


Optical Engineering
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Lecture – 20
Chromatic Aberrations and Aberration correction

Good morning. So, we have been looking at Aberrations and till last week we were looking at the monochromatic aberrations. We ended the week by looking at how we could affect or change the monochromatic aberrations. So, for a system with a particular power without changing the focal length of that system, but by changing the shape we could greatly affect two we looked at two particular aberrations, spherical aberration and coma. And, you should have plotted that graph in the last lab exercise and seen that for a particular shape you were able to get both minimum coma and spherical aberration.

So, you see that there is a lot that you can do just by changing the shape. So, we are going to extend the idea of how we can control or change aberrations. But before we do that, we will start by looking at the remainder of the aberrations and so, today's class we look very briefly at also the chromatic aberrations.


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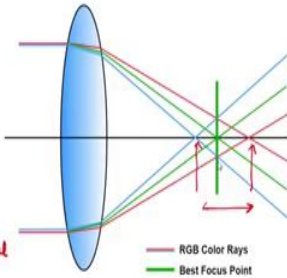
Longitudinal Chromatic Aberration

- Colour fringing throughout the image
- Longitudinal Chromatic Aberration reduced by stopping down the lens. Fast lenses have more LoCA



Longitudinal / Axial Chromatic Aberration

$n \propto \frac{1}{\lambda}$ → most optical materials



Chromatic aberration we discussed earlier arises because the refractive index and the wavelength are related. So, when we have been talking about a lens or a lens with a particular focal length, we never mentioned wavelength, but of course, that equation the focal length equation has the refractive index in it and if the refractive index is going to have different values as it will for different wavelengths of course, the focal length is going to be different for different wavelengths.

So, what is shown here is, the longitudinal chromatic aberrations. So, by now you know that means, we are talking about the aberration or the error along the optical axis. For a positive lens what you will find is that the lower wavelengths tend to focus closer to the lens than the longer wavelengths and that means, you have a spread along the optical axis over here. So, here you can see it is spread out nicely. To demonstrate this the blue is focused over here, the red is focused over here and so, you have a spread over here a longitudinal spread of the focus over here.

Why does this arise this way? Why is it that the blue rays the shorter wavelengths tend to focus closer? It is because most optical materials have n and λ kind of inversely related ok. So, n is inversely related to λ for most optical materials. Now, what does this mean when you look at an image which has been taken with a system that has or suffers from longitudinal chromatic aberration? And, you can imagine if your image plane were as indicated here by this green line, the green light of the middle wavelengths have come to focus here.

So, you will have a sharp point which is yellowish greenish, but you will have this blue red haze around it. Why is the blue red haze? When the blue is focused earlier and it has spread at that plane the red has not yet focused so, it is spread at that plane right. So, you will have this yellowish greenish focal point and then this bluish reddish haze.

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And, so, often you will see images with this kind of purplish tinge and this is a clear indication of longitudinal chromatic aberration. It is quite common actually to see this. Our mind or our eyes tend to correct for a lot of things. So, you do notice you know most of the picture looks well focused. So, you might not notice this effect. Now that you are aware of it I hope that you will notice it no. I do not know if that is a good thing or a bad thing, I made you now aware of how bad pictures are actually. But, the point is you can see that this blue red haze around a focal point is because of the longitudinal chromatic aberration.

This is because the image plane was kept here. I could have kept the image plane at where the blue focus was right, then the blur is going to be all the higher wavelengths. I could or push the image plane to where the red focus is there and then the blur is because of all the shorter wavelengths right. So, the overriding color that you see as a blur is going to depend on where you actually put the image plane ah, but you can see that the circle of least confusion is the central point and that is typically where people will take their image.

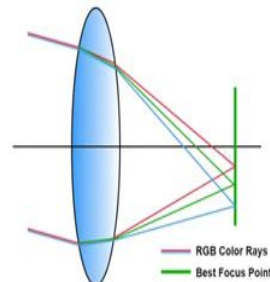
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Lateral Chromatic Aberration

- Colour fringing at the edges of subjects
- Lens having slightly different magnifications for different colours – off axis point

Lateral / Transverse Chromatic Aberration



Lateral chromatic aberration on the other hand is when you are looking at light coming from an off axis point and you can consider it again a variation of magnification, but I am like coma it is not a variation of magnification with aperture, but it is a variation of magnification with wavelength right. So, different colors are coming to focus on this transverse plane at different points and so, you will get a spread in that image plane and it is really you who is saying the red light which has been magnified to this point, the green to this and the blue to this. So, it is a variation of magnification with respect to color ok.

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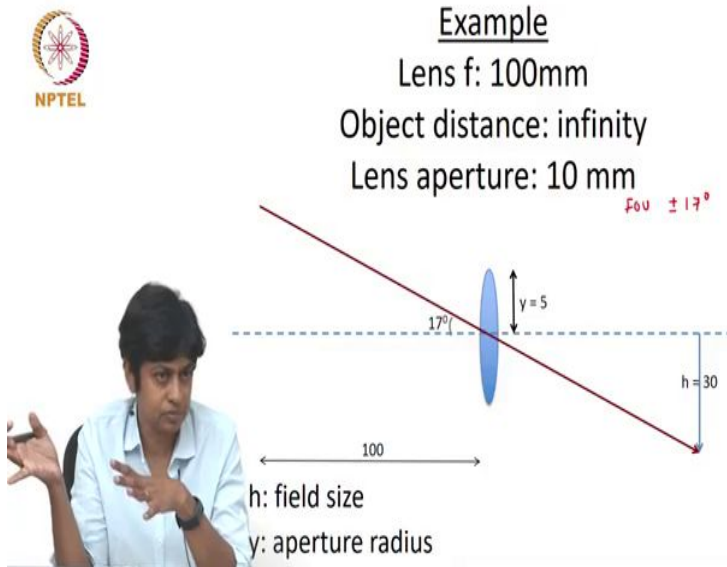
Chromatic Aberrations: Transverse/Lateral



Again, how will it affect your image? You will see that. So, the upper image here it has been taken with the system where there is minimum or no chromatic aberration and the lower picture shows the chromatic aberration and you can clearly see this kind of rainbow effect because each color is coming to focus at a slightly different location and it is very clear. The aberration is very clear. You can see how it overall gives the image a very blurred effect right. So, you really it is not that you have nice clear images, you have this very blurred effect caused by different wavelengths coming to focus at different locations.

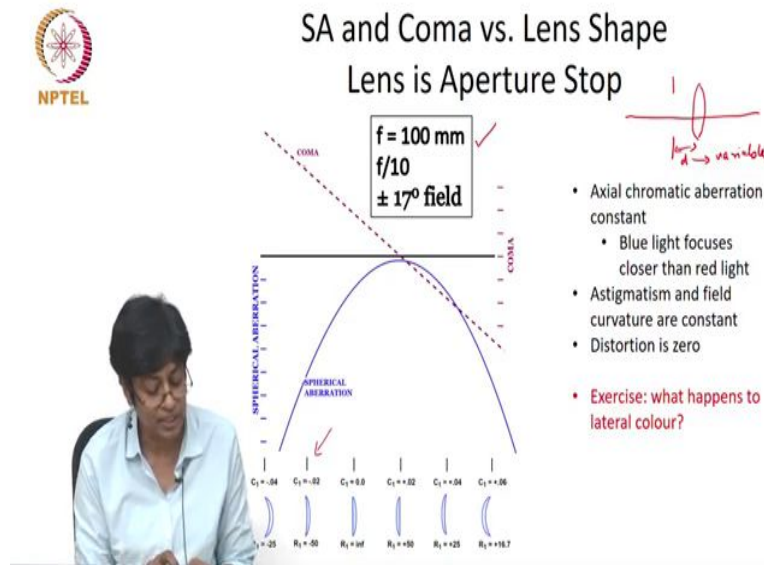
I do not want to go into much detail of the chromatic aberrations ah, but what I want to now do is spend some time on how we would correct these as well as other aberrations ok.

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So, let us go back to the example we were looking at in the last class. In the last class we said let us take a particular lens a single lens and these were the same conditions for that lens. It was focal length 100 mm. We are assuming the object is at infinity, but you have a field of view of plus minus 17° . So, that is this system that you are studying. This was the system you have looked at and then you change the radii of curvature such that the focal length stayed at 100 mm, but you had different shapes and then you could observe the different aberrations for those different shapes or rather how they varied with respect to the shape ok.

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So, we are taking the same system. Now, this was the graph that you should have plotted or something similar to this. We have only looked at coma and spherical aberration. Did any of you look at any other aberrations? The reason that I asked you to plot only coma and spherical aberration is because shape does not affect other aberrations right. So, if I were to plot what are the other aberrations we could have looked at? We have astigmatism, we have distortion, we have field curvature, we have the mono sorry, the chromatic aberrations right ok.

So, it turns out that if I look at the axial chromatic aberration the shape is not affected, sorry the aberrations are not affected by the shape of the lens. So, I did not ask you to plot that although now you could go back and look at that and convince yourself that changing shape does not change the chromatic aberration. So, there was no purpose in plotting it here except perhaps to see that it is unaffected by the shape. We are not saying that chromatic aberration does not exist for a particular optical system and these are what I mean by optical system.

What is its speed, what is its focal length, what is its field of view – for a particular optical system there will be some axial chromatic aberration, there will be some astigmatism, there will be field curvature. But, if you were to change the shape of the system of the lens, these aberrations would remain constant; they are not going to increase or decrease. So, there was no purpose in plotting it here. So, this graph and I want it to go and try it out, take any one of

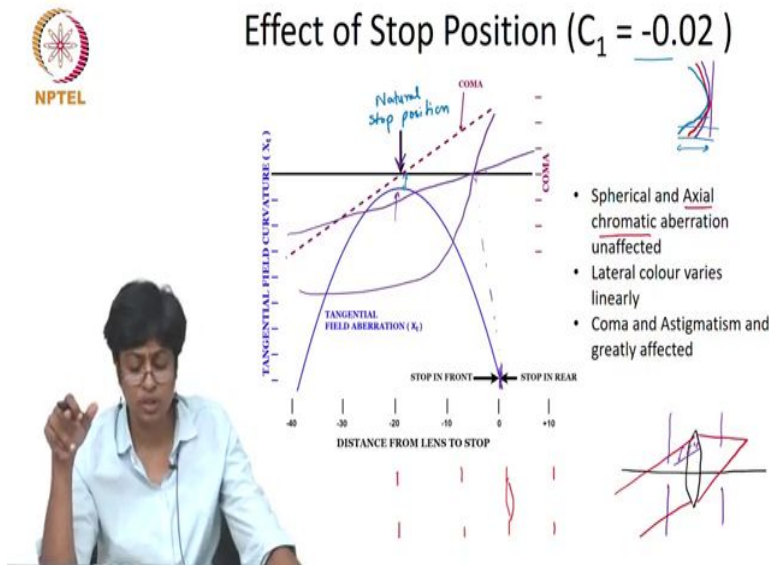
these aberrations and vary. Do you know just add that data along to what you have plotted earlier, convince yourself that yes shape is not affecting these aberrations.

On the other hand, I wanted to do something to affect these aberrations. So, I have looked at shape, but we had listed out a whole range of different parameters that we could play around with to affect the aberration, shape was just one of them. The other one very important tool is the stop itself. Now, in this example of course, there is a stop in this example if you do not say anything, if you do not add anything extra the lens mount or the lens diameter itself acts as the stop of the system. So, though we have not explicitly said anything, the other point I should add is that this optic in this optical system the aperture stop is at the lens itself.

So, now if I want to play around with the aperture stop and find out its effect on aberrations or if it has an effect on aberrations what I will do is now separate the aperture stop from the lens. So, I will add an aperture stop to the lens and will vary one parameter of the aperture stop. I am not going to vary the size of the aperture stop. I will say I have an aperture stop of a certain size and now, I vary its position with respect to the optical axis ok. So, you understand what we are setting up? We are now saying our system is not just an optical axis with the lens, but it is an optical axis with an aperture and a variable now for us is d , d becomes a variable.

I have drawn it here with the aperture before the lens, but of course, the aperture could even be after the lens. So, the aperture can be anywhere along this optical axis ok. Now, how does this affect the aberrations and which aberrations does it affect? We saw shape did not affect a bunch of aberrations right ok.

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So, if you now vary the stop now this is a graph plotted by varying the stop. What does it mean? Only two aberrations are plotted here again. So, this shows you the coma, this shows you the tangential field curvature. So, blue is tangential field curvature, this is coma. What I used? What are the different points plotted for? So, this is the position where the stop is at the lens itself. So, this axis here indicates the distance of the lens from the stop ok. If I have to stop in front of the lens so, in this case this is the lens and the stop is here itself -10 means the stop was moved to this position, -20 means the stop was moved to this position and so on +10 means the stop was moved to after the lens ok.

So, in every case we are taking the same optical system, but I have to choose if we have a range of shapes here. So, this is the graph that I am showing you now was taken for this particular case if we have picked the case where the coma is not 0. We have not picked that case because we want to start out with some value of coma to see how it gets affected by moving the stop around. So, we are taking this case all the other parameters of the system are the same focal length, speed, field of view, but you have taken this shape and we are moving the stop before the lens, at the lens, after the lens and for every case plotting the aberrations.

Again, I have picked some aberrations. Now, normally when we want to compare graphs we take the same parameters. So, I studied spherical aberration and coma before this exercise. Logically, I should study spherical aberration and coma after the successes. Why have I not

done that? The spherical aberration is an on axis aberration I can put the stop wherever I want. It is not going to affect the spherical aberration. So, I could plot it, but I would not get any information from it. It is unaffected by the stop position right.

So, spherical aberration, axial chromatic aberration are unaffected by the stop position. Axial chromatic aberration is affected by the size of the stop, right? If I were to change the size of the stop I would affect the axial chromatic aberration. So, stopping down a system is one way of reducing the axial chromatic aberration, but here we are now each time in these graphs we are varying one parameter and one parameter only.

In the previous graph, we varied the shape. Here we have picked a particular shape and we are varying only the location of the stop, not even its size. So, axial chromatic aberration is not affected, spherical aberration is not affected, only parameters that greatly affected a coma and the tangential aberration.

So, astigmatism and coma are affected and you can see they are affected a lot by the location of the stop and it should not be surprising to you because if I look at say this is my optical axis, this is my lens and I have rays coming in from an off axis point. I think I drew something like this in the last class as well. Which set of rays reach this point depends on where the stop is. If I put a stop here I am cutting off these rays; if I put a stop here I am cutting off a different set of rays. So, the stop location greatly controls the path of rays through the lens which rays actually make it through the lens to the image and this is true because it is coming from an off axis point.

So, coma and tangential aberrations are astigmatism are greatly affected and the moment I say greatly affected by it also means this becomes the tool or one of the tools by which to control or minimize these particular aberrations. I would not stop position as a means of controlling spherical aberration. If you went to OSLO, set up a system and made your variable thickness where it was setting the distance of the location of the stop and you set the operand I am optimizing for spherical aberration, you can optimize all you want the spherical aberration is not going to be minimized because that is not the right variable to choose to minimize spherical aberration ok.

But, coma and astigmatism will be greatly affected ok. It turns out lateral color is going to look something like this. I may not get the graph right, but something like this I have a linear variation; distortion is something like this ok. Actually this 0 is matched with this 0 distortion, goes to 0 this should be at this point sorry, right. So, this is the point where the stop is at the lens itself or you could think that the mount or the rim of the lens acts as the stop and then you have put the stop either before the lens or the stop is after than it is ok.

Now, you see in this case all that you did was move the stop and then measure the aberrations in each case and you have plotted here the two aberrations are vary greatly depending on the stop position. And, they are both off axis aberrations coma and astigmatism ok. I am saying astigmatism is off axis assuming you have a circularly symmetric lens. So, the asymmetry is coming because is the point is an off axis point ok. You can see that just by moving the stop you have found a place indicated by this arrow where the comma goes to 0.

So, for a given optical system there is a natural position where the comma goes to 0. At that same position you find that the tangential field aberration is at its minimum value it is not 0, but it is at its minimum value. Remember, what was tangential field you know astigmatism? This is my planar paraxial plane, but this is my actual paraxial image. It is on a curved surface not a flat surface and we said the sagittal will be like this and the tangential would be like this.

So, at this distance I am measuring what is the error of the tangential right, but if I put the stop at a certain location basically I am cutting off these rays. So, you can see that I am reducing this error right. So, at this particular distance from the lens in front of the lens when the stop is this distance in front of the lens you have the minimum tangential error and it is also where comma goes to 0 ok.

Because you have this position where comma goes to 0 and you could take any. This was plotted for the case where we took a particular shape, you could take any of those shapes right. I pulled out one particular shape, but you could take any of those shapes, keep all the remaining parameters constant and plot this graph. The curves would look slightly different, but you would have in each one of those curves a location where coma went to 0 ok. So, it would not necessarily be at the same distance in front of the lens depending on the shape the

location of the stop where coma goes to 0 will have a different value, but there will be a place where coma goes to 0 ok. Therefore, this is called the natural stop position.

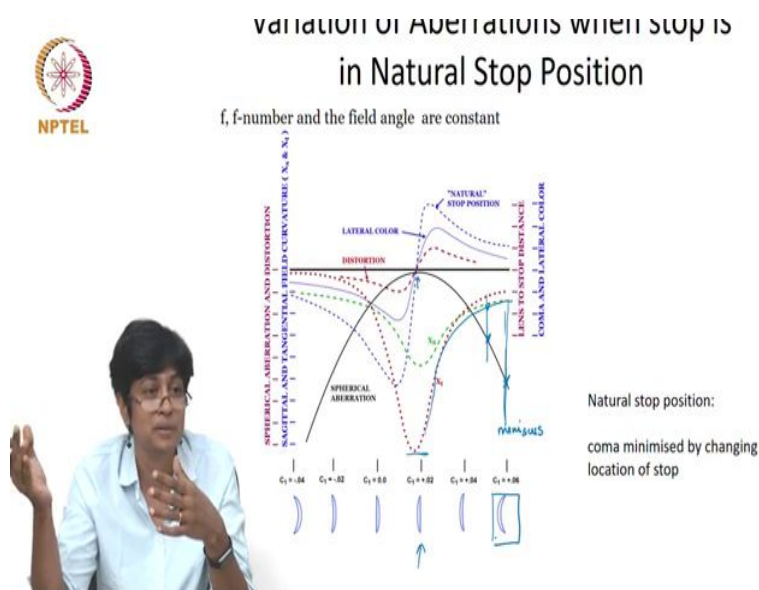
So, what I will ask you to do in tomorrow's lab class is to go back and re-plot that exercise and tomorrow's class is to go back and re-plot the aberration curve, but last time all you did was very shape and then plot the aberrations. Now, I am going to ask you to vary the shape, move the aperture stop to the position where coma is 0; in other words move the aperture stop to the natural stop position for that shape and then plot the aberrations.

Which aberration will you not plot in the exercise?

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You will not plot coma because coma is 0 right. So, if the last class you definitely plotted coma, but now we are saying each time move the stop to the natural stop position, you are going to have an operand saying select coma and make coma 0 and your variable for that will be stop position ok.

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So, if you do that this is a kind of graph you will get. Now, we are looking at all the aberrations in the previous exercises. We looked at only spherical aberration and coma because the others stayed constant with respect to lens shape. So, they there was no purpose

in varying or plotting them. They all had some value, but that values rate constant. So, you did not plot them. But, now you are doing several things you are going to change the shape and for a particular shape you are going to find the natural stop position and then you are going to look at all the aberrations. So, of course, they do vary and you can see that there is a lot of variation, there is a lot of information in this graph ok.

So, let us try to look at what all these lines mean. First point to notice we have not plotted coma here because we are looking at the case when the aperture stop is in the natural stop position, in other words we are saying coma has been minimized. So, there is no need to plot it, but every other aberration is plotted here. In addition you have a curve this blue curve which tells you where is the stop in each case and you can see for this lens, this is the stop is at the lens and that means, for all these shapes the stop is in front of the lens and for all the subsequent shapes the stop is after the lens ok.

Now, let us look at we have plotting spherical aberration here because we are taking shape as well into account right. So, you can see that the spherical aberration does not look very different from its previous graph. It should not, because basically what has changed spherical aberration here is only the shape not the stop position. So, it is the same as the previous graph. However, the other coma is not plotted and we have introduced all the other aberrations as they are affected by the stop location.

So, let us take spherical aberration and this tangential astigmatism ok. Now, you can see where the spherical aberration is minimum? I say minimum, but if you look at the value this is the least negative value. So, value wise it is maximum, but it has the minimum absolute value which is important in terms of how badly it affects the image. So, here is where I have the minimum aberration where I see the minimal effect of spherical aberration is at this point here.

You might think well that is a good place to choose a lens that is good. When I say good place what does it mean I am saying let us choose this shape. Many aberrations are 0 at that point, spherical aberration is minimum at that point. But, astigmatism is not minimum at that point in fact, astigmatism is maximum at that point right. So, if I was designing a system

which was meant for on axis imaging, I would happily choose this shape. What shape is it? It is almost plano convex.

I would choose this shape because I have minimized lateral color, I have minimized distortion, I have minimized spherical. In fact, this distortion lateral color is 0; I am and the natural stop is at the lens itself, spherical aberration is minimum. So, this is a good system to pick if you are trying to do axis imaging, but astigmatism comes from off axis points. So, if you were trying to do off axis imaging, if you knew your system had a wide field of view and you are going to have astigmatism in your system, the fact that the value is over here means that this is not a good shape. So, a better shape is when this curve where this curve reduces. Now, where does it reduce? It reduces the meniscus lenses meniscus.

So, if I was going to do imaging in a system where I know off axis rays are going to come in where I expect to see astigmatism, I would rather pick a meniscus lens than a plano convex lens knowing that the spherical aberration is not 0 there. So, I might not pick this place because I have a large amount of spherical aberration. Maybe I will pick something here and I compromise between astigmatism and the spherical aberration ok.

So, this in fact, is the shape that is used in these very inexpensive cameras right whether you have one lens and that lens typically will be a meniscus shaped lens. So, you are compromising between the spherical aberration and the astigmatism because those lenses expect a slightly wider field of view. They are not only going to be used for on axis purposes, you expect a slightly wider field of view.

So, plotting graphs like this help you to decide depending on your imaging system right. You look at this and you decide what is important based on the kind of imaging I am going to do out, how I expect the light to be entering my system, what is the required field of view, I would go and say which are the aberrations that will not be affected or affected less or where are they minimum let me pick that shape let and in this case we are saying we are not asking where shall, we put the stop we are saying the stop will always we put at the natural stop position.

So, you got rid of the coma and remember, we saw how bad an aberration coma was. So, it is really good to know that there is some justice in this world. We are given a bad aberration, but we are also given a way to fix that bad aberration ok.

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Aberration variation with Aperture and Field

Aberration	vs. Aperture	vs Field Size (or angle)
(long.) Spherical <i>LSA</i>	y^2	
(trans.) Spherical	y^3	
Coma <i>C₁</i>	y^2	$h \rightarrow$
(long.) Petzval		h^2
(trans.) Petzval	y	h^2
(long.) Astigmatism and field curvature		h^2
(trans.) Astigmatism and field curvature	y	h^2
(Percentage) distortion		h^2
(long.) Axial Chromatic		
(trans.) Axial Chromatic	y	
lateral Chromatic		h

Example
 Increase aperture $1.5 \times \rightarrow$ new LSA $(1.5)^2 L_1$
 decrease h to $0.5 \times$
 new coma $(1.5)^2 / (0.5) \times C_1$

So, let us say you were given a system and you have looked at or calculated all the aberrations of that system and now you want you make some slight change in the system ok. So, what do I mean by slight change in the system? You do not change the focal length of the system, but let us say you change the apertures. So, the aperture you started right you have calculated aberrations for a system with a certain aperture, a certain field size and a certain focal length.

And, you have now got the values for all these various aberrations for a particular optical system. Now, you go into that system and you change y and you change h . It turns out you do not have to recalculate everything because the way the aberration is dependent on aperture and field size, if you have already calculated those values, you can just use the relationship between the aberration and that particular parameter to recalculate.


So, let us take for example, a case where you had a certain aperture and let us say. So, in this example let us say you increase the aperture so that it is 1.5 times larger than what it was originally right. How does its longitudinal spherical aberration vary? So, this is let us say

longitudinal spherical aberration. If earlier the longitudinal spherical aberration was some let us say L 1 units of length then the new LSA will be 1.5 times L 1 ok. So, you do not have to go back and redo that calculation.

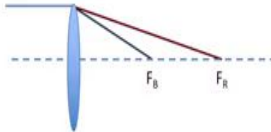
Or on the other hand, let us see the coma you have increased aperture by this much let us say you also decrease h to 0.5 its original value. So, what is the new coma then? If the original coma was C 1, then the new coma is y^2 . So, it will be 1.5^2 because it is squared and it will be 0.5 coma. It is only that into C 1 ok. So, I just wanted to not bother too much about this table, but I wanted you to always keep in mind that these things are not unrelated.

You are arriving at the aberrations for a system given that you have a certain aperture, you have a certain field size, you have a certain focal length you can also find out versus focal length and so on. If you change one parameter slightly, the aberrations are going to change in a very specific way because they are related to those parameters of the system ok. So, that is the main point to take away from this table ok. You do not have to recalculate and redo everything, you are not changing the system completely, but you are affecting the aberrations ok.


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Axial Chromatic Aberration




- Convex lens f_B left of f_R
- Angular deviation (θ_D) function of n
- $n \uparrow, \theta_D \uparrow, \lambda \downarrow$



So, let us look at one correction in some detail. So, axial chromatic aberration we saw that for a positive lens, the blue wavelengths come to focus closer than the red wavelengths. And that

is because typically n and λ are inversely related for optical materials, which means that the angle of deviation is going to also increase as the refractive index is ok.

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Correction: with two lenses
Combination must preserve focal length

- Combine two lenses

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

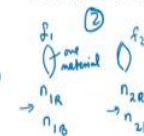
$$\rho_1 = \frac{1}{f_1} - \frac{1}{f_2}$$

$$\frac{1}{f} = \frac{(n_1 - 1)\rho_1}{d_1} + \frac{(n_2 - 1)\rho_2}{d_2} - d(n_1 - 1)\rho_1(n_2 - 1)\rho_2 \quad (1)$$

Suppose $\frac{1}{f_B} = \frac{1}{f_R}$ \Leftarrow in terms of ref ind for blue & red light

$$\frac{(n_{1R} - 1)\rho_1}{d_1} + \frac{(n_{2R} - 1)\rho_2}{d_2} - d(n_{1R} - 1)\rho_1(n_{2R} - 1)\rho_2 = \frac{(n_{1B} - 1)\rho_1}{d_1} + \frac{(n_{2B} - 1)\rho_2}{d_2} - d(n_{1B} - 1)\rho_1(n_{2B} - 1)\rho_2$$

are in contact; $d = 0$,

$$\frac{\rho_1}{\rho_2} = -\frac{n_{2B} - n_{2R}}{n_{1B} - n_{1R}} \quad (2)$$


Now, how do I make a correction? Clearly to do a correction, I need to add something to my system. So, we are going to make the assumption or say let us see if we can do the correction with two lenses. I keep in mind that this combination must preserve the focal length.

So, I have a system and I want to have a desired focal length, but I want to minimize axial chromatic aberration. So, I want to now say I am going to add another lens and adjust the individual focal lengths or maybe the distance between them such that the overall focal length of the system stays as my requirement, but I have reduced an aberration. And, in this case the aberration we are looking to reduce is chromatic aberration axial chromatic aberration.

Now, you can work it out the focal length for a system of two focal with two lenses and a distance d apart. So, imagine you have two thin lenses with the distance d apart going to be this equation, I can write this in terms of the individual focal lengths and this ρ_1 is nothing, but the shape factor is ok. So, it would be $\frac{1}{r_1 - r_2}$. So, this is nothing, but f_1 this is nothing, but f_2 and I have basically rewritten this equation expanding it in terms of the refractive indices and the shape of those individual lenses.

Our goal here is that blue light and red light should focus at the same point. I want this combination to give me the same focal length for two different colors and that is the condition I am setting over here. So, I will now write this equation. So, 1 in terms of refractive index for blue and red light so on. That is all that equation 2 is I have written equation one for red light; that means, I have taken the refractive index n_1 as n for red light and then the right hand side of the equation is the same equation 1, but now it is written in terms of the refractive index for blue light.

To make it simple at the moment to get the idea across let us take the case where $d=0$ and if I do that I can reduce this equation into this. So, we are saying what is this equation saying if I take the ratio of the shape of the two lenses that I am using in order for them to this combination to have the same focal lengths for blue and red light, the refractive indices must be have this particular relationship $n_{2B} - n_{2R}/n_{2B} - n_{2R}$ ok.

Now, I have chosen one lens, this is lens f_1 . It has a certain refractive index for red light, it has a certain refractive index for blue light. I have another lens, I am just drawing it has a convex lens. I do not know I am that please; do not think it is a convex lens. We do not know anything about whether f_1 is positive or negative. We are just taking a lens with a certain focal length. This has another, so, this has this one glass it is made of one material each of these is made of one material.

The material of the first lens has n_{1R} for red light, n_{1B} for blue light and the material of the second lens has n_{2R} for red light and n_{2B} for blue light. And, if the values of these refractive indices are such that they satisfy this equation then in principle I should be able to get ok. But, it is not that simple to just get two materials like this. So, what else can I do?

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Correction

Ratio of component focal lengths (yellow light - between R and B)

$$\frac{\rho_1}{\rho_2} = \frac{(n_{2Y} - 1)f_{2Y}}{(n_{1Y} - 1)f_{1Y}} \quad (3)$$

Equating equations (3) and (4)

$$\frac{f_{2Y}}{f_{1Y}} = \frac{(n_{2B} - n_{2R})(n_{1Y} - 1)}{(n_{1B} - n_{1R})(n_{2Y} - 1)}$$

Dispersive powers $\frac{n_{2B} - n_{2R}}{(n_{2Y} - 1)}$ and $\frac{n_{1B} - n_{1R}}{(n_{1Y} - 1)}$

Abbe Number or V-number $\frac{(n_{2Y} - 1)}{n_{2B} - n_{2R}}$ and $\frac{(n_{1Y} - 1)}{n_{1B} - n_{1R}}$

$$V_d = \frac{(n_d - 1)}{(n_F - n_C)}$$

large V-number \Rightarrow less dispersion
denominator $n_{2B} - n_{1R} \rightarrow$ small

Combine convex and concave

$$f_{1Y}V_1 + f_{2Y}V_2 = 0$$

$$\frac{f_{2Y}}{f_{1Y}} = -\frac{V_1}{V_2}$$

So, let us go back and say let me just take these two lenses and send some wavelength through them ok. So, here I have looked at if there was a red light going through if sorry, this red and blue light going through going through, here if there is red and blue light going through the second lens, now I say through each of these lenses I am going to send a third wavelength, yellow and I take the ratio right off. So, this is nothing, but the ratio of the focal lengths for a third wave length and that wavelength is yellow. So, now, I am saying the refractive index of the first light when yellow light goes through is n_{1Y} , refractive index of the second lens when yellow light goes through is n_{2Y} .

So, now, I have an equation here, which is a ratio of the shapes of these two lenses; in terms of refractive indexes of red and blue light. Here I have a relationship for the same ratio but, now in terms of the refractive indices of the yellow light going through these two different lenses. So, I can equate equations 3 and 4 and end up with this relationship. And, doing so gives us an insight into one of the parameters used to classify glass materials or optical materials. If you take this divided by this why am I picking those terms? Because those are all the refractive index terms for, the glass n_2 .

What is the second focal lens, what is its refractive index for blue light, what is its refractive index for red light, what is its refractive index for yellow light? So, that is this relationship; similarly, I could take this and this and that is this relationship. So, I have the first glass f_1

and of course, it will have three different refractive indices for each of these wavelengths and I have the second lens f_2 and it has three different refractive indices for the same three wavelengths. This ratio when we invert it is called the V-number or the Abbe number of this material.

Now, look at this ratio, what is it telling us? If I look at the spread of wavelengths I have blue, yellow and red right, that is how the wavelengths are spread out. And, this relationship is giving us the difference between the yellow or the middle refractive index and one divided by the difference between these two refractive indices. Now, if I wanted a material with less dispersion right; what is dispersion? What is dispersion?

Student: Spread of different wavelengths.

So, it is the spread of different wavelengths right. When will I have less chromatic aberration? I am going to have less chromatic aberration, if the wavelengths do not spread out; the less those wavelengths spread out the better for me. So, I want a material with less dispersion. What kind of V-number should I have, a large V-number or a small V-number? Look at the relationship exactly so, we want to have a large V-number right. So, large V-number means less dispersion, why? Because when do I get a large V-number? When this is $n_{2B} - n_{2R}$ Ideally it should be 0.

If it is 0, it means a refractive index for red light and blue light are the same. So, whether blue light or red light goes through a lens it will see the same refractive index therefore, it will have identical focal lengths. So, ideally n to be $-n_{2R}$ or $n_{1B} - n_{1R} = 0$. Now, if it was 0, your V-number would be infinite. For naturally occurring optical materials n_{2B} or the refractive index for blue light and the refractive index for red light this is like one of those crystal things: sea shells sea shells I am getting my R's and L's all mixed up.

These two refractive indices have to be as close to each other as possible, for optical materials you will never have a case where they are equal. But, you do have in some glasses where the values will be closer than for other materials right. And, we have just been going into OSLO and saying picking a glass like BK7, now what you will find is you need to be able to play

around with the glass types. We said two lenses, so I want to pick these two lenses such that the combination gives me less dispersion. So, we are getting towards how we can do that.


So, remember this equation right if I can rewrite this in terms now of the V-numbers. So, if I am able to arrive at this somehow in my optical system then I know that I can get a system where blue light and red light come to focus at the same point. Now, either I could do this by saying I should have materials where the dispersion is opposite, one has positive dispersion, one has negative dispersion or I will choose and what does that mean? That means, in some cases, n_d is never going to be less than 1; refractive index is never going to be less than 1.

So, the numerator is always positive, I can never make this numerator negative. You might think somehow I need to get this to be negative, but again that is not natural for our optical materials. This is all it is going to be positive as well. So, how do I get negative dispersion? I do not get it from the V-number then I will get it from using two different focal lengths – a positive focal length and a negative focal length. So, my combination then is going to be a convex lens and a concave lens.

One point to note is that the V-number is a number that is assigned as a standard number to any glass type. So, if you now go and look at OSLO and open up an optical catalog, you can go look at the parameters of the glasses and one of the parameters they will give you is the V-number. So, the V-number you can see is written in terms of three refractive indices – yellow, blue and red light. So, they use standard sources and at those specific wavelengths is where they find the V-number of a particular glass type ok.

(Refer Slide Time: 47:04)

Correction

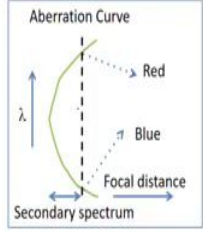


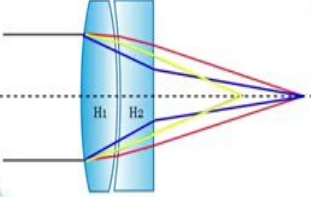
- Abbe number
- Helium d line: 587.6nm
- Hydrogen F line: 486.1 nm
- Hydrogen C line: 656.3 nm

Typical V-numbers

- Dense Flint: 20
- Polycarbonate plastics: 30
- Light crown: 65
- Fluor. Crown: 85

$$V_d = \frac{(n_d - 1)}{n_F - n_C}$$





For BK7

$n_D = 1.519$

$n_F = 1.522$

$n_C = 1.514$

$V = 64.9$

So, these are the sources. So, the helium d line is 587 nanometers, hydrogen F line, hydrogen C line, these are the three wavelengths these are the standard sources that are used to calculate the V-number of any optical material ok. Typical V-numbers for glass types optical materials most the dense flint, light crown, fluoride crown these are all glasses, this polycarbonate plastic is a plastic. But, these are the V-numbers and you can see you have the range from 20 to 85. And, the BK7 glass that we keep using these are the refractive indices at these three sources and it has a slightly higher and V number of 64 ok.

So, we will continue with this tomorrow. What have we done in today's class? We looked at how we moved from looking at mono chromatic aberrations to chromatic aberrations right, we did that very briefly. But, then we went back to those aberration curves and looked at how does aperture position and how does shape, how do they affect the aberrations, which aberrations they affect. And this is part of the exercise you will do in tomorrow's lab is to plot and show which are the aberrations that get affected by these parameters and how much they get affected by these parameters ok.

We then in the latter part of the class took one of the aberrations, we took the aberration of axial chromatic and we said how do we correct it. So, we are looking now at how to correct it and that is how we will continue in tomorrow's class.