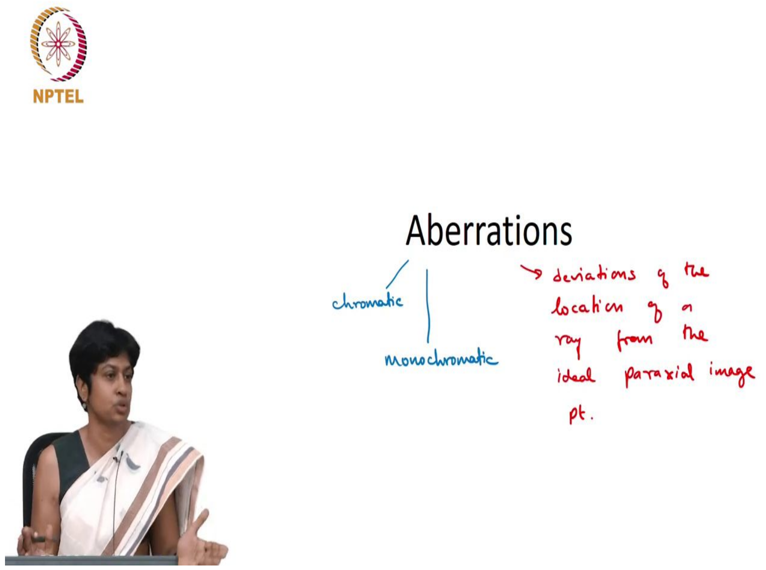
Optical Engineering
Prof. Shanti Bhattacharya
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture – 16
Aberrations

(Refer Slide Time: 00:16)



So, we have looked at first of all, what is focusing, if we go back a several classes, and then we said what is the surface that will give us good focusing, what since we cannot get that surface very easily we made an estimate of another surface, and said how well will that surface the spherical surface give us images. We looked at when you have systems with several lenses in them, how do you analyze those systems.

But finally, the point and we all of this we have talked a lot the last matrix we looked at was an imaging matrix. So, what does that mean? It means we are tracing rays or we are talking about rays that start off from a point on the object and should all the rays from that point should reach one point in the image plane right.

Now, for many different reasons all the points from one point on the object do not always go to the same point on the image ok. And that is why we have aberrations ok. So, you can call aberrations as basically, the deviations of the location of a ray from the perfect paraxial point, from the ideal paraxial point, paraxial image point.

So, we expect if everything were perfect the ray should have ended up in the image plane at a certain point, but instead of landing up at that point it lands up at a different point and that distance, that separation or that error is what we call an aberration ok. Now, we classify aberrations because, by doing so it makes it easy for us to understand how to correct them ok. We will not accept, we do not say we use a spherical lens too bad we are going to have to live with aberrations. We say yes there are aberrations, now how can I correct them, what are the different things we can do to correct for them?

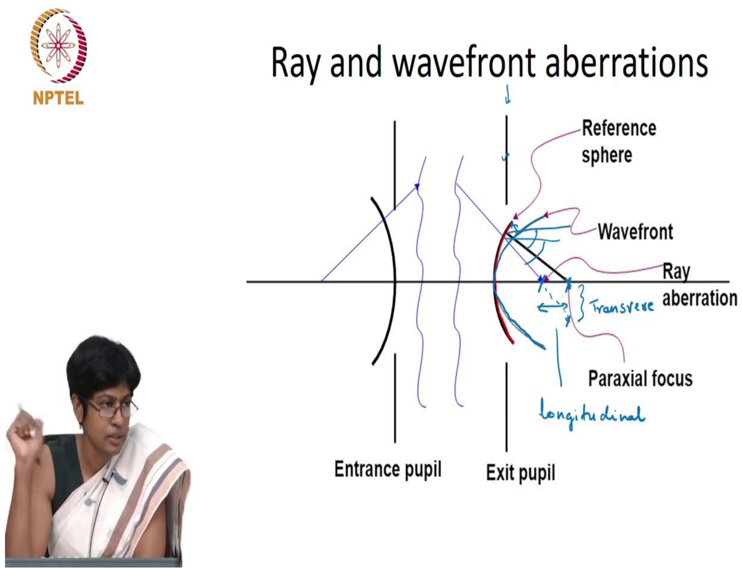
So, classification one needs to do that then is to understand these kinds of rays or under these circumstances you see these aberrations and doing certain things is going to be ok. So, the 2 broad classification of aberrations is you have chromatic aberrations and monochromatic aberrations. Why do these come about? Why do they come about? Obviously, chromatic is to do with wavelengths. Why do you have chromatic aberrations? Think of your imaging equation.

Student: Wavelength depends on the.

The wavelength depends on the refractive index and every imaging equation that we have looked at; every matrix that we have looked at; obviously, has a refractive index in it. So, the moment you send a different wavelength through the matrix; the matrix itself is different for every wavelength because it is a function of the refractive indices. So, while you may arrive at a ABCD matrix for a system, actually though I never said it before the very fact I said n or n dash it meant we were doing it for one wavelength, we were arriving and calculating it for a particular wavelength right.

So, the perfect image position for an optical system is different for every wavelength, it is not the same. So, the moment I send a number of different wavelengths through the same system, it is possible that if the separation in those perfect imaging positions is very large you are going to see that and then you clearly have a very large chromatic aberration. So, you deal with these 2 differently because one even if your system is perfect just the fact that different wavelengths are going through is going to cause aberration.

(Refer Slide Time: 05:04)



So, those are the chromatic and those are the ones we will deal with later, what we are going to deal with first are the monochromatic aberrations. Now, there are many different ways at looking at an aberration. I told you that, we talked about the aberration is basically that displacement of the ray from it is an ideal position, but how do I quantify that? It turns out there are several ways of quantifying that you could think I could quantify it in terms of an angular deviation; I could quantify it in terms of a transfer.

So, in this plane the ray should have been here well in this case the ray should have been here, but instead the ray is here. So, this is a longitudinal error right. So, I can measure it as a length, but as a longitudinal along the optical axis. On the other hand I could continue this ray to this point and say it should have been here, but it said it is here this is now a transverse or a lateral error.

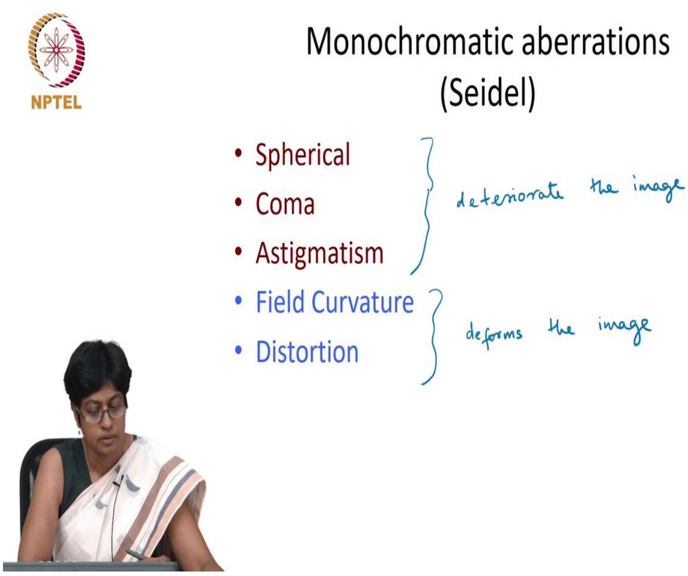
So, I could use units of length to quantify the error, but even there it could be a longitudinal or transverse length that I measure, it could be that the angle with respect to the optical axis, the angle with respect to the optical axis is this, but now the ray is coming to a different point. So, it is a angle is different, so I could use an angular representation.

So, I can quantify it in many different ways and in fact, if you look at Oslo you will find it quantifies it will give you the numbers in all these different ways ok. While we were doing ray optics it is also very useful to look at the wave picture right. So, I could also quantify the wave error. Now, what do I mean by the wave error? If I had a perfect imaging wave it should be spherical in nature the surf the wave front should be spherical, but because it has an aberration it is not spherical.

So, in this image you see they have taken this reference spherical wave, but the actual wave front has a different shape. It is this wave right. And I could then quantify the error as the deviation between these 2, between the perfect reference spherical wave front and the actual wave front.

And Oslo will also give you the wave fronts. Why is it? That a useful wave of looking at the error you can very easily picture how the wave front looks and that gives you immediately an idea of the kind of effect you are going to see in your image ok. So, these are all different ways of quantifying or studying the aberrations in the system ok.

(Refer Slide Time: 08:19)

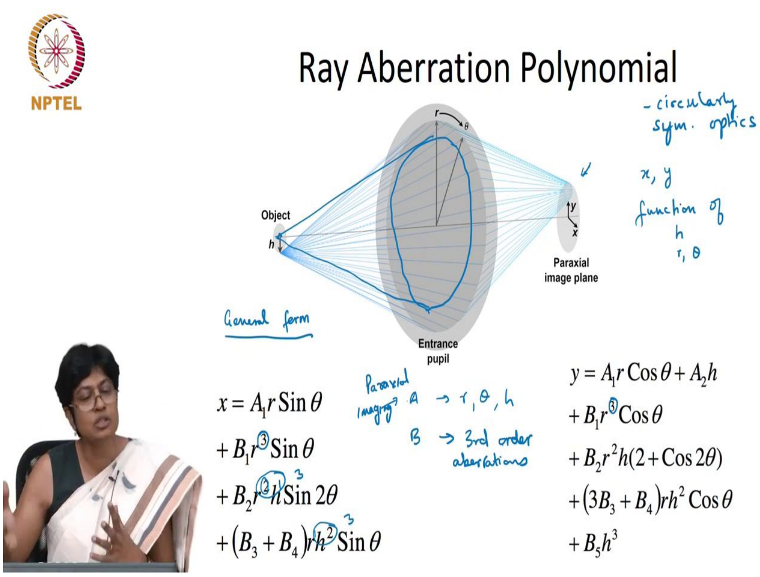


So, we will start with the mono chromatic aberrations, they were studied quite extensively and codified by this person Seidel. So, they are also often called the Seidel aberrations. And there are 5 aberrations Spherical, Coma Astigmatism and then Field Curvature and Distortion.

Now, I have written them in different colors because, while they all affect the final image these 3 actually they deteriorate the image; that means, you do not get a good quality focus or a good focused image, even if you do you will not get a focused image throughout the image plane there may be regions of the image that are focused, but there will be other regions that are not focused ok.

So, they actually deteriorate the quality of the image. These 2 on the other hand field curvature and distortion do not deteriorate the image. You always have a focused image, but then you can think of it as if the image is stretched ok. So, it is focused, but it does not retain the same scales and ratios with respect to the object ok. So, it kind of just deforms the image. And we look at these one by one.

(Refer Slide Time: 10:00)



Now, in order to look at them we need to make some definitions right, because again we said we recall an aberration or we quantify an aberration is the same where the ray determines how much the ray deviates from the ideal position.

Now, what are the parameters I am going to use to decide those things? So, in this figure we are taking and to make it simple we are looking circularly symmetric. So, let us make our assumption here as we are dealing with circular circularly symmetric optics. One of the reasons for doing that I assume that I can always rotate, so I have an object here you can see an object of height h.

I can always rotate the object such that, that height of interest to me lies along the y axis of the system and because it is circularly symmetric it does not matter right. So, that helps in our analysis. So, the parameters of interest are the image point in the paraxial image plane that is what is shown here. And we want to find out where that ray that is coming from the object hits the image plane.

In other words, what is the x and y coordinate of the ray as it hits the plane? The image plane. And this x and y are functions of, the coordinates of the ray from starting from the object and traveling through the optical system. So, what are the coordinates there well one is so it is sorry, this is the function of h which is nothing but the object height and r and theta where the ray crosses the optical system.

Of course, I want to be able to arrive at some equation like this, but it is not possible to arrive at an equation like this for any system maybe for a very simple system you could actually work out equation that will tell you what is the x position of a ray, what is the y position of a ray. But you can imagine for any slightly complicated system with more than 2, 3 surfaces you are not going to be able to arrive at an equation very easily.

So, what Seidel did was to use some ideas and the fact that we are using symmetric systems to arrive at a general form of equations; we are not doing those derivations in this class. So, the equations that I listed here are just the general form of the aberration equations. And you can see that you have one equation for the location of the ray in the x axis according to the x location and one for the y location.

Now, they are very similar equations they have a set of coefficients A B and I have not written higher order terms, but they do have higher order terms as well and you will notice that the 8 terms are they are just a function of or they are coefficients of just r and theta some trigonometric form of θ.

Whereas or r of h, the B terms if you notice the this is order 3, this is order 3 this is 2, but this is 1 so this is again order 3 right. This is 1 this is 2 so, this is again order 3. So, the B terms give you all the third order aberrations. And in fact, I can write 5th order, 7th order and so on. You do not have the even orders here because of the circular symmetry, you could change this equation if you tilted your optical system or your lens and introduced a symmetry into this system, because you have taken symmetric the even orders disappear ok.

So, do not worry too much about these equations. The I the idea I want for you to get is how do we quantify aberrations and we are going to quantify aberrations by saying, I expected the ray to arrive at the perfect paraxial image point, it did not it landed up somewhere else in this plane the paraxial image plane. And I the coordinates of that point are x and y and they are related in general, 2h r and θ h being the object height r and θ being the point on where the plane is the entrance pupil of my optical system. And in terms of these parameters I am able to arrive at a general form of what I expect the aberration equations to B ok. So, I just want it to have that idea in mind.

Student: Mam how do you derive into?

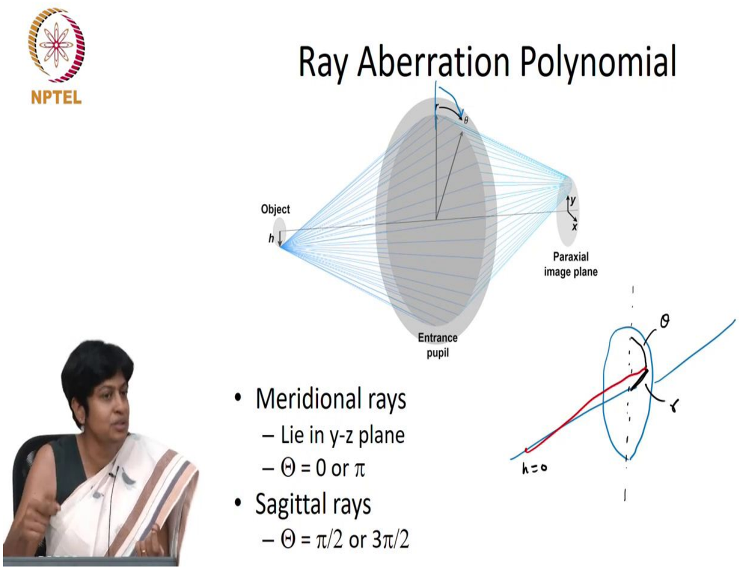
You can derive for a very simple system you cannot. If you can, you would derive them; you could derive them using some form of matrix optics, but it would get very cumbersome, if you are doing it for a complicated system. So, here he has derived it. So, you take some things that you know into account for example, you will know that any point axial object point will also lie in the axis and the image space you know that any point axial.

So, any ray coming from an axial object point and traveling through any, so let us say I take a ray from here, and it travels through any region, it is very bad right, travels through any region in one annulus it is circularly symmetric. So, any ray that goes either to this part of the lens or the same annulus, but a different part of the lens all those rays you know will image at the same point.

Because it is symmetric that must be the case. So, you take these symmetry rules into account and you can trace ray through it and get it in terms of this, but it very quickly becomes complicated if you are trying to do it for more systems with many surfaces ok. So, this is just to give you an idea that the x look coordinate is going to depend on r h θ

But you can break up that dependence into a part that is dependent on just the A terms all have to do with just Paraxial imaging. In fact, A 1 turns out to be the transverse magnification, B 1 is the spherical aberration, B 2 is I think coma, B 3 is astigmatism. And so you are seeing it deviates from here so much because of coma, deviates so much because of astigmatism. So, taking all those ideas you arrive at this.

(Refer Slide Time: 17:50)



Now, again I need to be specific about the types of rays that are going to go through this system. So, let us revisit our definition of the meridional ray. We said it is the ray that lies in the y-z plane, and maybe this figure makes it a little easier for you to understand.

We are now saying it is in addition we it is the same thing, but make it easier to picture. We are saying think of it as the rays that have angle θ is either 0 or θ ok. In the terms that we have given over here you could alternately look at what we call the sagittal rays that lie in a plane perpendicular to the meridional plane and; that means, those rays will have θ equal to pi by 2 or 3 $3\pi/2$ ok.

Why do we break it up into these kinds? Because, it turns out meridional rays will contribute to certain kinds of aberrations sagittal rays to other kinds of aberrations. θ is the angle from the y axis here; so take 1. So, if maybe it is easy if I draw this is my lens and I am drawing an ellipse because I am showing it to you at a tilted thing, this is the optical axis and you have a ray that starts off. Let us say from an axial object point and that ray happens to go here.

Now, because it has gone there , it is coordinated. This is the entrance pupil of our system in this case if you consider it to be a single thin lens it is an entrance pupil exit pupil aperture stop. What are the coordinates here? Well it is angle from the y axis is this is θ and it is distance from the origin or the optical axis. This is r and h is 0, because we have taken an on axis point ok.

So, this tells us what the coordinates of that ray as it enters the entrance pupil of the system ok. It can think about it like this: we are saying every ray that comes through the system we would like it to go to the paraxial image point, but some rays make it to that point and some rays do not. And our contention and what you are going to see is that why do some rays make it to the image point and others do not well it depends on how they enter the system, where they enter the system?

If they enter the system with r 0 or θ was 0 h, 0 they are all going to make it to the paraxial image point they are perfect paraxial rays. The moment the wave it starts moving away from the paraxial ray. We know that they are going to have aberrations and we are saying this is saying that the amount of the aberration is going to be related to where they entered the; enter the optical system. In other words what was the value of r and θ at the entrance pupil of the system? Ok is that clear.

Student: (Refer Time: 21:21) Symmetric (Refer Time: 21:25) of the system. So, that (Refer Time: 21:30) in this case if you are (Refer Time: 21:40).

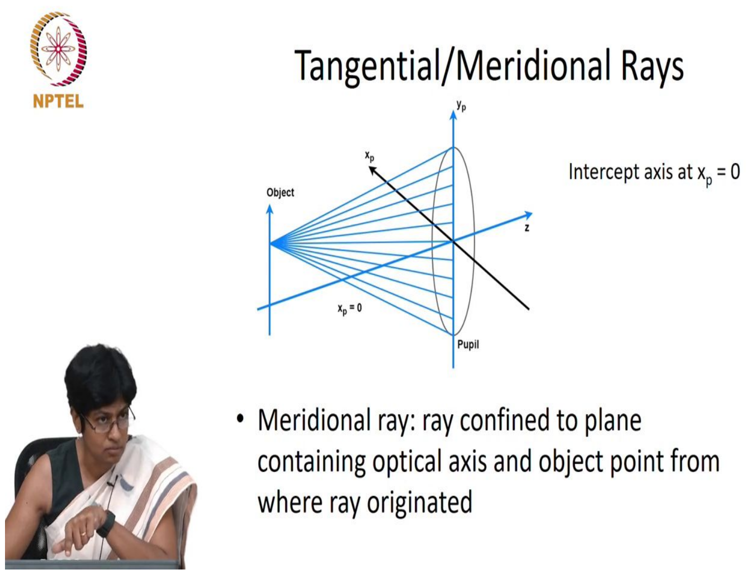
So, his question is because, it is circularly symmetric we can always rotate the system. So, why would I bother about θ You bother about θ because I can rotate the system when I am dealing with a set of rays coming in one plane, but actually in an optical system I have rays going out in all planes. So, when I am analyzing all of those rays I cannot rotate it any more, I could rotate it and get one set of rays to fall in a plane.

But then I have raised that up in planes perpendicular to that planes, not perpendicular to that and they all will be affected by the optical system differently. It is not just r θ h that affect the image or you can think of it is not those terms directly it should not be surprising if you think about it. When a ray travels through an optical system, if the ray c is symmetry, you expect that the ray will behave in a certain way.

But if the ray c is A symmetry or A set of ray c A symmetry, it should not be that surprising that they behave in a different way. And depending on their r θ h they are either going to see the optical system as symmetric or not. We are using a symmetric system that has an axis of rotation, but if a ray comes from and off x is point one ray hits the top of the lens, one ray hits the bottom those 2 rays are seeing completely different they see the optical system in a different way right.

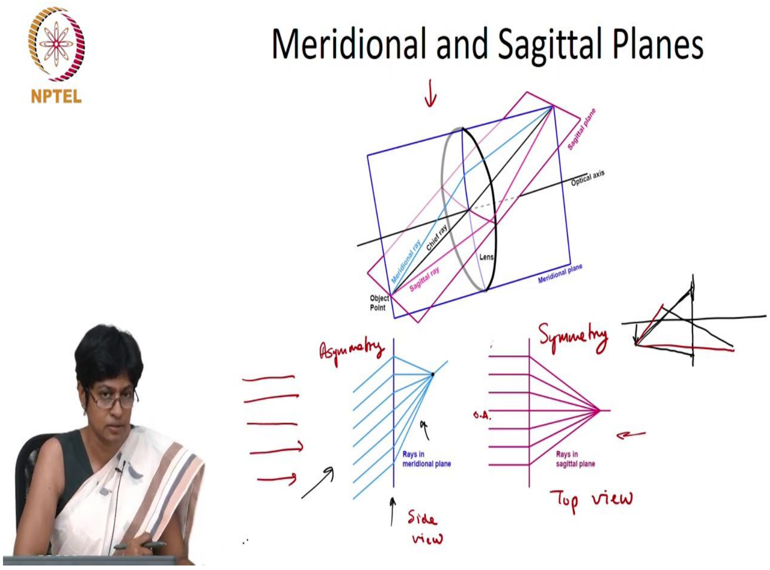
So, the way the ray comes into the system can introduce a symmetry and does introduce a symmetry and that causes one set of aberrations ok. It will become clearer as we do the definitions.

(Refer Slide Time: 23:34)



So, just to get these concepts clear again we said meridional rays y z plane and here we have a set of rays coming from one object point and these all lie in the y z plane they all have the same value of θ. But they all have different r values.

(Refer Slide Time: 23:58)



And we define a sagittal plane, the sagittal plane is what is in this I do not know, if you call it pink. It is in a plane perpendicular to the meridional ray sorry meridional plane. So, you have the optical axis here, you have the lens and then you have this sagittal plane perpendicular.

Now, below I am showing you what I was just talking about. In this case you can consider that this is your lens, this is your object and you are looking at the rays coming from this point, the extreme point of the object. Now, you can see that the rays coming from this extreme point are all coming from the same object point, but this ray lands up at a certain angle on the optical system and this ray lands up at a different angle of the optical system and that is what is shown over here.

Here they are not coming from a point they are coming from infinity. So, you have a parallel set of rays, but you can see that the way they are incident on your optical system is different, they bend differently here to form an image point here right. These are rays in the meridional plane from that same point here; if I now drew the rays coming to the lens it is hard to draw this, but I will try ok.

Let us say I am now drawing what is happening in the plane perpendicular. And so, these rays are going outside words, they all so this is now; this is a side view and this is now a top view. I am looking down at the system from here and looking at the sagittal rays.

So, they are coming and hitting the lens from the side and getting bend they are hitting the lens at different points this is the on axis ray this is the ray traveling along the optical axis these are the rays coming to the side of the lens, they are all getting bent like this, but you can see they all see the lens in a symmetric way. So, it should not be surprising that this could cause some aberration because these rays see the lens is symmetrically.

So, there is an A asymmetry over here whereas, there is asymmetry over here. And in fact, the aberration astigmatism arises because of asymmetry that comes from the way a ray is incident on the lens ok.

Student: (Refer Time: 26:38).

Yes you are right. Your question is if this is the case.

Student: (Refer Time: 26:43).

I had rays coming in like this or.

Student: (Refer Time: 26:44).

Parallel to the optic axis. You are correct. In that case, I will not see a symmetry and the error would be different. Yeah then that is identical to this case actually right.

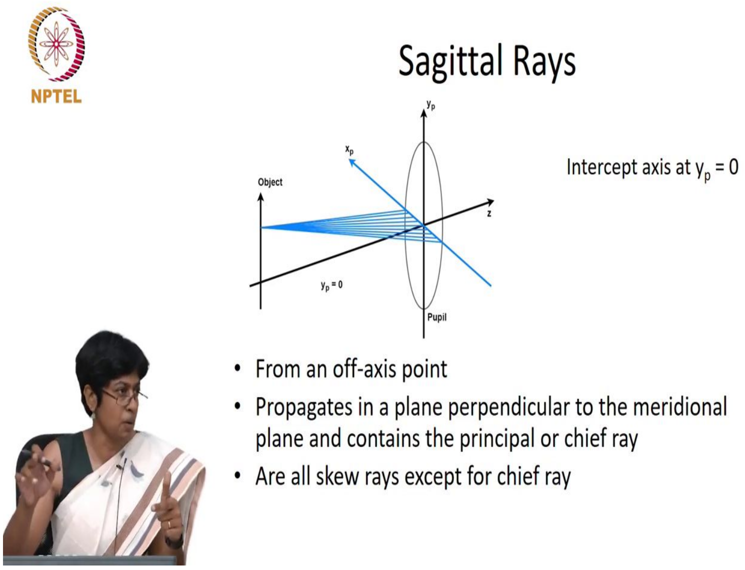
Student: (Refer Time: 27:00).

So, you have.

Student: (Refer Time: 27:03).

Some aberrations which occur only for off axis, so in fact, I broke up the aberrations into monochromatic and chromatic, but in monochromatic we will first look at the on axis aberrations and then we look at the off axis aberrations. You had another question? No ok.

(Refer Slide Time: 27:24)



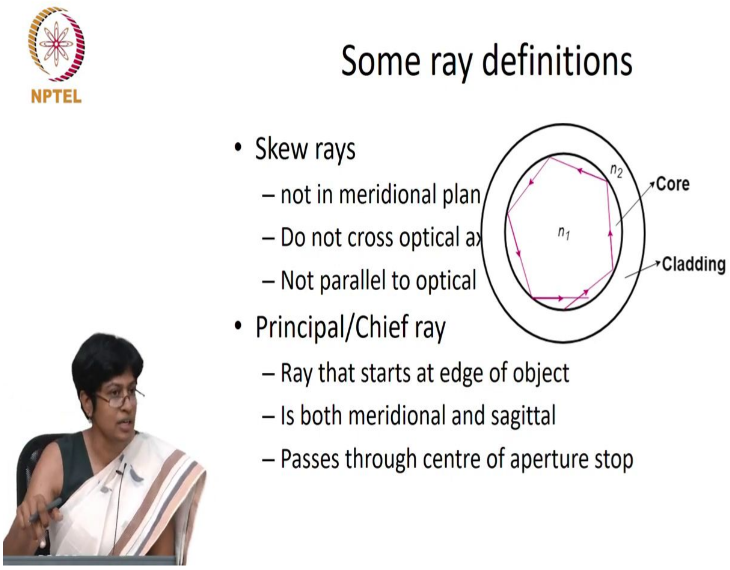
So, the sagittal rays come now you notice here, the sagittal rays even though they are coming from an off axis point they still see the system symmetrically right. And that is why there you see there is the difference in the final image depending on how the rays have traveled to the system and to make it easier for us.

We will say these are meridional rays; these are sagittal rays and then immediately tell you oh they are seeing the system symmetrically or they are not right. You do not have to define those terms every time ok. Sagittal rays propagate in a plane perpendicular to the meridional plane and it also contains the chief ray; the meridional ray plane also contains the chief ray. So, that basically that intersection between these planes is where the chief ray is travelling ok.

Student: (Refer Time: 28:22) Chief ray (Refer Time: 28:24).

Yes, you are right. The chief ray is both the sagittal ray as well as a meridional ray. It is both of them, yes it satisfies the definition of both of them ok. Sagittal rays are skew rays except for the chief ray. Now, I mentioned skew ray sometime back, but I will refresh your memory skew rays are rays that crisscross the system. They do not go along the optical axis, they can avoid the optical axis completely and I think I have a picture that will help you understand that a little better. So, I have rays traveling within a fiber.

(Refer Slide Time: 28:58)



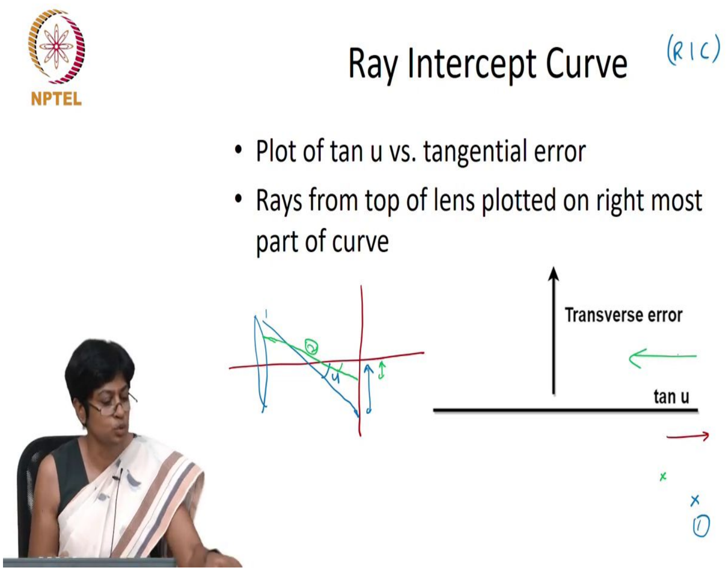
So, you are looking at the cross section of the fiber, these rays are all in different planes it is not in one plane. So, it is traveling like this, then traveling like this, but you can see they never travel along the axis. These are curious, ok.

(Refer Slide Time: 29:20)

So, curious are not in the meridional plane they do not cross the optical axis. They are definitely not parallel to the optical axis, except for the chief ray ok, agree or obey these different features.

The chief ray happens to be a sagittal ray and skew rays are also sagittal ray, but the chief ray does pass through the center of the aperture stop and is both meridional and sagittal as we discussed ok.

(Refer Slide Time: 29:53)



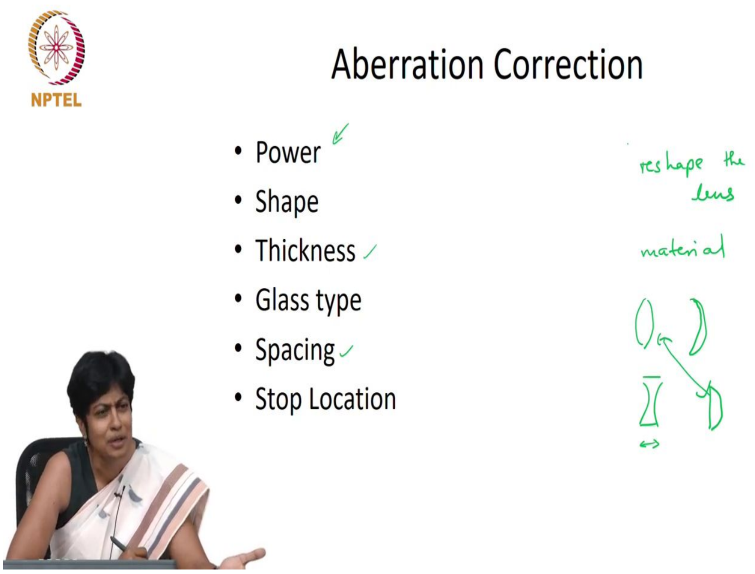
Now, since I want to quantify them there again you have seen those aberration curves in Oslo there are several ways of quantifying, there are all these nice curves over there. And by convention they do not give you the axis labels ever, they will just show you the curve and you must understand what curve it is and what is being plotted. So, this is one case where the teacher will not jump up and down and say you did not label the axis, they do not label the axis it is it is quite infuriating, but that is the way it is right.

But I might still ask you what the axis is to understand, make sure I know that you understand what they answer. So, in that case you will have to tell me what they are. One way of plotting aberrations is to plot something called the ray intercept curve and it is basically where the ray intercepts the paraxial image claim, but rather than plot it in terms of x and y, we plot it in terms of this axis is the tangent of the angle that remakes.

So, let us say I have this as my paraxial image plane. This is the optical axis and I have a ray that hits the plane like this. Then this is u and I am plotting tan u over here and this is the transverse error and we are saying maybe and it is negative you can see. So, this point would come over here ok. The standard way of plotting these curves is that you take the rays of this was my optical system, the ray that you plot on the right most of this r ray intercept curve is the ray coming from here.

So, this is ray number 1 and that corresponds to this point. So, the ray from the top of the lens, it is transverse aberration, is plotted against the tangent of the angle it makes with the optical axis on the rightmost part of this curve and often we will just call these the RIC curves. So, if I take another ray let us say that ray started from this point and it went over here this is the secondary. So, it has this angle, it has this transverse aberration, so maybe it comes over here ok. So, you plot in this direction, it will clear up, you see more of these curves and we plot more of them.

(Refer Slide Time: 32:40)



Our goal is that we want to correct for the aberrations. This is not. We do not just want to observe them and quantify them and say oh look, I have so much of spherical and I have so much of this. We want to quantify them and say I have so much of this aberration how do I correct it? Right. Now, can you think of some things we could do to correct? What can we do that we could correct? What optical operations are what could I do with my optical system to help correct? The lens can be made symmetric ok.

So, I will rephrase that I will say maybe we can give away an answer, maybe we can reshape the lens right, you still have spherical lenses, but they do not all have to have the same shape right. I have one of the answers that came up: you can play around with the power of the various lenses in the system right. What else?

Student: (Refer Time: 33:54) Refractive index.

You can play around with the material. He said refractive index, so maybe play around with the material ok. What else?

Student: (Refer Time: 34:07).

How do I change my optical system? So, wavelength is not a way to change my optical system. Remember the goal of correcting an aberration is you have designed an optical system or you want to design an optical system that works for a certain wavelength or range of wavelengths and should have a certain power, correcting for aberration does not mean you change your requirement, for your requirement how do you get the least aberration?

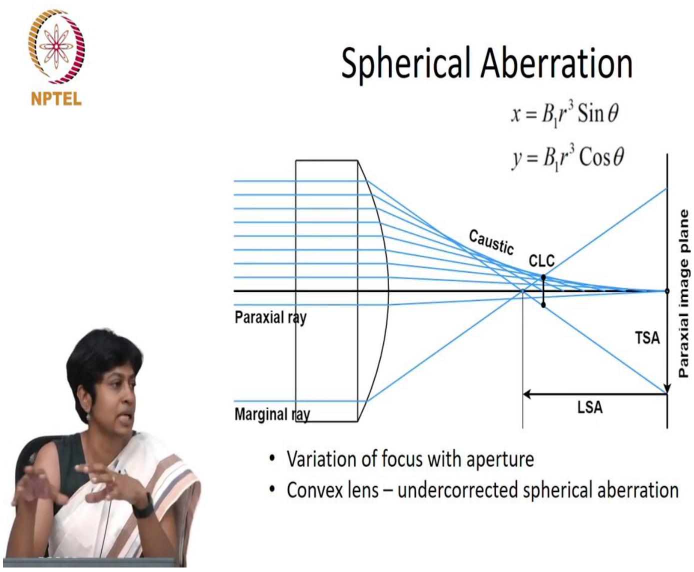
So, even here when I put power I do not mean that you change the power that you require or what is required of the optical system. I mean here that if you have multiple elements you can change the relative power. So, the overall power of the system is the same, but maybe you had lenses f_1 f_2 and you change it to f_1', f_2', f_3' such that total power is the same, but because you have changed the individual powers the way the ray bends is different and therefore, an aberration is less ok.

So, correcting for an aberration is not changing your requirement, it is for your requirement how I get the best out of the system. So, a number of things that you could do when we said shape I can take a convex lens, I could take a meniscus lens, well it does not look like that, but you get what I mean you could take a concave lens.

You could take a planner convex lens, you could have all the lenses with positive at least you can they could all have the same focal length, but the shape is different, you could change thicknesses within your system. So, when I say thickness now it does not mean just the thickness of this lens you now understand it also means the distance between opticals. So, I can play around with that, glass type material that is refractive index spacing's right.

So, I can change the thickness of the lens if I am talking about thick lenses ok. And I thought you would all tell me this because we talked about this a lot over the last few weeks. You can change the top location ok.

(Refer Slide Time: 36:30)



So, I think we would not start on spherical aberration because I do not have enough time, but what we will do in tomorrow's class is start looking at these specific mono chromatic aberrations in detail. So, that in the lab tomorrow you are going to look at an optical system, look at it as aberrations and from tomorrow those other crazy curves in Oslo have to make more sense to you and then your goal will be how do you reduce the aberrations of the system ok.

So, today's class we started with the continuation of the matrix optics and we took the system that we had developed, that converted and up the ABCD matrix of any optical system to that similar to a thin lens and therefore, helped us with our analysis. And we then took that equation today and pushed it further and arrived at the imaging condition for any optical system right.

After that we started looking at the aberrations; first of all and in order to do so we have not really got to the aberrations, but we spent most of the class saying well if I want to quantify the aberrations how do I quantify them, what are the parameters in terms I will use to quantify them. And then we looked at how well I want to correct them, what are the variables, what are the tools that I have available to me to help correct the aberrations ok. So, from tomorrow's class we will look at the specific mono chromatic aberrations.

Thank you.