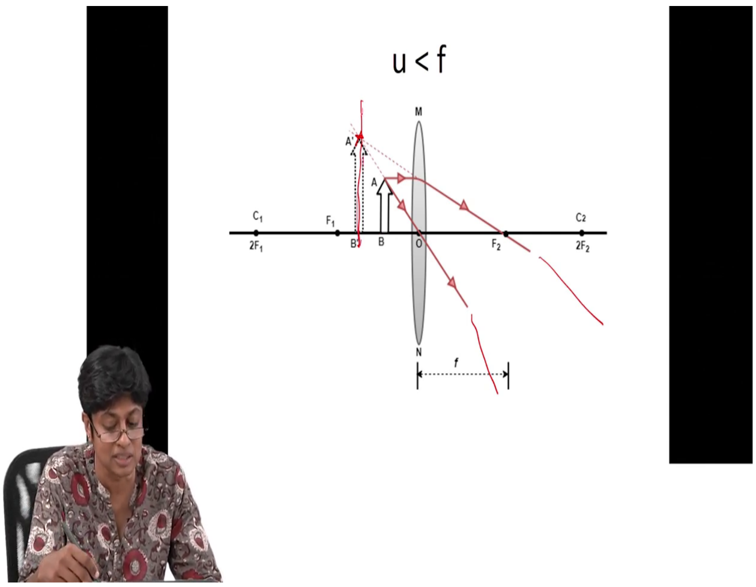

And you can see this is the last case; when the object moves to a distance less than focal length the rays now they are still of course, going through the lens, but they are diverging here. They are not going to converge to a point. They appear to come from a point over here and that is why we say it is a virtual image.

It is virtual because, we would see it with our eyes as if it was coming here forming here when actually the rays are diverging over here you could not place a camera or screen and capture, you could not place a screen here and capture that image you need extra optics to capture that image.