Optical Engineering
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Lecture – 26
Gaussian beams

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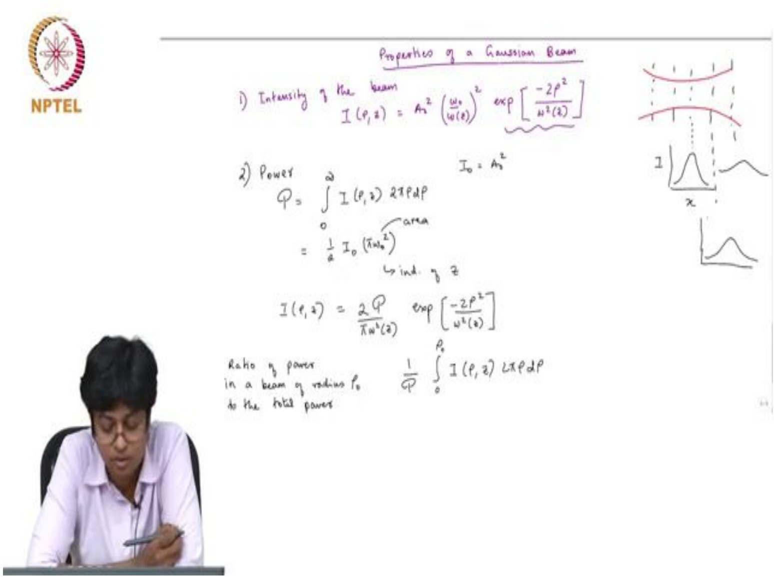

So, with that let us start looking at some of the properties of a Gaussian Beam. So, of course, one thing you are going to be interested in because you are looking at Gaussian beams is going to be the intensity of the beam.

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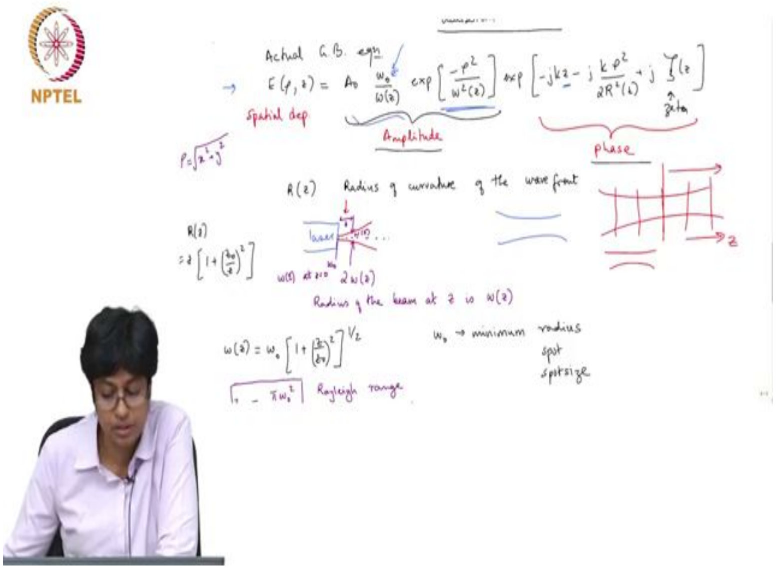

Now, I have that whole expression for the amplitude, right. That is this expression. So, to get the intensity you basically have to take this is $E(\rho, z)$ you take E E^*, right. So, if I do that the entire phase is a complex term that is going to disappear, so the intensity is basically be the amplitude squared, right. So, I am going to write out the square of this amplitude term, ok.

So, I had say I which is of course, a function of ρ and z is $\frac{A_0{}^2\omega}{\omega_0(z)}$ and this exponential is not a complex exponential that is why it does not disappear, but when you square it you will have that 2. What does this term do to the intensity? It tells you how the transverse intensity varies, right. And you can see that while it is a function of z, right the form of this expression is the same irrespective of the value of z, the form is exactly the same.

So, if you were to look at a Gaussian beam propagating, so again I will draw that standard picture irrespective of which plane you look at, you could look at any plane. At any plane the cross section, let say I took intensity versus x is going to be a Gaussian shaped. What will change if I go to another plane, is not the form not the shape, but of course, where the peak occurs, right.

So, if I am at this plane where the minimum spot is, this is what I would get, but if I traveled further the beam would diverge. So, at this plane what you would get is still Gaussian, but more spread out and if I went to this plane still Gaussian, but even more spread out, ok.

Remember we started this whole discussion on the Gaussian beam by saying the plane wave is not a practical wave because its wavefront spreads out to infinity cross section, its wavefront spreads out to infinity in which could only be true if it had infinite energy. The Gaussian beam is where you can see it rapidly fall, right. So, it does not spread out to a, that high intensity does not spread out to infinity.

But even here it asymptotically goes to infinity at the. So, it is never really that even in this description we are not saying really that the beam ends somewhere. The way I am drawing it, it is actually it is going to if you look at this expression it actually continues till infinity, ok.

However, what we would be interested in is what is the power contained in a certain radius of the beam, cross section of the beam. I know that the beam has this shape, I know that even this beam tends to spread out to infinity, ok, but how much of the beam power lies in the extremities of the beam, right that is what is of interest to me.

So, we are not always interested in intensity, but we are also interested in power, ok. And how would we arrive at the power? Where we would integrate this intensity equation radially, so from 0 to infinity and if you do that you will end up with this expression, ok. I naught is nothing but this A naught squared. So, it is the peak intensity.

Now, it should not be surprising that the expression that you have arrived at is independent of z, because we are assuming here that as this wave propagates there is no loss. So, if the wave had a certain power in one plane, it has exactly the same power in any plane as it propagates, the difference is though that power is spread over a different area, right.

So, the expression you get is independent of z and in fact, you can consider this term that I put in brackets sort of as an area. So, if you were to calculate the power when the beam was at its smallest waist could consider that the spot had an area of pi omega naught squared, right and a peak intensity of I naught and this power then gets distributed as the beam propagates.

Sometimes you will see the intensity rewritten in terms of the power, ok. So, it will be in terms of this power pi; sorry, this is squared here, this is squared here also I missed. So, sometimes you will see it written in this form, but what is really of interest is often the ratio of power in a beam of radius let say ρ_0. So, it is the ratio of power in a beam of radius ρ_0 t to the total power, ok.

So, you are finding out 1/ P and you are taking now, you are calculating the power, but you are not calculating the power in the entire beam now, you are calculating the power in a certain area of radius a circular area of radius r_0, ρ_0, ok.

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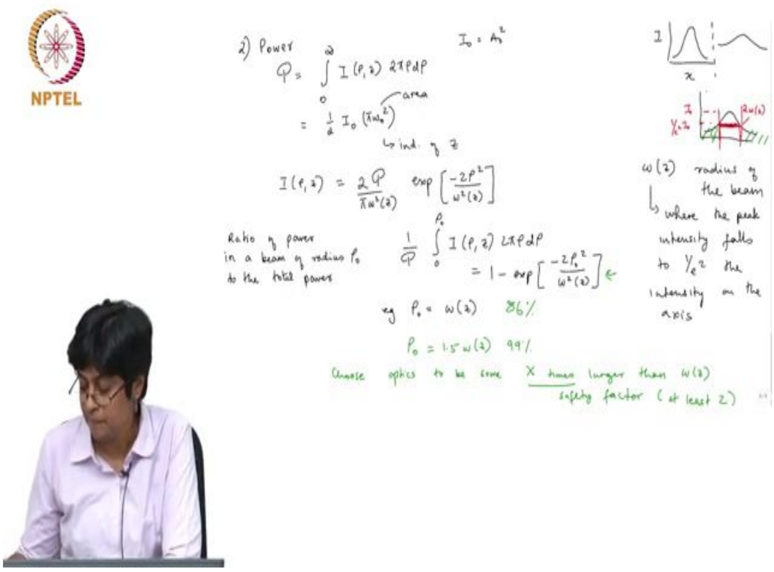

So, we were looking at the ratio of power in a beam of radius ρ_0 to the total power. And again if you work out this expression you will get this. Now, why is this important? We are always interested in building as efficient systems as possible. You do not want to generate a beam, and then lose half of the power of that beam.

You want to ideally always use all of the power of the beam. If I look at this expression for a Gaussian beam, ideally if I want to use all of the power since my Gaussian beam also actually goes to infinity. I might say well my optics has to be infinitely large to capture all of the power, but we know that unlike a plane wave where the power is spread uniformly from the center all the way to infinity the Gaussian beam is not the power is not distributed in that way its distributed according to the Gaussian shape that means, most of the power is lying at the optical axis.

But we want to quantify that better and this expression that we have written down here helps us to quantify that. So, how would I use it? I could say this expression was arrived at saying what is the power in a circle of radius rho naught compared to the total power of the beam.

So, let us take the case where $\rho_0 = \omega(z)$, ok. What is $\omega(z)$? We have said $\omega(z)$ we consider the radius of the beam and in fact, I can give you a better definition of it is where the peak power or the peak intensity falls to 1 by e squared the intensity on the axis, ok.

So, if I look at this as I_0 and then I am looking at where it has fallen to $\frac{1}{\sqrt{eI_0}}$, that let say it is that then I will say this is what I call the beam radius omega well this is $2\,\omega(z)$, ok. There are different definitions because the radius I could define I could say go to $\frac{I}{2}$, take the radius at $\frac{I_0}{2}$ that is also a definition of radius, but in a Gaussian beam omega of z refers to where it has fallen to 1 by e squared. It is the radius at that value, ok.

So, let us say I take ρ_0, I choose the radius to be this size. That means, I am only capturing the power in this region, I am throwing away this power. We are discarding; that how much power falls in this region where $\rho_0 = \omega(z)$ and if you substitute this into this equation over here you will see that 86 percent of the power of the Gaussian beam lies within this region, ok.

Now, suppose it is you had got a value 5% you would then say this definition of radius is a pretty bad definition. I am saying this radius gives me a size idea of the size of the Gaussian beam, but I am saying what I call the size of the beam only contains 5 percent of the power then I would consider that to be a bad definition of the size of them, right. $\omega(z)$ if I choose $\rho_0 = \omega(z)$ then I get 86 percent of the power.

Now, this you might still say this is not good enough, right. It is still you are losing 14 percent of the power, but this is the definition by convention. I could look at a higher value I will say let us take rho naught to be $1.5\,\omega(z)$, right and then you will find 99 percent of the power lies within that range, ok. So, you are not constrained to design your optics and say the equation tells me the radius of this beam is omega of z. So, all my optics in the system will have radius omega of z.

This just gives you an idea of the size that contains at least 86 percent of the power. Really when you are designing a system and you are considering that a Gaussian beam is traveling through it, if you calculate the propagation of the beam through the system and you find out that its largest $\omega(z)$ 5 centimeters when you are picking your optics, you might say well I will pick optics which have a radius 10 cms, right.

So, you would choose optics to be some x times larger than $\frac{\omega}{z}$ to ensure you capture more power, right. And this x times we call this the safety factor, right. So, you want that size of your optics to definitely be larger than $\omega(z)$, and the safety factor is usually at least 2, ok. Usually choose it to be at least 2 times larger than the largest $\omega(z)$ in your system, ok.

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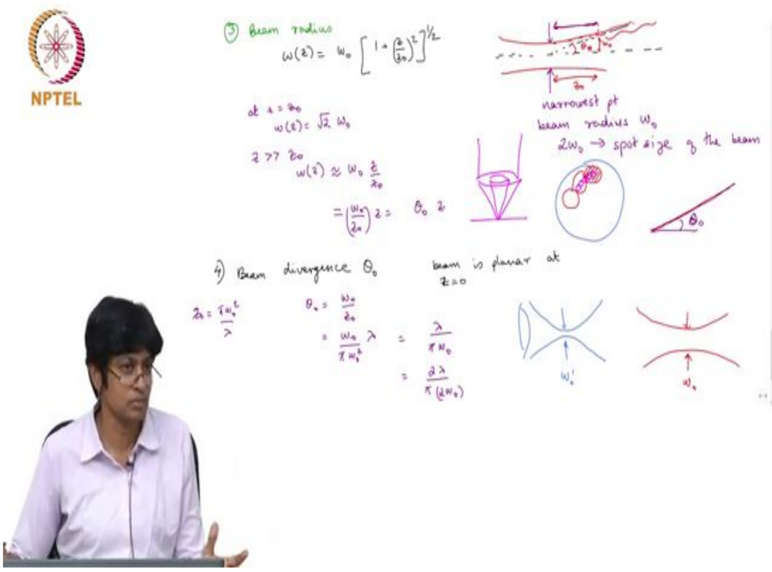

The third parameter of interest of course, is the beam radius. So, we already had written out this expression in terms of omega naught. So, again let us just say this before, but just to make it clear we are seeing at the narrowest point the beam radius is $\omega_0(z)$ and $2\omega_0$ is called the spot size of the beam. This changes, $\omega(z)$ changes as the beam propagates.

So, let say at $z = z_0$, $\omega(z) = \sqrt{2\omega_0}$. And for z very much greater than z naught I can neglect the 1, so I will have omega of z is equal to sorry z by z naught. And I can consider this omega naught by z naught to be equal or to give me an idea of the divergence of the beam, ok.

So, let us look at divergence a little closer. What do I mean by divergence? Now, the Gaussian beam as it propagates it has a planar nature at z equal to 0. So, the beam is planar at $z = 0$. Let us draw the optical axis.

If I were to draw a line, see that beyond a certain point from in this region up to this point if I were drawing a straight line from the center of the optical action outward that is kind of going in parallel with the Gaussian beam at a distance the outer edge of the Gaussian beam at a distance that happens in this region.

But for a particular distance that does not happen. This is nothing, but the beam divergence. This is roughly ω_0. So, the Gaussian beam does not have a constant divergence, right. If it had a constant divergence that means, I would draw the beam one side of the beam spreading out like this and if I looked at this it would be nothing but a straight line and I could say this is theta naught, right.

If the divergence is constant I could draw a straight line along the outer edge of the beam and define an angle theta naught. I cannot do that with the Gaussian beam. Why? Because in this region you can see that the difference between the straight line and the outer edge of the beam is going on changing.

It is after a point that it becomes constant. What is that point? That point happens after the distance z naught. And what is the width? What is the radius of the beam where that happens? While I am looking at, I am taking it as omega naught I am taking z naught and I am defining the angle in terms of those 2 parameters, ok.

So, the beam divergence theta naught is omega naught by z naught. But if you remember we had $z_0 = \pi\omega^2/\lambda$. So, I can replace that here, and often people want to write this in terms of beam diameters. So, you might see it written like this, just to ensure that it is in terms of diameters.

So, look at this expression carefully. A theta naught is telling you what is the divergence of the beam and the moment you use the word divergence with the Gaussian beam keep in mind that the divergence is varying across the distance z naught. So, that is one more pointer to what z naught. z naught is the distance over which there is no constant divergence. After z naught I can consider the beam to have a constant divergence, ok.

Theta naught gives me an idea of the divergence of that beam, but theta naught is not independent of the other parameters of the Gaussian beam. It depends on the wavelength of the beam and its initial spots size omega naught. It is inversely proportional to omega naught. What does that tell me? If I had a beam with a very small omega naught as opposed to a beam with a larger omega naught.

So, this is ω_0. This is let us call it ω_0' just to show it is different. These are 2 different Gaussian beams, and we are looking at something that has been done. Maybe there is a lens over here, something has been done so that the spot is occurring where we see it.

And the spot where we see it for what I have sketched in blue omega naught dash is very small and the spot for the red light is larger. And what should be evident from the figures is that I have done something. I have used some optics to focus on a spot very small. But in doing so what has happened to the divergence the divergence has increased.

So, here is an inherent trade off when you are working with Gaussian beams. You may want to have a very very small spot size. Let us go back to our example of the DVD player. How is the light picking off information from the DVD? Well, it is focusing a spot of light on the disc. If this is the disc and this is the spot of light that is focused that means, I can only have one bit of information in this area.

So, I will have one bit of information there, one bit of information there, one bit of information there, how much data I can put on this disc is limited by the spot that I can focus too. If I took the same disc and I now said the spot is focusing on this region. When I could put another piece of information here another piece here another piece here you can see I can put much more information.

So, the goal in many optical systems is to get as fine a spot as possible. You want to focus it down as small as possible, but I do not do that without some cost. And the cost here is if the spot is focused to a smaller and smaller value, its divergence is larger. Now, you might ask what, I want to capture or store a lot of information or readout a lot of information and I can do that if the beam size is very small; who cares about the divergence. But remember in the CD pickup you had a lens you had the disc and you were focusing on to this, ok.

Now, the finer you are focusing it means the larger the beam has to be, which means your optics has to be larger. So, then you are making your system larger in order to achieve smaller focus. In some systems that might be fine but in others not so. So, that is a trade off that you would have to work to say, if I make the focusing this small it means the beam is larger because the divergence is larger and the optics I need to say I want to make it parallel the optics to control that larger divergence maybe I need more power.

So, I have all these different parameters I will have to take into account, I cannot. So, what this is telling means I do not have absolute control over every parameter of the Gaussian beam independently. I can control the beam radius, but that is going to do something to diverge or I can control the divergence, but that is going to do something to the beam radius.

These are inherently linked, ok. And they are inherently linked also. So, z naught is a parameter you can say that the beam is generated and when it emerges from the source for a particular wavelength and for a particular omega naught, this divergence is inbuilt into the beam and z naught is a parameter that relates to that, ok. So, I hope the concept of z naught is becoming clearer, ok.

So, say I wanted a highly directional beam, I want to send you know people have sent a beam experiment I do not remember decades ago was done where they said aimed a laser beam at the moon and there is a after the polo mission a retro reflector was placed on the moon. So, they sent a laser beam to the moon and it reflected off that retro reflect and came back. I mean the beam to travel the distance from here to the moon and back.

Now, of course, the retro reflector placed on the moon is not 10000 kilometers big. I am putting a mirror, a finite sized mirror, so you might say I need the spot size to be very small. But the beam has to travel this distance and then for us to detect it back here it has to travel all the way back. So, there I have a tradeoff. I have a finite sized reflector, so I want a small spot, but I have a large distance of travel, right.

So, then you have to see what is the divergence and the beam size I have to manage, so they both come to acceptable values, ok. And I cannot say, I want this theta naught and I want this omega naught and I will get both of them, it is not possible, ok.

So, if I wanted a highly directional beam it actually means I have that means, I have a small theta naught a relatively small theta naught. How do I achieve that? What kind of beam should I have? What should my omega naught be? It has to be larger. So, in other words, if I want a beam to travel along a distance I need a fact beam, which means I need larger optics blah blah blah, right, ok. So, just keep that in mind, ok.

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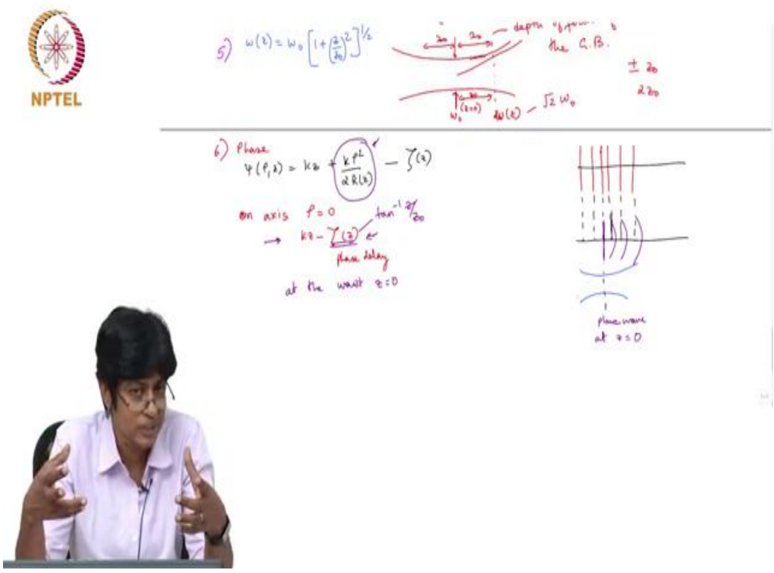

So, let us get to z naught itself. So, we have looked at let me let me write this expression again, ok. This is the beam waist at and this we define as $z = 0$ and if you go to a distance of z_0 so this is z_0, then we have $\omega(z) = \sqrt{2\omega_0}$

And we also said this is the point where you can imagine that the divergence becomes constant. So, another way of considering z naught is to say z_0 is the depth of focus of the

Gaussian beam because you have this gradually varying divergence and then it becomes this constant larger value, right.

So, this is exactly the same concept of depth of focus that we talked about earlier, within this distance plus minus z naught or some people might just say $2z_0$ So, you can of course, go z naught in this direction. You can move within this region and the beam behaves or has more or less the same behavior in this region therefore, I will say this is the depth of focus for the Gaussian beam.

What does it mean? If I look at, ok, let's say these are the wave fronts of the plane wave, right, this should be equally spaced, right. What does the Gaussian beam look like? Well, if this was the beams, these are all equally spaced, ok, right. What do the wave fronts look like for a Gaussian beam? It is going to look very similar over here.

Why? Because zeta of z is tan inverse z by z naught, at the waist z is 0. So, at the waist the phase is the same as the plane wave, there is no deviation from the plane wave, but as the wave propagates this delay goes on building. So, if the plane wave exists here the Gaussian beam is delayed, so its wave front is going to exist a little delayed, ok.

I am exaggerating it, it is not so delayed, but it is a little delayed, ok. So, wherever I expected the plane wave to be all the axis the Gaussian beam is delayed and so you will see the wave front crossing later, ok. So, this term, this zeta of z is basically introducing a phase delay compared to how a plane wave would propagate. Does that make sense?

Now, this expression was for what is happening on the axis, but I have this term here. There is this term you clearly see it is giving some curvature. So, it is not, the Gaussian beam is not just that is why I drew the wave fronts curved, so the Gaussian beam is not a delayed planar wave, right on the axis it is delayed from a plane wave in this way, but off axis I have to take this into account and this gives a curvature.

There is no curvature z=0, because $\rho = 0$ at z=0. So, this term does not play a role there. So, there is no curvature there. So, the Gaussian wave acts as a plane wave at z equal to 0, ok. This is an important point. So, I will come back to this.

So, what we have done in today's class is start looking at the properties of the Gaussian beam, ok. We have looked at the intensity. We have looked at the power. We have looked at the relationship between the inherent terms omega naught, z_0, and λ and the significance of meaning of those terms. So, z_0 is the Rayleigh range or the depth of focus θ_0 is the divergence of the beam and we have seen that we do not have independent control of these parameters. If you control one the other one is going to change, right, ok.

We are now finally, looking at the phase and of course, phase is all important because phase is what happens as the beam propagates and affects the propagation. It is why the beam has the behavior it has because of its phase structure. If I want to change this beam and make it into another beam, I have to change its phase nature, right.

That is a lot of the work we do in the lab here is to design and fabricate elements that change phase in a particular way and therefore, control the kind of beam that you are generating. Why? Because that beam becomes useful for a certain application. You want to capture a cell and make it twirl around and twist around, I need a beam which has angular momentum, a regular beam will not have that.

How do I control that? I make sure the phase of the beam has angular momentum, ok. So, phase is very important and we have just started looking at the phase of the Gaussian beam to see how it is different or how it is similar to both that of a plane wave and a spherical wave, ok.