Optical Engineering
Prof. Shanti Bhattacharya
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 12
Principal Planes

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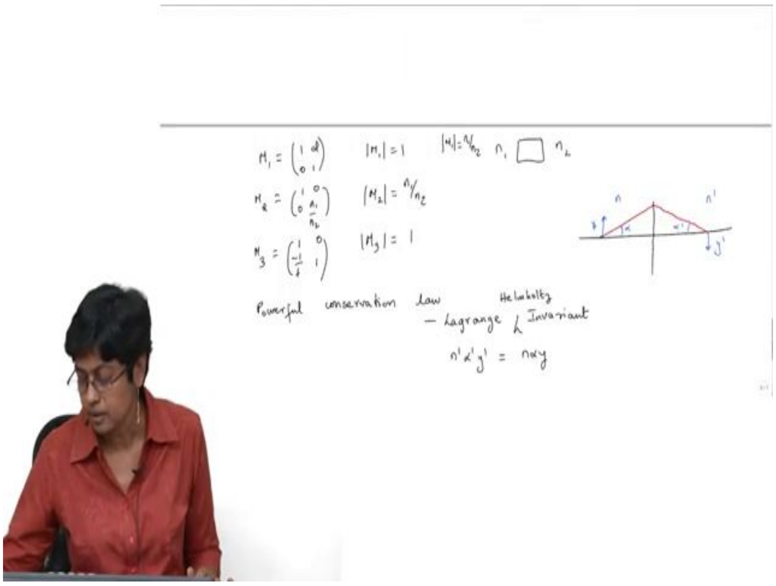

Is every system that we are going to deal with going to be a thin lens?

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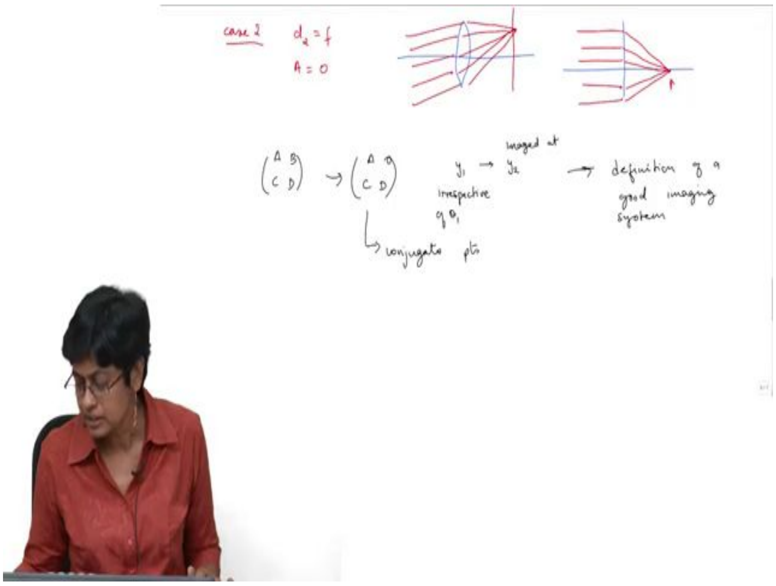

And, the answer to that is obvious that is not the case. And, even in the exercises you have done so, far in OSLO the moment you put a slight thickness to the lens, you saw some numbers changed. And, any real system, any real lens you are never going to have a lens with no thickness, there is going to be a thickness. It may be very very small compared to the other distances in your system, but it is never really 0 right.

So, even if I am trying to analyze a single lens, I cannot always neglect the thickness that is 1 thing. Secondly, many optical systems consist of several lenses, several imaging elements or several optical elements. So, is there some way I can arrive at an equation that helps me analyze these systems in a relatively simple way?

Now, why did we start with the thin lens, why did we make that assumption that we can ignore the thickness? Because it reduced the equation we had got the imaging equation we had got to a very simple equation. And, it was very easy to make lots of analysis based on that simple equation. And, all our distances were measured from the vertex of the lens right. We said focal lengths are measured from here object distance, image distance and everything was measured from there and it was very simple to analyze your system.

So, let us say we want to have a similar equation for a thin lens equation for a thick lens ok. If I could somehow have any one single equation, that allowed me to analyze a thick lens. I could analyze it in the same way I analyze a thin lens; that means, I will say consider the focal length from this point, consider the object distance from this point. If I calculate I will say the image distance is the distance measured from this point.

So, I somehow want to take the matrix that I have arrived at the cascade and matrix that I have arrived at for a system with several optical elements in it. And, convert that somehow into a matrix that represents a thin lens, it is not a thin lens; but I want to use an analysis that I was able to use with the thin lens on this system.

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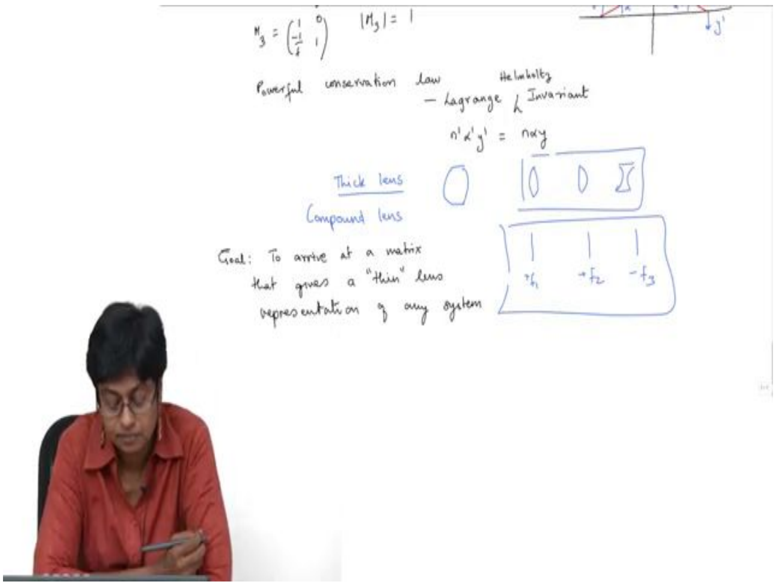

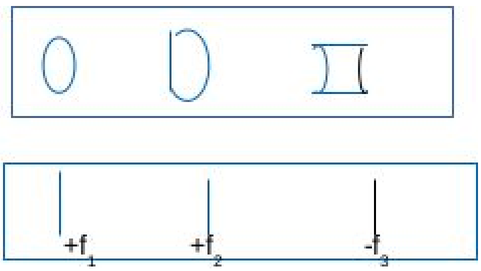

So, I have got a thick lens and a thick lens does not just mean a lens whose thickness I cannot ignore, it can also mean I have a lens with a system like this and I will say I want to consider this as one lens. The way I have drawn it here each of these lenses is also not a thin lens, because I have put some thickness there. I could not even say let me take a system which has a number of thin lenses and I will say this is $+f_1$, this is $+f_2$, this is $-f_3$. While each of these individual lenses is a thin lens together the system plus the distances again I can consider as a, what I say is a thick lens ok.

Use, so you will say thick lens or you could say compound lens. So, what I want to do, because I think it will make my job of analyzing the system easier is to arrive at. So, my goal is to arrive at a matrix that gives me a “thin” lens representation of any system. Excuse me. Now, how am I going to do this?

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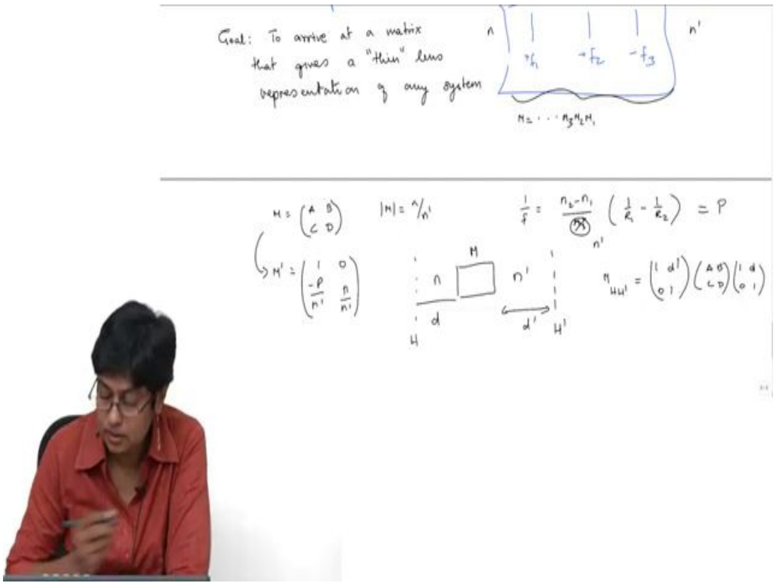


If I had a system as I have drawn. So, there is an M_1, there is an M_2, there is an M_3, and it could be any system. So, maybe I am cascading more I mean, in fact, here I have to take the distances into account. So, you will cascade more matrices, but finally, I end up with the matrix M, that matrix M represents this system.

I am now saying, let M equal this and calculate and I have this system matrix, what I know is that the determinant of this matrix is n by n dash where n is the refractive index here and n dash is the refractive index here we just discussed that right. So, that is one constraint I know and I can not violate that, what I am saying is how do I convert this to some new matrix M dash which has these coefficients. Now, I have slightly modified this to nothing, but the thin lens matrix. I have slightly modified it because as I mentioned in the beginning of the class, we derived the thin lens formulation assuming you have the same refractive index before and

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, |M| = n/n', |M'| = \begin{pmatrix} 1 & 0 \\ \frac{-p}{n'} & \frac{n}{n^1} \end{pmatrix}$$

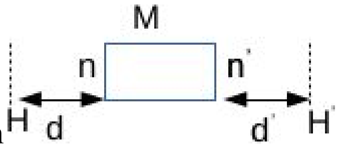
after the lens right.

$$\frac{1}{f} = \frac{n_2 - n_1}{n_1} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Now, I want to modify M and get an M', but I do not want to modify my optical system. I want to say can I somehow convert this matrix to the form of a thin lens, but I do not want to change my optical system right. I am not saying you change the system, I just want to change the way I analyze this system.

So, if that is the case, how do I convert one matrix to the other What element can I add clearly to convert it I must change something, but I cannot change it by adding another lens. Because, if I add a lens I am changing the matrix itself, I am changing the system itself. I cannot add up a curved mirror. A curved mirror will change the optical system itself. What could I possibly do? What operation can I carry out that does not change the fundamental nature of the optical system?

And, all the optical operations that we have looked at, what is the operation you could possibly do that does not change the fundamental operation of the optical system? What were the different matrices we looked at yesterday corresponding to optical operations? You can either distance right. I can add a distance that does not change the power of the system.

So, I am going to take a guess and say let me take whatever my optical system is. This is my optical system 

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it is a black box. I do not know what is in it. It has the matrix M, I am saying the refractive index n before it and the refractive index n', after it and now I am saying, let us consider that I am adding a distance d before it and a distance d' after it and I will call this plane H and this plane H'. So, I am now going to calculate the matrix that takes into account these additional distances. What will that matrix be well I will have? So, let us call that nH nH''

right, the ray first sees this distance. So, the first matrix will be $M_{HH'} = \begin{pmatrix} 1 & d' \\ 0 & 1 \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$

The next matrix will be of course, the system matrix and the next matrix will be 1 d dash 0 1 ok. Now, if you multiply this, what will you get? Let me just write it ok.

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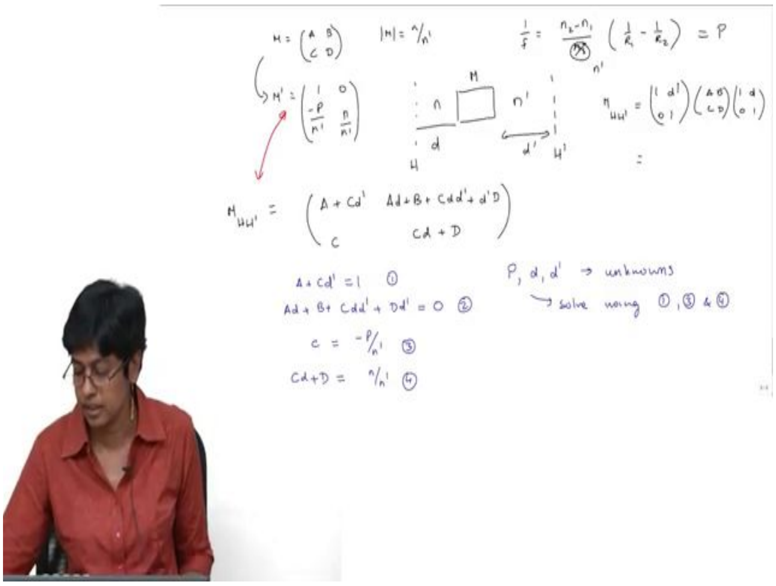

$$M_{HH'} = \begin{pmatrix} 1 & d' \\ 0 & 1 \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$$

$$M_{HH'} = \begin{pmatrix} A + cd' & Ad + B + c + d'B \\ c & Cd + D \end{pmatrix}$$

$$A + cd' = 1$$

$$Ad + B + cdd' + Dd' = 0$$

$$A + cd' = 1, \tag{1}$$

$$Ad + B + cdd' + Dd' = 0 \tag{2}$$

$$C = -\frac{\rho}{n^1} \tag{3}$$

So, my

$$Cd + D = \frac{n}{n'} \tag{4}$$

when I multiply it is finally going to give me this and that is the product of those three matrices ok. Our goal was to make this a thin lens matrix. So, now, I will equate these two matrices the coefficients or the arguments of these two matrices right. So, that will give me four equations. So, what are those four equations? I know that A plus C d dash should be equal to 1, this long expression here Ad plus B plus C d d dash plus D d dash is equal to 0, this is a conjugate matrix.

The next one is C is equal to minus P by n dash. And, finally, C d plus D is equal to n by n dash. Now, what are the unknowns of this in this case? I have three unknowns basically right. I have the power of this system. And, of course, I have added some distances. I do not know what those distances are. So, these are the unknowns. So, I have three unknowns and I have four equations.

So, I can solve for these unknowns and then I can put them back into the fourth equation and check I have not violated anything ok. So, the second equation is a rather long one. So, we would not start with that, we can solve for Pd small dD dash using this equation, this equation and this equation, so 1 3 and 4 ok. So, we are going to solve for these using 1, 3 and 4 clear up to this point.

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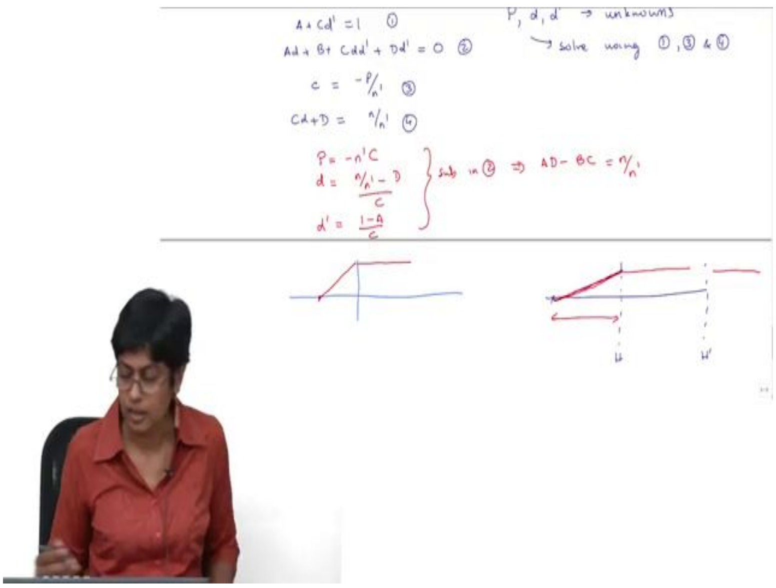

So, if you actually do that solution you will end up with these terms P is equal to minus n dash C, small d is n by n dash minus D by C, and d dash is equal to 1 minus A by C. And, if you substitute these three solutions into equation 2 you will end up with so, if you substitute in 2, you get AD oh sorry AD minus BC nothing, but the ratio of the refractive indices right. It is AD minus BC is nothing, but the determinant of the matrix right.

And, if you substitute it please do that later, you can verify that this is what you get. So, you know that you have done everything correctly. So, what have we achieved? We have found out the location of two planes. How do those two planes help us? They have tremendous benefit, now in analyzing a compound system. Why, because I can treat this compound system like a thin lens, what does that mean? It means, I can say from these planes, I will define focal lengths, I will define object image distance right.

So, if I take if we took the thin lens the way we have taken it earlier. So, we have taken a thin lens and drawn it like this, we said if this is a focal point any ray from that focal point travels to the lens and will emerge. Now, I am saying I have a compound system, it has a number of optical elements in it, but forget all that somehow based on all these elements I have defined two planes right. Let us say 1 plane is here and one plane is here right this is my plane H, this is my plane H dash.

Now, I will say if this is the focal length of this. How do I just trace a ray forget this system, I can remove it, I can erase these, that erases very nicely. I can erase these and I can then say, I will imagine that a ray traveling like this sorry I wanted to change color, a ray traveling like this will emerge parallel. Now, it does not mean that within the system that is what is happening right.

But, I will draw a ray to this point and then I could draw a ray coming out as parallel. We will see in more detail as we go along, but basically what I am doing by doing this is saying I can, I do not have to look at individual components anymore, I use these principle planes. And, in fact, I will now define a focal length from this point to this plane earlier, we defined it to the vertex of the lens right. Now, I will define it to this plane ok. So, let us do that in a little more detail.

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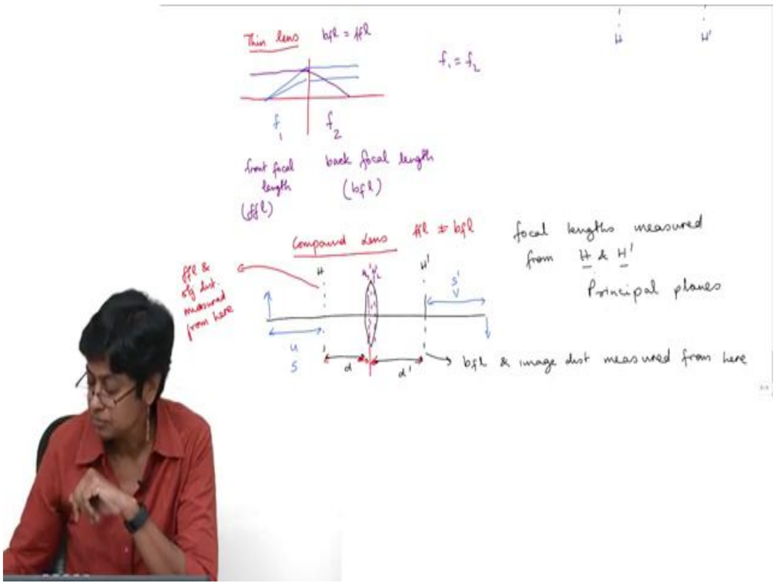

So, if I look at let us take a thin lens again ok. Yeah, he had the optical axis we had our thin lens, if we had rays coming from this point, they emerged parallel we call this f. If we had rays coming in parallel they focused to a point we call this length L. We might have said f 1 f 2, but for a thin lens f 1 is always going to be equal to f 2 ok. And, more specifically this is always called the front focal length, this is the back focal length and you may see this abbreviation.

So, for a thin lens right back focal length is always equal to front focal length. However, now in our representation of a compound lens right. Let us say, I draw. I am drawing a single lens. Here is representative of my compound lens, it may be a combination of lenses and distances, it is rippling, maybe just a thick lens that is my representation. We have now defined two planes.

So, let us say those planes are here called this H and this is H dash. I have a distance d, I have a distance d dash, I have an object distance let us say that is my object and my object distance is measured from the plane, I have an image distance and that is also measured from the plane. You know sometimes u sometimes different variables are used.

What is the focal length in this case right? What are the focal lengths in this case? Well I am going to measure focal lengths not from the vertex of the lens, but focal lengths will be measured from h and h dash and these are called the principal planes of my optical system. Soul this is h dash, so, this plane effective focal length and image distance measured from here.

And, similarly this plane is the effective focal length, an object distance measured from here. In a compound lens front focal length need not be equal to the back focal length. And, in fact, you could ask me I have made the distances d and d dash rather vaguely, this end is clear, this end and this end are clear. They are from the principal planes, where up to what point do I measure them on the set. Again, if my lens over here were a thin lens, then I would just say you will measure them from the vertex of the lens. And, since it is a thin lens, the vertex of the lens for the front and the back are the same. Otherwise, if it is actually your optical system the lenses are thick lens it itself will have that lens itself has principal planes right.

So, I might say the principal plane of this lens. If this lens is as shown and not representative of a system with many different optical elements in it, then I will say this is the h of the lens, this is the H dash of the lens. And, I would measure distances from the principal planes; planes of that lens, but we can keep it simple right now and just see d and d dash are measured from the vertices assuming that it is a thin lens. Then, I will ask you a question have you noticed in Oslo that there is a term called effective focal length.

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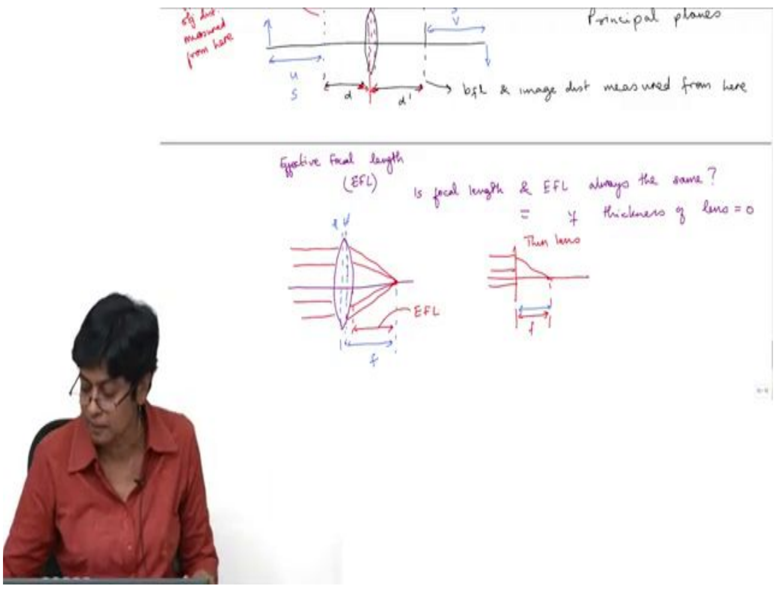

So, right on the spreadsheet the design spreadsheet in the rightmost corner has EFL right.

So, we have something called Effective Focal Length or short form EFL. Now, you have a little more knowledge about all of this. Do you have any idea what that effective focal length is? Did you play around to see how you had focal length in your last exercise. You have to design a lens to have a particular focal length, the focal length and the effective focal length always the same value. So, my question is focal length and effective focal length always the same, if the thickness of the lens is 0. So, the focal length and the effective focal length are equal if you make the thickness of the lens 0. So, go back and try that today when you are in the lab ok.

So, effective focal length is actually a definition. So, let us say I took a lens and it is not a thin lens ok. So; that means, I have rays that come if they are coming in parallel, I do not know what happens inside, but let us say that we know that they will focus to let us say there is no aberration. So, they are all focused to this point here and this is nothing ok. Now, this is where we say focal length right.

So, focal length actually now you must redefine, because how do I write the focal length of this lens? For a thin lens, I would have said it is the point rays are coming in like this, this is the optical axis, they are bending like this. So, this distance is focal length for a thin lens, but now we are no longer dealing with the thin lens. So, what is the focal point now, how will I say what is the focal point now, from the principal plane. So, this lens you will be able to calculate for this lens alone, what is the principal plane there will be two principal planes right H and H dash.

So, the focal length will be the distance called effective focal length. So, it is the distance from the second principal plane to the focal point that is the focal length. When you design your system this was actually the distance you were designing. If you took a thickness of 0 you would have been designing this distance, but in any lens which has a thickness, you are actually when you give a number as a design. So, focal length is this, the distance that you are telling Oslo to control is the distance from the principal plane to the focal point. Now, that is not always something easily measurable, because the principal plane as you see in the diagram I have drawn in this case happens to lie within the lens. So, something that is easier to measure is the distance from the vertex.

So, this is the back focal length. And, so, you will see that is why if the lens has thickness these two numbers are going to be different, because one is being measured from a principal plane, one is being measured from the vertex of the lens, in a thin lens the vertex and the planes all lie at the same from on the axis they all lie at the same point and so all the distances are measured from the same point, but; obviously, if it is a thick lens that would not be the case it is. So, the question is how do we see the principal planes, how do we?.

Student: Focal lengths (Refer Time: 25:23).

So, you will have to calculate it, you will have to calculate it for a particular system. So, Oslo will tell you so, either if you were manually calculating you can that is what we are going to do now I am going to work out an example where we calculate the location of the principal planes, but again in Oslo they are doing that calculation and if you go to one of the menu commands, it will give you the location of the principal planes ok.

So, you can do a calculation and get the principal we have done. can we just do the calculation we used, the derivation that I did just little while earlier. Finally, we arrived at expressions for P d and d dash right, small d and d dash d and d dash are nothing, but the location of the principal planes. So, you can calculate them for any system. And, how were they functions of ABCD?

So, if you are given a system matrix ABCD, you are saying for a matrix and you do not care what is inside that, but you are told this is the system matrix using that you are calculating where the principal planes for this system exist right.

So, let us do that. I have an example that I want to work out here.

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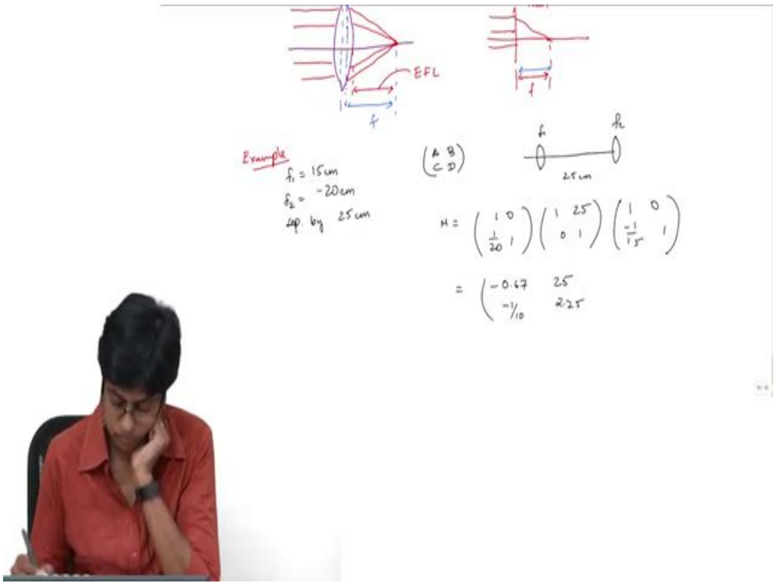

So let us say you had. So, consider you have 2 lenses, 1 which is 15 centimeter focal length another one is concave. So, it is minus 20 centimeter focal length and it is separated vertex to vertex by 25 centimeters ok. And, let us say it is surrounded by air, everything surrounded by air is ok. So, first I want you to calculate the ABCD matrix of this system ok.

So, I have basically f 1 I have a distance of 25 centimeters and then I have a lens f 2 right. So, what is the system matrix for this? So, of course, it sees the second lens later. So, it is going to be 1 over 21, it is seeing a distance of 25 and sees the first lens first so 15 that is the system matrix for this system right. I will give you the answers you can check them later.

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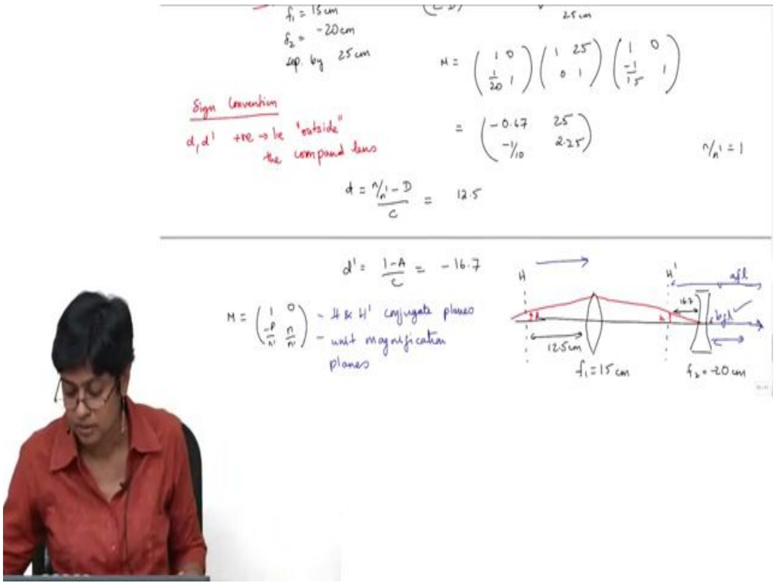

Now, we go back to that derivation we had done and we use those expressions. So, we had found out that d that was a location of the first principal plane right, d is equal to n by n dash minus capital D by C, that is what we got right. In our case by the way n by n dash is 1, because we have air everywhere. So, that ratio is 1. So, this works out to 12.5 d dash was 1 minus A by C and this works out to minus 16.7.

So, now again we have to be careful with our sign convention ok. There are as usual in optics many different sign conventions. So, you have to be very clear about which one you are using or if you are reading up something somewhere you have to be clear about which sign convention they are using. If the distances are so, we are going to use the sign convention we are going to use, if d and d dash are positive right, we say they lie outside and we will see what I mean by this outside the compound lens ok.

So, clearly if they are negative they lie within the compound lens. In this case one of these distances is positive; one of these distances is negative. Therefore, one of them lies outside the compound lens; one of them lies inside the compound lens ok. Now, what exactly is the picture we are going to draw over here I will start with my optical axis. I am always so pleased when I get anything that looks somewhat like a straight line. Then, I have a convex lens right this is f 1 is 15 centimeters not exactly drawing these things to scale ok. And, then I have a concave lens here f 2 is equal to what it was 20 centimeters minus 20 centimeters all right.

What are the distances we have calculated so we have 12.5? So, I am going to say it is 12.5 this is the location of H, this distance is 12.5, again if I considered f 1 to be a thin lens I will just say it is from the vertex. If, however, f 1 itself was a thick lens this distance of 12.5 would be from the first principal plane of the lens itself, that lens would have its principal planes.

Right now I am going to just draw it from the vertex ok. The other distance d dash is negative; that means, by our sign convention it lies within the compound lens and so, I am going to say it lies over here 16.7 units away or everything is in centimeter. So, this is centimeters away again either from the vertex if f 2 is being considered to be a thin lens or from the second principal plane of f 2, if f 2 is a thick lens is that clear right. So, this is my H dash.

This entire derivation said that we have got the matrix in this form right. So, it is a conjugate matrix, because that second coefficient B coefficient is 0. So, in other words we are saying H and H dash are conjugate planes. And, another interesting thing is they are unit magnification planes; that means, if I was tracing a ray let us say I have some ray going through the system like this bends over here it is going through the system like this, the height at this plane. Let us call it small h and the height at this plane would be equal. So, there is no magnification when you compare the heights of these two planes, they are unit magnification planes, this is my back focal length, this is the effective focal length right.

So, they can be very different and clearly you can see in a system just two lenses there will be a very big difference, when you were doing the exercise last week if you had noticed, when you kept the thickness of the lens 0, back focal length and effect or focal length and effective focal length were the same. If you increase the thickness and you might not have increased it very much, I do not remember what the exercise is called for, but maybe you give the lens a thickness of 2 millimeters or 3 millimeters, you would have seen a small difference between back focal length and effective focal length.

But, in a system with compound lenses, you could see that the back focal length and effective focal length could have a very large difference between them right, because the principal plane lies well within the system ok. We will so, in today's class we have just gone over some of the ideas of using matrix ray tracing methods and from those ideas developed a matrix that allows you to analyze any optical system in terms of a thin lens; that means, making the analysis simpler ok.

In doing so, we defined some very important planes called the principal planes ok. And, what we will do in the lab classes I will just cover some more points about these planes, which will allow you to do the lab class, today which is on designing and studying systems, which are thick lens systems ok. We talked and a few classes back about numerical aperture and we showed you some figures to say how we define it and we said n sin alpha.

So, where would you think of I mean you might ask if my lens itself has to be a material of a different refractive index. So, if I have a lens clearly I am choosing something other than air, but that is not the only place I change refractive index. If you go to numerical aperture since you asked I am just bringing this up this is an aside.

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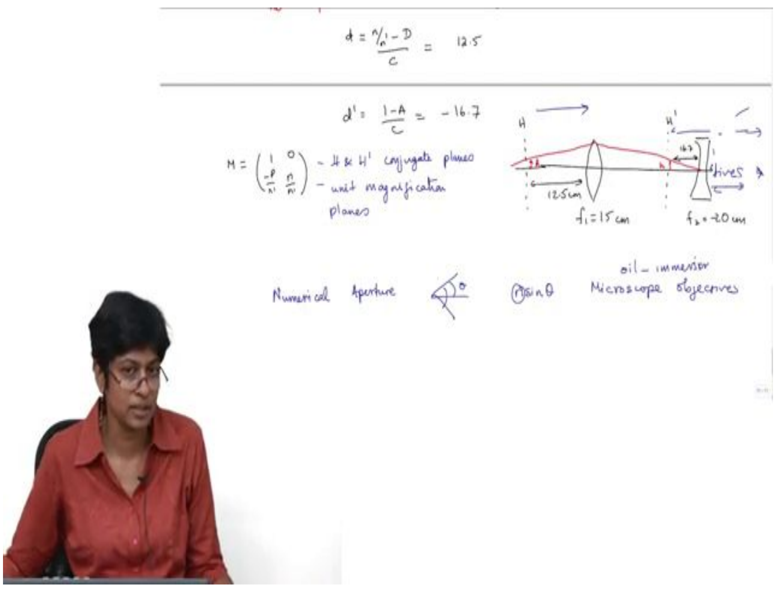

If you go to the numerical aperture of a system remember numerical aperture related to how much you collected light right?

So, if you look at the numerical aperture, it was this angle coming from an axial object point n sin theta, where theta was this angle. You could increase the amount of light you collected by changing n. And, so, you actually can get microscope objectives, they are the optics which are right close to the sample they are collecting light from the sample and you can get oil which are called oil immersion microscope objectives.

So, they actually have oil in this region before their collecting lens. So, that the refractive index is high and the amount of light that is being collected is high. So, that is one way of increasing how much light you collect ok? So, while in our cases in the kind of systems we are designing here, normally we will see this air surrounding right and in many cases the image the object space and image space have the same refractive index, that need not always be true.