Optical Engineering
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Lecture – 11
Ray Tracing Matrix – Part2

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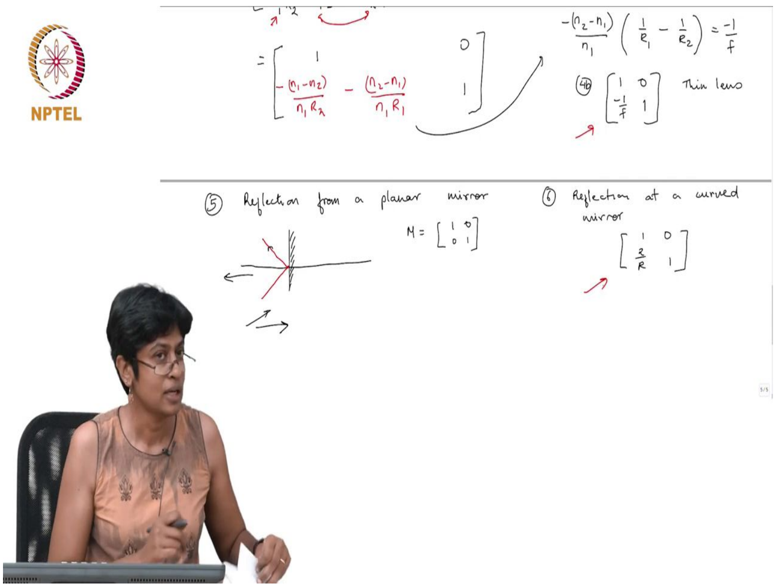
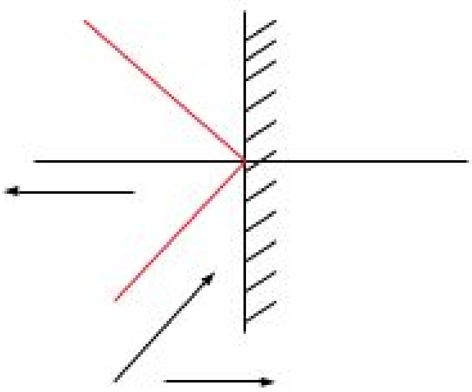


Reflection from plannar mirror



So, the next thing would be to look at reflection from a mirror. So, if you have a planar mirror optical access, incident light reflected light right; I am just going to give you the matrix now, which is $M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, height of course, does not change and the angle is also the same. You might think there might be some sign convention to worry about; but keep in mind that, this is defined saying if the general travel of the rays in one direction, we consider this as the general travel that is the optical axis in this case, for the reflected ray this is the optical axis, ok.

So, always take the general direction of the ray it took after, ok. So, that is my ray map A B C D matrix for a planar mirror. You could, of course, carry out a similar operation as we did for refraction at a curved surface and work out for reflection at a curved surface, right. And it should be no surprise to you that if you do that you will get. So, this is the matrix for reflection at a curved mirror, it is going to be $\begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix}$ where R is the radius of curvature of the mirror.

Now given you an exercise to do something with the beginning and I am very optimistic that you all have done it by now. Work out all the equations for a mirror, right. We have been working out in the class, imaging conditions for a lens and so on and I had asked you to repeat that whole exercise for the mirror. So, if you have done that, you will know that the focal length of a mirror is given by 2/R; I am sorry the power of a mirror is given by 2/R, right. So, it is not surprising that this matrix and this is really very similar, right.

The c term, the term that changes the angle is the only term of which has a value other than 1 or 0, right. So we looked at some standard matrices, we have also looked at how you would trace a ray through a system by cascading those matrices. And you can now see this is very powerful, very easy to do calculation, to use a tool like MATLAB for example, and just give the A B C D parameters for an optical system with any number of elements in it.

And you just need to take into account the matrix for every element; keeping in mind the travel through homogeneous medium requires an element to be treated as an element. So, let us have sometimes, let us do an exercise which uses what you have learnt. And see it is not just that you can trace rays with this method, but you can also learn things about the system, you can use information; the fact that the A or the B or the C or the D parameter has a certain value, a specific value or meets a certain condition.

Means, you can make a guess or not a guess, you can estimate or you can make some knowledgeable informed decision about what is happening to the rays in the system, ok. So, the exercise I want you to carry out is to find the cascaded matrix for this system.

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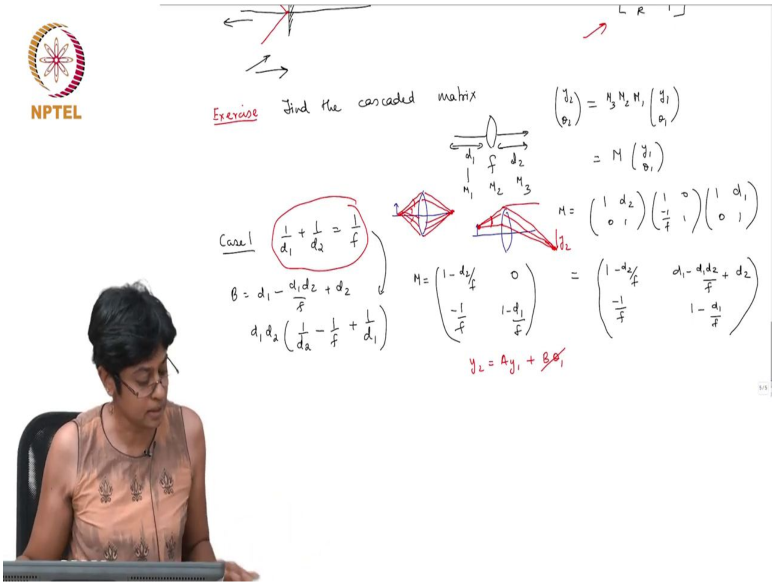

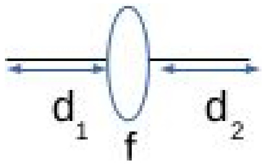

So, find the cascaded matrix for a ray that travels first through a distance d1, then through a lens of focal length f and then through a distance d2. So quickly do that matrix multiplication for me; keeping in mind that, if this is associated with matrix M1, this with M2, and this with M3, you are going to have to do it in this order.

$$M = \begin{pmatrix} 1 & d_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} 1 & d_1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 - \frac{d_2}{f} & d_1 - \frac{d_1 d_2}{f} + d_2 \\ \frac{1}{f} & 1 - \frac{d_2}{f} \end{pmatrix}$$

Now, let us see if we can get some further information out of this. Let us take the case where $\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{f}$ What condition is this that I have imposed? Is the imaging condition, right. You saw it as 1/u + 1/v =1/f, right. Now, let us say I say this happens to be true for the system. Now, if I could somehow apply this in this matrix that we have arrived at; can I get some information out of it?

Now, d_1 and d_2 only both appear in the term B. So, let us pull up that term B right which is

$$\frac{1}{d_1}+\frac{1}{d_2}=\frac{1}{f}$$

$$B=d_1-\frac{d_1 d_2}{f}+d_2=d_1 d_2\left(\frac{1}{d_2}-\frac{1}{f}+\frac{1}{d_1}\right) \quad B=\begin{pmatrix} 1-d_2/f & 0 \\ -1/f & 1-d_1/f \end{pmatrix}$$

How does having the B coefficient? And now forget how I arrived at this, I am asking you what is the significance of having the B coefficient as 0. What does that mean? What does it mean when you say, an optical element or an optical operation has B equal to 0? What does that mean?

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B effects, which parameter y2, right. $y_2 = Ay_1 + B\theta_1$ and now we are saying B is 0. So, in effect we are saying we are removing this term always. Other words $A_2 = Ay_1 + B\theta_1$ what does that mean?

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Irrespective of the angle coming in, the ray is going to some point, right. And we should not be surprised; because if you look at this relationship, this is the imaging relationship. What does the imaging relationship tell us? Let us draw a figure, let us say I have my optical axis a lens and I have an object and I am tracing rays let us say from the axial point.

So, I am tracing rays and if this object and I am looking at the point where imaging is happening, right. Irrespective of the angle of these rays, irrespective of θ_1, they all land up at this point. And that is what you expect for a good imaging system. All rays coming from one object point should arrive at the same image point, they should all have the same height.

So, irrespective of θ_1, I can have an infinite set of rays, they each have a different θ_1; but if this is a good imaging system they will all arrive at the same point in the image plane. I do not have to do it on an axis, I could look at this as I have an optical system and I have an object and I am looking at all the rays coming from this point.

And they all are getting let us say; let us say it is imaged here, they are all getting imaged here. So, everything from this point is imaged here, they all have the same height y 2 irrespective of the angle ok. That information I have got from looking at the coefficients or saying this particular coefficient is 0; what does it mean, ok. We can look at one more thing before we end the class and we can say let us take the case where d_2 is f.

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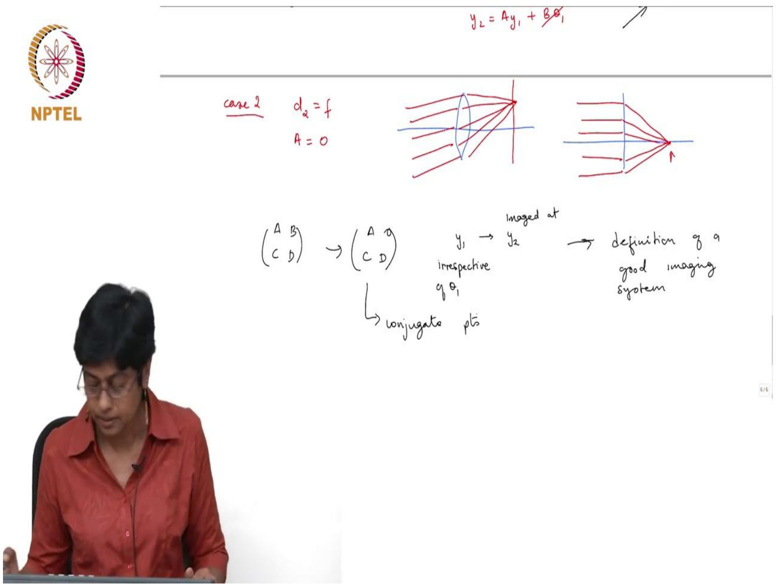
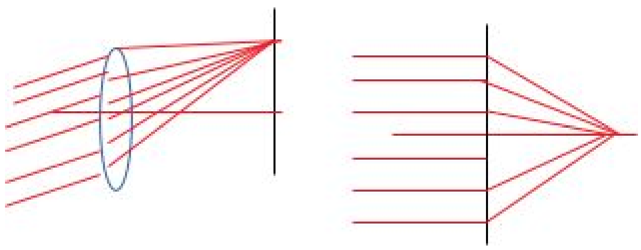


So, let us take a case; no I am not considering the first case anymore, let us say we take this case where d_2 =f, that will make coefficient A go to 0, right. Now, what does that mean? If I have an optical system, I have a bunch of rays coming in, they all have different heights; but these different heights do not matter right, they are all going to get focused at the focal distance, ok.



Maybe, I can make it clearer if I do the on axis case, where I say all of these have different heights, but they all get focused to this point. Irrespective of the height of the ray, they will all reach this point here with a different angle. So, that is why this coefficient is not zero here. So, they are all reaching this point here with a different angle; but it does not matter what height they come in.

And if I look at it what does d_2 equal to f say; you are saying the image plane is at the focal plane and or that distance d_2 is nothing, but the focal distance. And in this particular case I am looking at rays coming in parallel. So, irrespective of the angle they are all going to get focused to the focal plane, ok.

So, the idea today's class was to show you that, you can do ray tracing in a very simple manner using these matrix operations; you can arrive at a matrix for every optical operation; standard optical operations may be a combination of matrices and you need to carry that out once and get the combined matrix and henceforth apply only the combined matrix.

If you have a combined matrix for a system and you look at the places where A goes to 0 or B goes to 0 or C or D, you can learn something about the system, right. Or you can say if I apply this condition this parameter goes to 0; so under this condition I will make it work like a good imaging system or parallel rays will do this so on, right. So, it tells you a little bit of what you might need to do in order to make that optical system with that matrix behave in a certain way ok, that is all for today's class.

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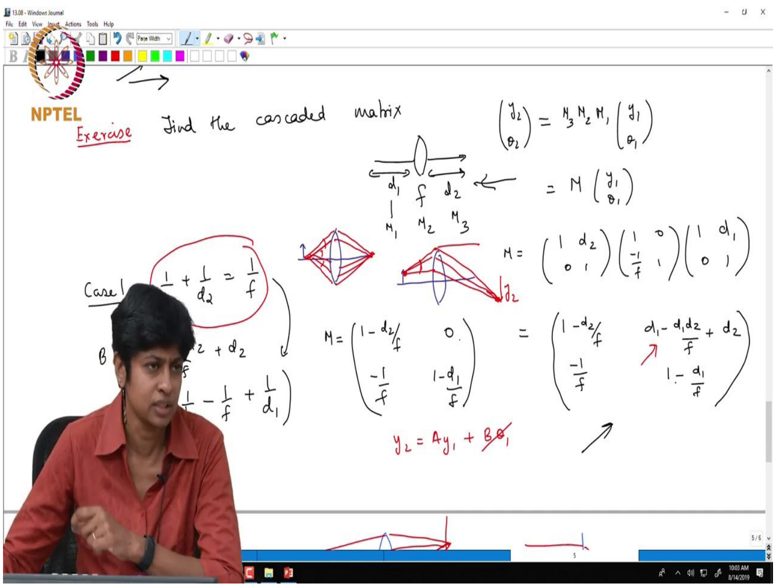

Good morning. So, in yesterday's class we looked at tracing rays using the matrix method, right. And we developed matrices for certain operations and I gave you some as exercises. We also looked at how you can cascade various operations and that in effect is the ray tracing procedure; because when you cascade matrices, you are. In fact, saying this is how the ray bends and travels through the optical system. So, we ended with a system shown here, I have gone back to yesterday's notes and you see you have a lens of the system here, you have a lens with a distance before.

And after it and we arrived at the matrix, the cascaded matrix for the system which gave us this result over here. And then we said let us apply a certain condition, we applied this condition 1 by $d_1 + d_2 = 1/f$; doing so, turning one of these coefficients to 0. And we also looked at the implication what does it mean if that coefficient is 0, right. So, I want to go back to that, because though we looked at it; what exactly it is important enough to take another look at this matrix and we said when the coefficient B is 0, what does that imply; it means that, any ray with height y_1 , this is a ray with height y_1 at the object claim, right.

It is going to get imaged at the image plane at y_2 , irrespective of the angle; that is what that B equal to 0 means. And that is the very definition of a good imaging system, right. If all rays from one object point get image to the same image point that is the definition of a good imaging system, right. So, we are seeing all rays from y_1 , irrespective of θ_1 ; θ_1 can have any value, they get imaged at y_2 , they may be coming to y_2 , with a different angle, but they are arriving at y_2 ,. And this is the definition of a good imaging system.

So, we can say that this matrix with coefficient B equal to 0 gives us the conjugate points. This matrix that we developed in the last class, was the matrix for a space, a lens, a space; and d_1 d_2 are conjugate points and we know that, now because B = 0, ok.

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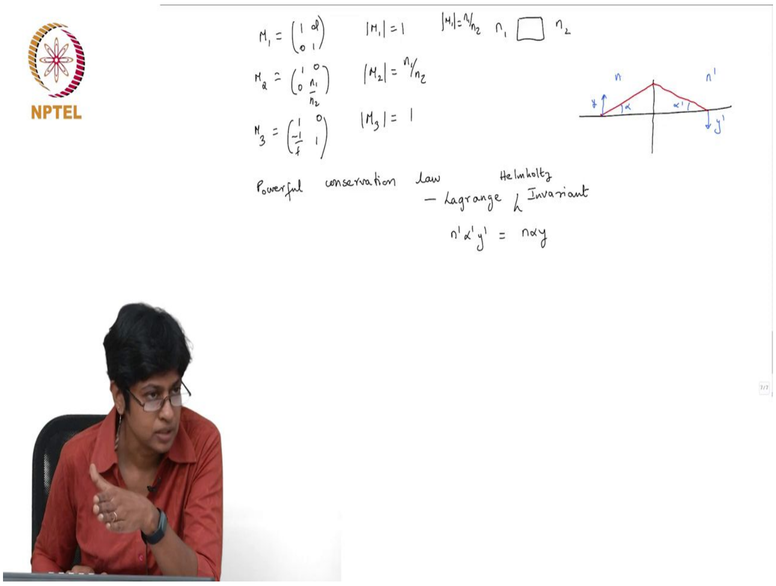


Now, another thing you might not have noticed. But, if you look at the matrices we developed yesterday, we did a matrix for travel through free space. So, if you travel through a, the ray travels through free space of distance d; this was the matrix that you got. The determinant of this matrix is 1.

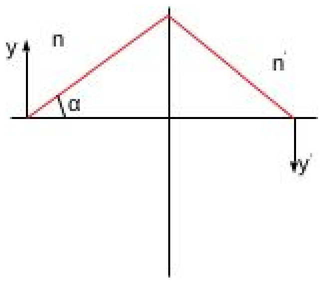
$$M_1 = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \qquad |M_1| = 1$$

$$\begin{pmatrix} 1 & 0 \\ 0 & \frac{d_1}{d_2} \end{pmatrix} \qquad |M_2| = \frac{n_1}{n_2}$$

$$\begin{pmatrix} 1 & 0 \\ \frac{-1}{d} & 1 \end{pmatrix} \qquad |M_3| = 1$$

If you did the matrix for refraction we had 1 0 0 n 1 by n_2 and the determinant of this matrix was n_1 by n_2 right. If you did the matrix for a lens we had 1 0 minus 1 by f 1. So, a thin lens of focal length of the matrix a determinant of this matrix was 1, ok. And if you go back to these derivations, wherever you have the determinant 1 we actually get that determinant; because we made the assumption that the refractive index in the starting space, in the object space and the refractive index in the image space were the same. In fact, we said it is n_1 , n_2 n_1 or air n_{1air} , right.

So, if you go back to those derivations and if you actually take your optical system whatever it is, and then say I have n 1 here and I have n_2 here. The determinant of all of these matrices will end up being n_1 by n_2, ok. So, you can go back and verify that of course, with the refraction at a surface because you have those two; you cannot say the same medium, because then you would not have any refraction. So, there the odd result comes automatically; but in other cases if you do not pick the same medium for the object, in the image space, you will end up with a determinant which is a ratio of these two refractive indices.



And this leads to a very powerful conservation law in optical systems. We are not going to go into the details of it, you can derive this law using matrix methods and maybe I will give you an exercise in which you do that. But I want to mention the law, because I want you to be familiar with it, ok. So, it gives rise to a powerful convention: lot has several names, sorry conservation law, it has several names; it is very popularly known as the Lagrange invariant or sometimes it is called the Lagrange Helmholtz invariant, ok.

And if you use matrix techniques and this fact that, the determinant always works out to be the ratio of the starting and the ending refractive indices. This law is going to look like this. So, invariant means something stays constant and here it is as a ray travels through the system something stays constant, ok. And the constant turns out to be the product of these parameters, let me make it clear what I mean by these parameters.

So, again I have a lens I am going to draw. So, I am drawing it as a line and I have a ray traveling like this; this is an axial ray, I have an object, this is y, this is α, I have an image; actually means, this is y dash, this is α, dash this is n dash this is n. So, what is this constant is the; product of the refractive index in that space, the angle that the axial ray makes with the optical axis and the dimension of the field at that point; that stays the same throughout the optical system ok.

So, you can derive this using matrix methods, where do you think this is going to be useful. So, you think about when we wanted to calculate magnification of a system. To calculate magnification we always needed to find out the height odd as we took the ratio of the heights and objected to an image right. So, you always have to find out location of the image, height of the image, sorry where it is and derive these parameters to get to magnification.

Here by tracing a single ray, I am able to get some information about the optical system; just because there is a constant, I say if I know something about this ray somewhere without having to figure out it is. Something in object space I am able to use this constant and we find out parameters in image space, ok. So, I will try to give you some exercises. So, you see the power of this law, but just be familiar with it, you might give and see it in OSLO; they will give you some number saying this is the Lagrangian variant of your optical system, ok. So, I just want you to be familiar with this.