Optical Engineering
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Lecture – 02
Geometric Optics Basics

We are continuing with this course on Optical Engineering. So, there is a quick recap of what we did in yesterday's class. We looked at basically the importance of studying the subject irrespective of what background or field you may be working in and we started by looking at the postulates of geometric optics.

So, we said light travels in a straight line in a homogeneous medium. When will that not happen? If the medium is not homogeneous and that brought us to the next postulate which was every dielectric medium has a refractive index associated with it. This in turn brought us to the next postulate we said in geometric optics what is really important is not just the physical distance that the light is traveling, but the product of the refractive index into the physical distance, a parameter called the optical path length.

We came then to the final postulate which was Fermat's theorem.

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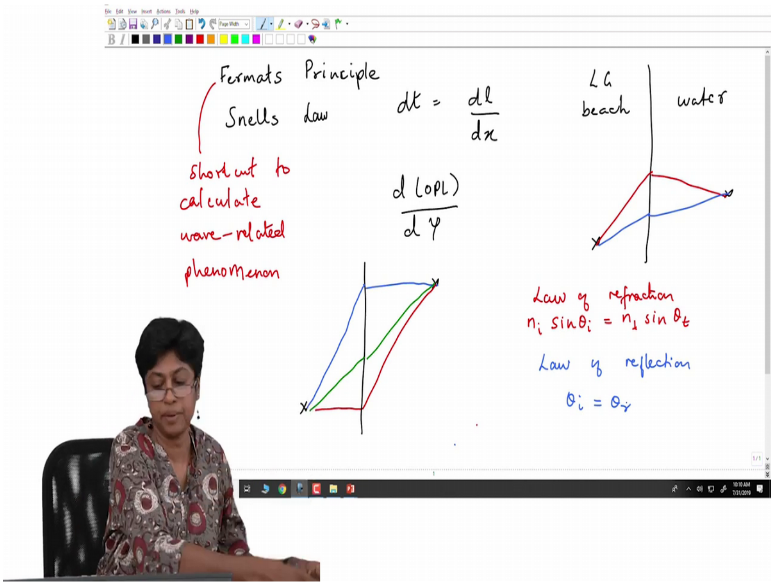

And, Fermat's theorem is basically that light will travel a Fermat's theorem a Fermat's principle is that light will travel on the shortest path or the path of least time. We then used this principle to arrive at a very well known law Snell's law.

Now, we also came to a very important observation when we use Fermat's principle to figure out the way light is traveling. We are minimizing the path and I can always minimize the path with respect to the optical path length itself or I could minimize the path with respect to the time. If however, you are taking it with respect to the optical path length then you would have

$$\frac{d(opL)}{d\Psi}$$

Now, I can look at this law that we all know. You have studied Snell's law all the way from high school right; you should be very familiar with it. Fermat's principle still does not really explain why Snell's law is true. It stated a theorem, we used that theorem and using that theorem we arrived at a well known law. So, that might give you the confidence that Snell's law is correct, still does not tell you why it is correct and really if you want to understand why Fermat's principle is correct you have to go back to the wave theory. You cannot escape Maxwell's equations, you have to go back to the wave theory and you are all supposed to have done a basic course in Electromagnetics or Wave Propagation.

You might have derived Snell's law in that course. Do you remember what you used there to arrive at Snell's law? You did not use Fermat's principle to arrive at Snell's law, but you would have arrived at Snell's law in some way. How, forget Snell's law. In electromagnetics when you talked about a wave hitting an interface and then you used Maxwell's equations to figure out what happened to that wave when it hit the interface what did you use to allow you to calculate that?

Student: We imagine boundary conditions.

So, that is right, you use the boundary conditions and when you use the boundary conditions a boundary conditions in electromagnetics allow you to calculate how much light gets transmitted into the next medium or how much light gets reflected and there you see already a big difference from geometric optics because we said this yesterday geometric optics allowed people to trace the path that rays took.

So, they could understand concepts or phenomena like reflection and refraction, but it did not tell them how much light get reflected and refracted. Whereas, when you go to wave optics you can not only quantify exactly and accurately not only the directions in which light will take, but also the amount of light that goes in a certain direction. And, you can use the boundary conditions to arrive at Snell's law and there you will understand why.

Now, we talked about Snell's law and generally if you say Snell's law you are referring to this equation $n_i \sin\theta_i = n_t \sin\theta_t$, where n_i is the refractive index of the medium of incidence θ_i is the angle of incidence and n_t and θ_t are the similar parameters in the second medium. This is actually the law of refraction.

Of course, we all know the law of reflection and again this can be proved both using Fermat's principle as well as wave theory that $\theta_i = \theta_t$ and a little later on today we will look specifically at how we define these angles because that is important.

Any questions till now? No. So, I want to go to the next part.

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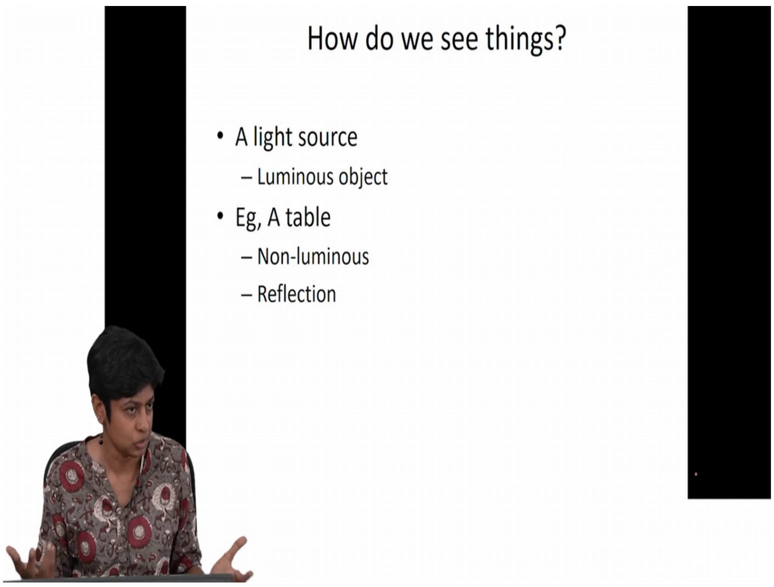

And so, we are looking at using reflection or refraction to enable us to design optical systems right and before we go down that path I want to ask you a simple question. It is a simple question. Maybe it is not a simple question, but it is a question that we should be asking ourselves because it is something that we are doing every day. How do we see things? Why do we see things? You are seeing things all the time, you are looking at me now, you are looking at the board, you are writing notes; how do you see things?

Student: When light hits the things so, that can be reflected so that we can.

So, he is saying that we see things because of reflection and that is right. So, if I am looking at a light source I could make a distinction I could make a further distinction. I could say if the object I am looking at has its own light a luminous object that light travels from the object reaches my eye and that is how I see it, but most of the objects around this table is not luminous, it does not have a natural light source. How we see it is that light from some source falls on the object reflects and comes to our eyes.

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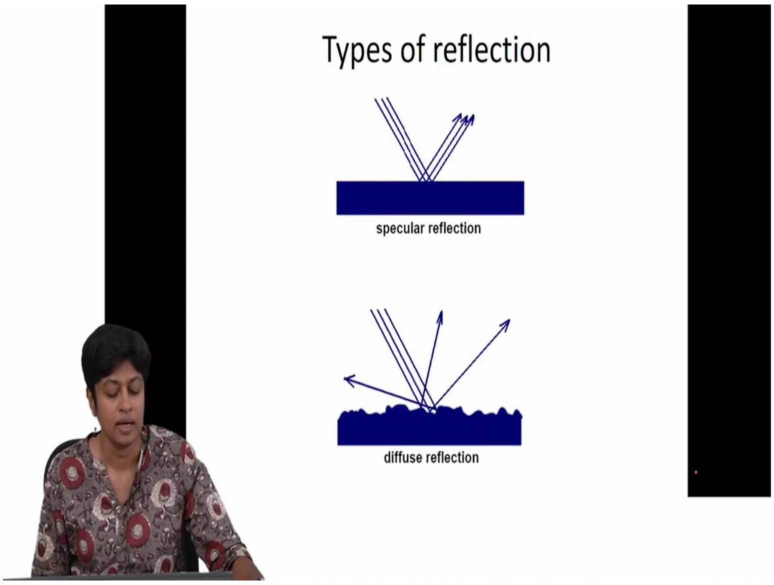

Now, when we talk about reflection, I can talk about very clean and neat reflection. The reflection that follows that nice that seems all reflection will follow Snell’s law, but from specular reflection you can easily see that Snell’s law is being followed. Specular reflection is a reflection you see from a mirror you have a nice optically flat smooth surface. However, most objects are not going to be mirror like and what you would get from most objects is diffused reflection what is shown in this lower picture here and yet whether the imaging system is our eye or a camera if the goal is to create an image we must work with this kind of reflection and get a good quality image.

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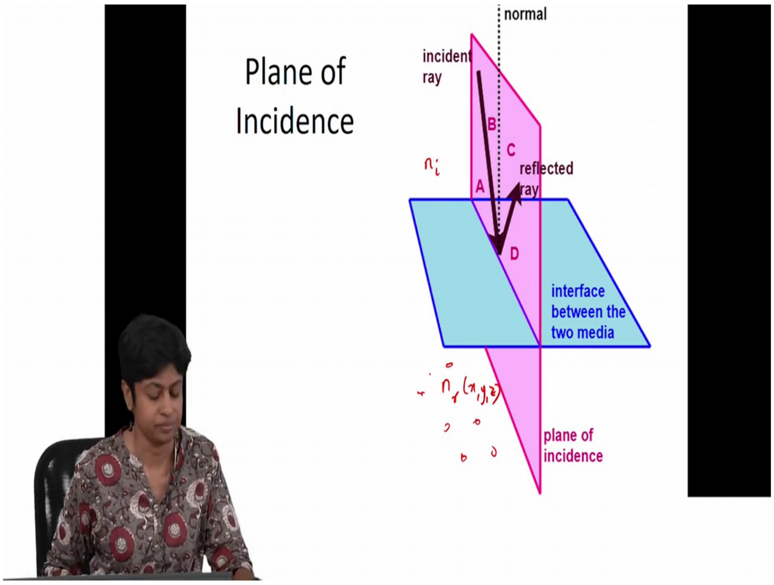

So, if I look at maybe before we go there I will just let us run through again the angles, the definitions. So, if I had a nice flat surface so, as shown in this green surface here let us say that is our interface and light is say reflecting off this interface. So, from the incident light there is a reflected ray and both the incident and reflected ray are going to lie in the same plane. If this interface is not 100 percent reflective then there is also some light that is traveling through and that is going to continue to lie on the same plane.

So, I have a plane of incidence which contains the incident ray, the reflected ray, the transmitted or refracted ray and the normal that lies in this surface that is it is the normal to the interface between the two media. So, if I have a diffused surface I can think of having many planes of incidence because a normal will lie depending upon the particular angle or curvature of the surface.

So, for every ray in a particular plane of incidence it will reflect in that plane and it will transmit in that plane and you can see this messy picture that you get down here ok. Again, this is just been shown as a 2-dimensional picture, but it could be reflecting in two different planes.

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So, this is a reflection of an aluminum foil, it is fairly shiny and reflective, but you can see it is not mirror quality by any means. You cannot really make out the person standing and the image has been taken. If it was a mirror you would I would not have to tell you here is a person standing, you would see that right. So, this is not specular reflection and yet we see the world around us every surface that is reflecting light is we are not hopefully you are not seeing images like this right; that means, our eye is able to take diffuse reflection and make sense of it make proper images out of it right.

So, very large part of this course is going to look at how do I design optical systems such that I take light that is reflected maybe as specular reflection maybe as diffused reflection or take light that has been transmitted and even when it is transmitted if the surface of the interface is not smooth, but has a lot of curvature and variation when it is transmitted it can the plane of incidence may be changing and therefore, the light is being refracted in many different ways.

For example, you can imagine you are using an optical system to image just below the skin. Light is traveling through tissue reflecting off traveling back through tissue. So, there are a lot of different phenomena that are going on there is some reflection refraction and there is a lot of scattering and yet the doctor has to see a clearer image to make a proper diagnosis.

So, this course will look a lot in the almost the first 6 weeks. We will look at how we design optical systems so that you capture this light in a proper way that gives you a proper image.

Just as a recap in yesterday's class we said what all can we do with light and we talked about three things that you can do with light you can.

Student: Ma'am.

Yes.

Student: How is diffused reflection different from scattering?

So, his question is how is diffused reflection different from scattering. So, when I talk about reflection I am talking about what is happening on the surface of an interface ok. Scattering is more a phenomena you talk about when light travels through a medium. For example, I gave a tissue light is traveling through this medium now the scattering happens because the medium through which light is traveling has particles in it and those particles have certain sizes depending on the next dimension with respect to the wavelength you get different types of scattering but, in reflection we are saying light hits an interface.

So, an interface means you had a medium with a certain refractive index, you have now a medium with a different refractive index or you have a medium where you which is the refractive index is a function of position. So, there is an interface right. So, like I have shown over here let us say this is n_i incidence and below this interface I have n_r ; n_r may be a function of x, y, z, but at this interface what happens? The reflection that happens here may be diffused if this interface is not a smooth polished interface, then I talk about diffused reflectance.

If this medium has particles in it of varying sizes of different sizes then as the light travels it is getting scattered by this particle. So, that is the basic difference and it is not that I get information from that light that is scattered it is not that I have lost that information, but I have to capture that light in some different way and our analyze it in a different way.

So, our goal then is to design optics so that we can collect light in a manner that makes it usable to us and I said I was going to write out what are the different applications main applications I guess the very broad applications anything with optics in it can be thrown into one of these buckets. So, if I think the first thing I would answer if I asked you what is an application of light, well, it is illumination the very first application of light is illumination. So, you would have illumination.

Yesterday when I asked for applications some of the first applications people said were camera, microscope, telescope all image right. So, the whole host of applications that come under imaging and of course, now there is a lot of use of light in biomedical applications in medical diagnostics. So, it is again imaging, but you are imaging in order to help with some diagnostic technique. And, the third applicant third broad area what would it be communication or let us put it let us stick with the I letters words information.

So, the projector here is displaying an image for you, it is giving you information now. Fiber optics which is used in the for internet we are using optics to transmit information and that data travels on the fiber in the form of light and that carries information right. So, there are million examples, but almost any application would fit into one of these 3 and imaging here is imaging as well as information. So, it is not that you can completely separate these three, but these are the three broad areas.

So, a very large part of this course is going to focus on the optics required for this for imaging.

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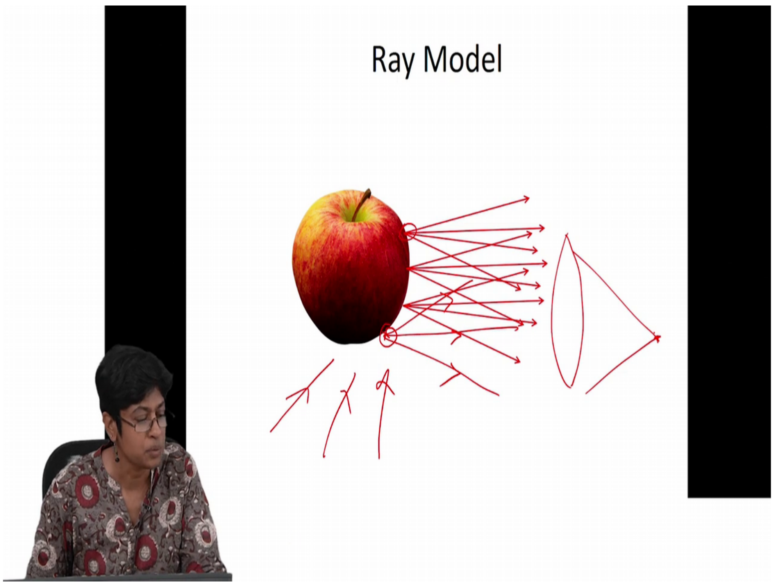

That the wave model is the one that really explains why things happen the way they do, the ray model does not explain why. It allows us to analyze and understand certain phenomena without going into the why without doing a lot of detailed maths. It works and that is why we use it and we use it for a lot of applications. You are going to spend 6 weeks using the ray optics model.

When we talk about imaging what exactly do I mean? So, let us say I wanted to take an image of this apple right. So, what does that mean? That means, every point on this apple says I want to imagine this point; now, if it is not a luminous object, so that means, light has to be incident on it from some external source. Let us say I have some external source light. The incident on it is an incident on the whole apple.

Let us just concentrate on this one point. There are a large number of rays that are being sent out from this point. All the rays from this point have to be captured by my optical system and focused to one point and this is true for every point on this apple. Every point and you can see how messy this looks, but if I want to create a good image I must capture all the rays from one point and my optical system must focus that down to a single point and that is the challenge.

Now, it turns out I can never capture all of the rays, it is impossible. I have a huge cone of rays that is being reflected on every point. So, my optical system at best will capture some cones of rays and we will see the implications or the results of that. All the rays coming from one point on the object must be focused down to one point in the image and this must happen for every point on the object.

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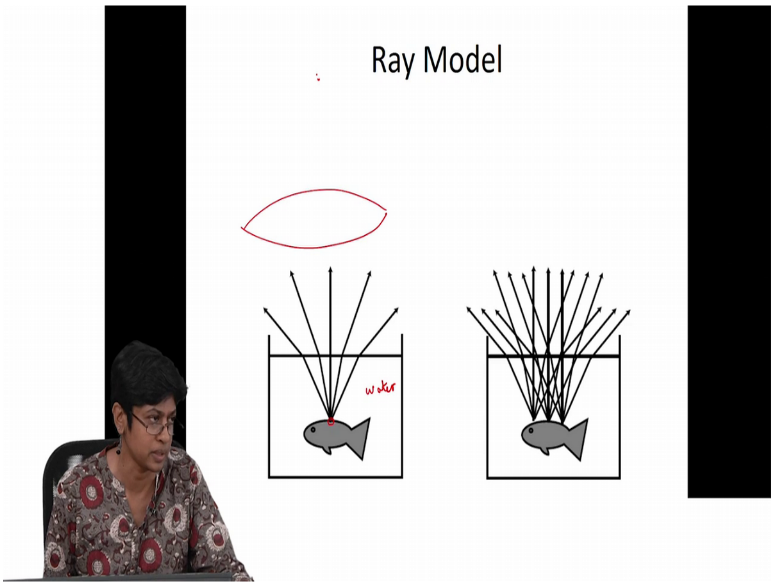

You can see it gets complicated when I have a system where in this case I have water here. So, not only are all these I here I have not shown the incident rays at all right. This fish is not a luminous object. So, obviously, light is incident on the fish and it is reflecting off the fish all the rays from this one point here and now they are going through an interface so, they are bending.

So, there is a lot more happening and yet my optical system whatever it is must take all these rays and focus it to one point over here. And, we must do that for every point on the object that is what my optical system must do and that is what you are going to learn how to do in the next few weeks.

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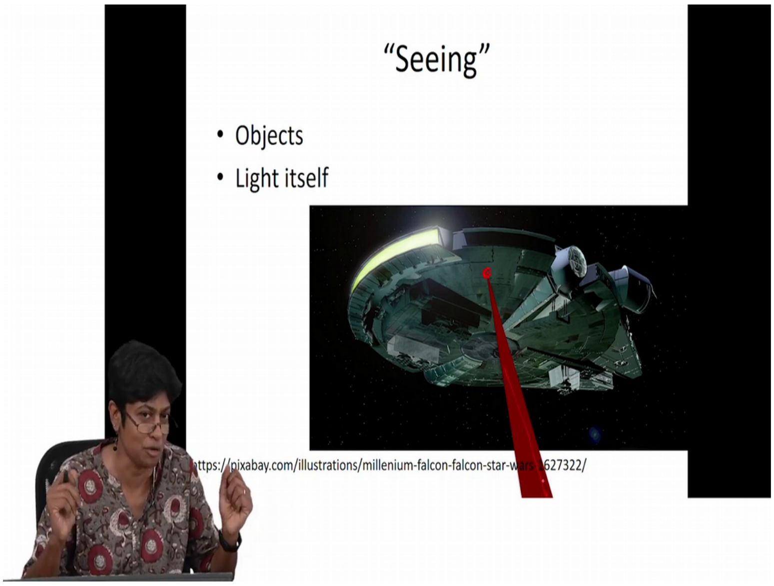

So, if my object is luminous it is clear how we see, if it is not luminous we need an external light source, but how do we see light itself ok, just a small a site to get you thinking. How do we see light itself?. So, how many of you watch science fiction? Star wars? You see images like this um? What is wrong with this picture? Star wars is famous for its light lightsaber. There is something very wrong with the lightsaber too, but will not go into the lightsaber.

What is wrong with this picture? This is a starship out in space and here is this laser beam. We will try to use laser beams for non-violent things, but here's a laser beam presumably as a weapon being shown somewhere what is wrong with this picture? You are saying something?

Student: (Refer Time: 20:11).

So, how do we see light? Actually we see light through scattering right. It scatters off particles in the atmosphere. So, say in this room right now it is fairly well lit, but say the room was dark and it is brightly lit outside and there is a gap in the curtain and some sunlight comes through you often actually see the ray of sunlight or we live in a beautiful campus. You go out for a slightly early morning walk, you will see the sunlight filtering through the trees.

Now, you could see it in different ways you could see the patch of light on the ground. That way how are you seeing that it is this reflection and that reflected light is reaching your eyes, but you often actually see the passage of light. Now, when you go for a walk on campus or if light was filtering through this room and you saw the passage of light you would see it because it is scattering off may be dust or particles in the air. It is because of scattering. Scattering is an important phenomenon. It is not always bad I would not see things if there was no scattering, sometimes.

But, in this picture this is out in space, there is no atmosphere. So, while you the light is leaving this place and maybe there is some scattering or reflection just as it leaves so, at the edge you may see some reflected light and the light may reach your eye because of that. So, you may know that there is light leaving this point, but you should not actually see this path of light because there is nothing for it to scatter off.

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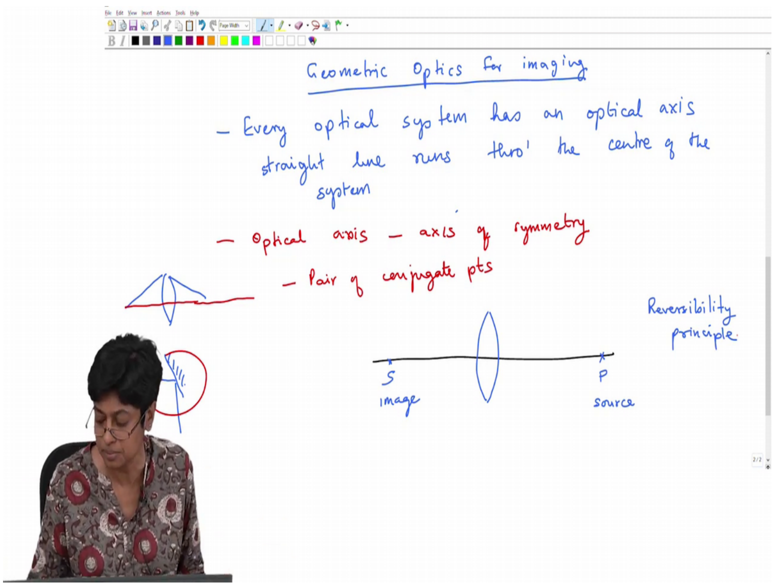

So, as I said we are going to concentrate on geometric optics for image. So, any optical system we start off by saying every optical system has what we call an optical axis ok. This is a straight line that runs through the system and I will be more clear and say runs through the center of the system ok. Sorry, what is the optical axis? That is the line through the system that if you sent a ray the ray would travel undeviated through the system.

So, if you consider a lens and if I had a ray that was traveling like this you know the lens is going to bend the ray right. If I have a mirror and I had a ray traveling like this you know that it is going to travel in some axis angle. The optical axis is that line which travels through the center of the system. I have not really drawn it through the center, but I hope you understand what I mean right. It is the path that a ray would travel undeviated through the system. I can use a mirror to change the direction of the optical axis ok.

In most optical systems or I should say in many optical systems the optical axis is also the axis of symmetry. This is because a lot of our optical elements are circular in nature. So, this does not have to be true I can have non-circular lenses, I can have cylindrical lenses for example,. I can have optics that are not circular in nature, but a lot of our optics are circular in nature take microscopes, cameras, the lenses will be circular in nature and therefore, more often than not the optical axis is also going to be the axis of symmetry. We can even think of it as the axis of rotation if you.

Any optical system has a pair of conjugate points. So, if I think of an optical axis, this is my optical system. I am drawing a lens that is representative of an optical system, it does not have to be a single lens could be a number of lens. So, if it is an imaging system; that means, if I have a point source S and it is getting it imaged at a point P that the conjugate points are S and P this means that if I were to put the source here the image would form here.

So, the optical path is reversible. So, this is the reversibility principle. Now, while the path is reversible; that means, source is at S, image is at P or source is at P image is at S the quality of that image need not be exactly the same. The locations will be the same, but you could design the optics such that it gives you a better image when light travels in one direction as compared to another direction.

So, this only tells us where the locations will be and that would not change and that is a pair of conjugate points, but do not think that means that the optical system works exactly the same way irrespective of the direction . Now, what is the goal of any optical imaging system? You want to collect light and create an image and you want to create as best quality an image as possible that that is your goal.

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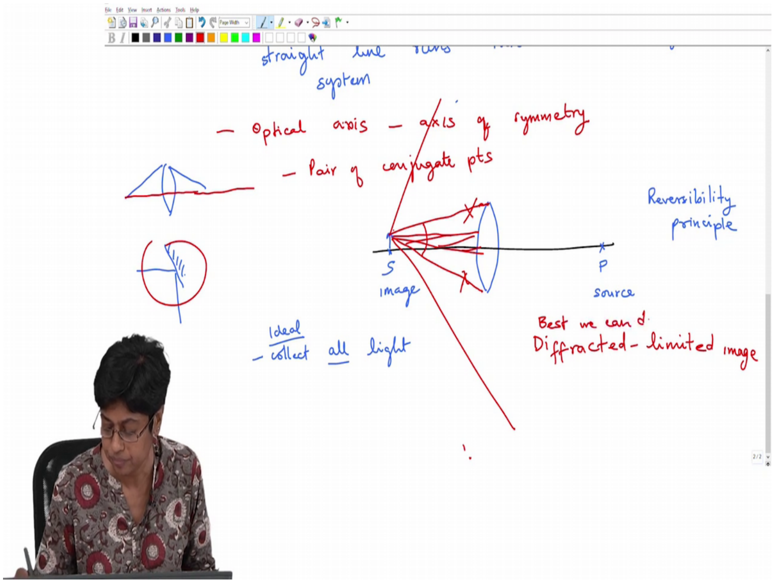

Ideally, I want to collect all light ok. So, this is the ideal goal. This is ideally what I wanted to, but I can never ever collect all the light. If you take this example over here, let us say I have an object over here I have light traveling. So, there is light incident and then there is reflected light and it is reflecting in a very large cone. Some fraction of that gets through this optical system.

And, you learn a few classes on that I you might think I have drawn this set of rays this cone of rays where the upper ray hits the top of the lens and the lower ray hits the bottom and this cone is the cone that will now travel all the way through the system. But, it could be there is some element later on in the system that blocks this part and this part. So, so actually could be some much smaller cone that is making it through the system ok. So, we will look at that later on.

But, the point to take is never do you collect all of the light reflected of the object you only collect a part of the light and the most that you can collect with the particular diffractive sorry, with the particular optical system we call that a diffracted limited image. So, that is the best you can do alright.

The amount that you collect is limited and we would not go into the details of that process in this course, but it is limited by a phenomenon called diffraction and that is why the best image you can get. If there is no nothing else wrong with the optical system, if your lenses are perfect, your mirrors are perfect, everything is in exactly where it should be, then you have a diffraction limited system so that is the best you can do.

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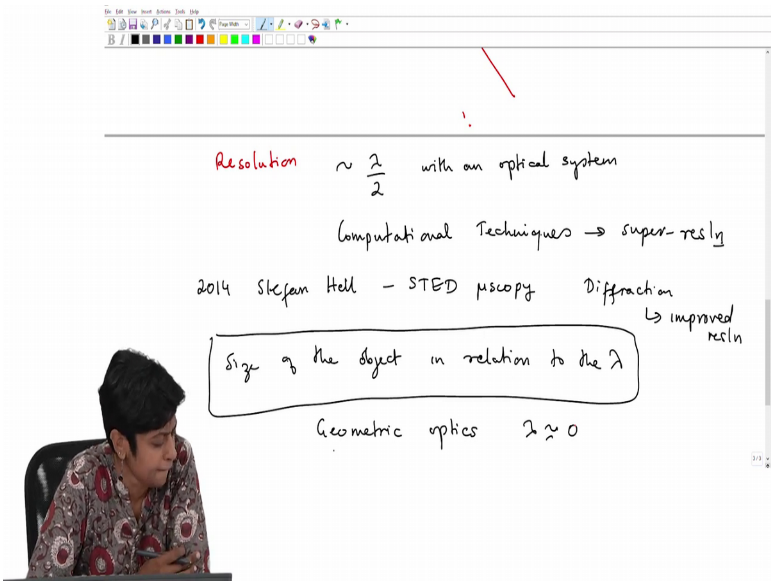

This also relates to the resolution with which you can imagine. Clearly I want my imaging system to have a high resolution. I want to capture the fine details in the object that I am imagining. Optical systems are limited by the wavelength of the light that you are using. So, the rule of thumb you can say roughly lambda by 2 where lambda is the wavelength; lambda by 2 is the best resolution that you can get with an optical system.

But, optical systems are not the only way you can image right. Do you know of any other ways of imaging, non-optical? You can use electron microscopy right. Again, the resolution is determined by the wavelength, but it is a wavelength associated with the electron and that is the orders of magnitude higher better right. So, in optical microscopy your best resolution with the very good optical microscopy microscope would be in hundreds of nanometers; with an electron microscope you can go down to tens of nanometers ok. You can use ion beam microscopy, again tens of nanometers; you can use atomic force microscopes, again very high resolution right.

So, you have many different ways of imaging, but in optics it is diffraction limited and you are stuck with around $\frac{\lambda}{2}$. Even here I have to say that in the last 10 years or so, people have taken optical images and then used computational methods to get what is called super resolution ok. So, then you can computation you will capture optical images with the best resolution you can and then you can improve the resolution using some computational techniques. So, this is a big area of research right now.

Funnily enough in 2014 a Nobel Prize was awarded to somebody called Stefan Hell from Germany for a technique called STED microscopy. It was actually awarded I think in chemistry the Nobel Prize right and he used diffraction. So, we say optical systems are limited by diffraction and therefore, you cannot get better images, but he actually used diffraction and improved resolution ok. So, it is very clever and once the technique is explained a very simple technique and you think you know, anyone could have thought of this, but of course, Stefan Hell did.

So, maybe if we have time later on in this course we will look at this technique in some detail, but in this case diffraction is actually used to develop some element that could be used to improve resolution. So, the main point is in an optical system wavelength is what will limit your resolution. There may be other ways of getting there are other ways of getting much better resolution, but in an optical system this is what your limit really is.

One way to think of this is if I had since the resolution is limited to $\frac{\lambda}{2}$ you can think of let us go to lower and lower wavelengths right and people do that some of the more sophisticated tools use ultra violet – UV, so, you are going to lower and lower wavelengths. But, of course, you cannot always use lower wavelengths right. How do you think of geometric optics then? We think of geometric optics as and even when I say lower wavelengths you are saying you are reducing the wavelength, but reducing it with respect to what right?

So, it is reducing the wavelength the wavelength you are using for imaging with respect to the size of the object that is being imaged. So, the size of the object in relation to the wavelength is what will determine your resolution. So, in geometric optics we say because you geometric optics you are talking and imaging of large with respect to wavelength you can consider $\lambda=0$ here ok. That is an assumption we are making because the wavelength is so small compared to the objects that we are typically imaging ok. So, that is an assumption that we are making.

So, if I am imaging objects in the order of nanometers I cannot make this approximation and geometric optics will not work there, I have to go to wave optics to explain what I am seeing right. You would have seen if you take a CD or a DVD right if you show hold it against the light you see it splits up into colors right. Their structures on that are in the order of hundreds of nanometers right it is not a geometric optics phenomenon that is splitting up that light. So, I cannot use geometric optics to explain.

So, only where that condition is satisfied can I use geometric optics and that is what we are going to do in the first part of this course. So, if I am talking about an imaging system we have already said that clearly any imaging system has to capture as much light as it can from any point from every point on the object and all the light from that point of the object has to form one point in the image plane right. If one point in the object plane forms ten points in the image plane I have a blurred image right.

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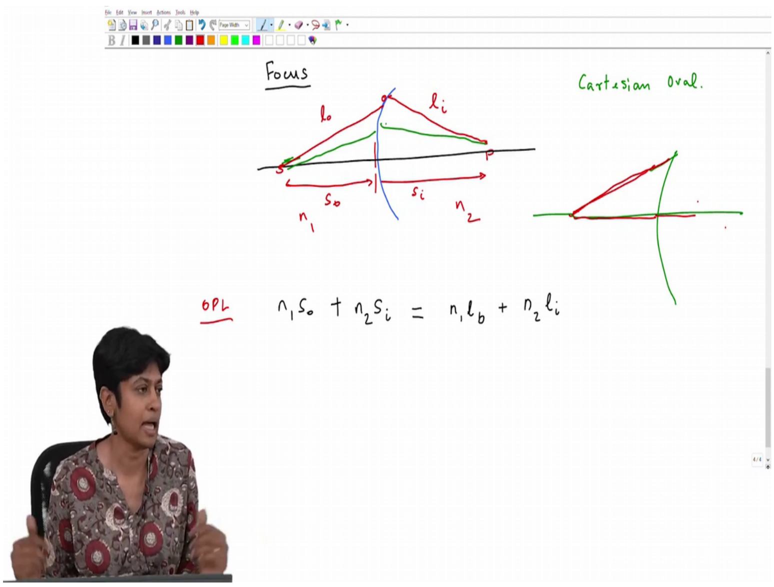

So, of course, what is very important in any imaging system is the focus. So, what element can I use that will provide the best focus um? So, let us take first thing I will do is always draw my optical axis then I am going to take just one interface and I would say I have a point source here, light travels from this point source to this interface. So, it is an interface; that means, I have refractive index n 1 here and right now I am not limiting it right. So, it is just an interface. So, after this interface it is completely another refractive index n_2 are talking about homogeneous medium media. So, it is n_1 on one side and n_2 on the other side.

This travels to some point over here. So, this point I will call S this is P and this distance to the vertex of this interface let us call it S naught again from vertex to here we will call it S let us call it S_i this is l_0 this distance let us call this O and this is l_i. The optical path length of the ray traveling along the optical axis right will be n_1. So, this is optical path length n_1 the refractive index into the physical distance $S_o + n_2 S_i$.

Now, our goal is every ray coming from this point S should arrive at point P. In other words, for the second ray that I have drawn this should be equal the path length should be equal. So, $n_1 l_0 + n_2 l_i$ and this should be true for any ray that I draw. If I draw another ray like this sorry from S and all the way to P. You agree with this? If every ray traveling from S reaches P and this condition is satisfied I am saying the optical path lengths are equal, right then I would say this interface that I have chosen it is not a flat interface, it is an interface with some kind of curvature. This interface, that I have chosen is doing the job of imaging.

Now does such an interface exist? It turns out it does right and the interface that satisfies this condition is called a Cartesian oval. You can think of how it works if you break it down you know look at this if you have. If you have rays traveling along the optical axis and a ray traveling at some distance, this ray hits the interface before this ray does right. So, the ray along the act enters the interface and starts traveling slower.

While this is traveling slower here this ray is still traveling faster in air. By the time it reaches this interface this ray has already traveled some distance within the interface right oh sorry, what is exactly happening? You can see that the wave that is hitting this interface is going to bend because this has traveled to this point this is still traveling at a higher velocity this is slowing down over here and so, there is the bending of the wave and that is what that interface is doing. That is what any optics does right it changes the shape of the beam that is incident on it.

It changes a wave that is diverging into a converging, it is creating that focus ok so, that that is the primary goal of what we are trying to do. So, I think we have run out of time. So, we will continue in the afternoon class at 2’o clock. Keep this image in mind we will continue from here right, but the goal is to shape the incident beam either to collect it and focus it to a point or shape it to give us some particular application.