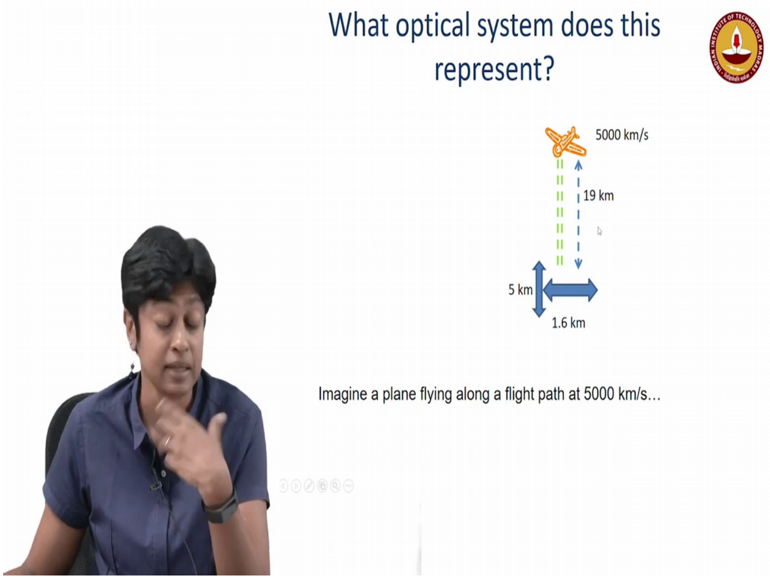
Optical Engineering
Prof. Shanti Bhattacharya
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture – 01
Introduction to Optical Engineering

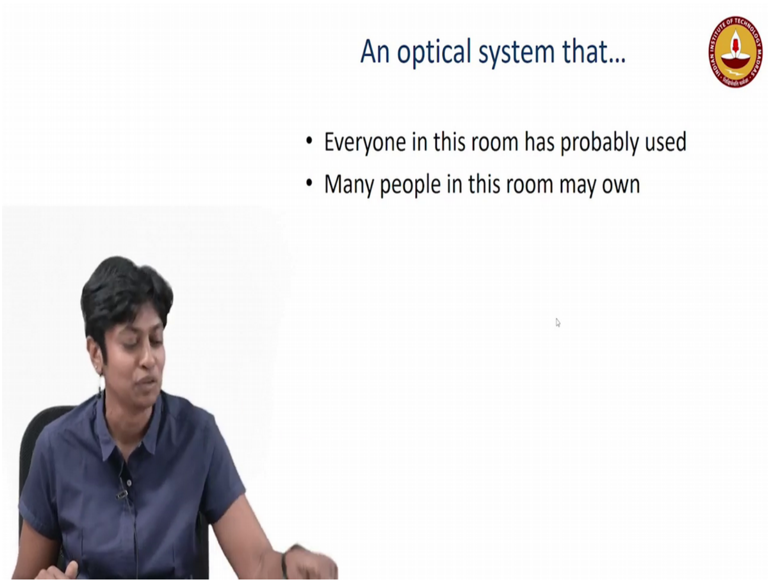
So, good afternoon, welcome to this course on Optical Engineering. This is a course offered for M Tech students and UG students, and we will see why it is important to learn the subject for people who do not have our basic optics degree. But, before I go into any specifics of the course, I want to start with a question, ok. I have a system here, an optical system here that I am representing through an analogy, ok. The analogy is that you have something in this system that is moving at a fairly high velocity and it has to move very accurately so, the numbers that are given here for the height.

(Refer Slide Time: 01:01)



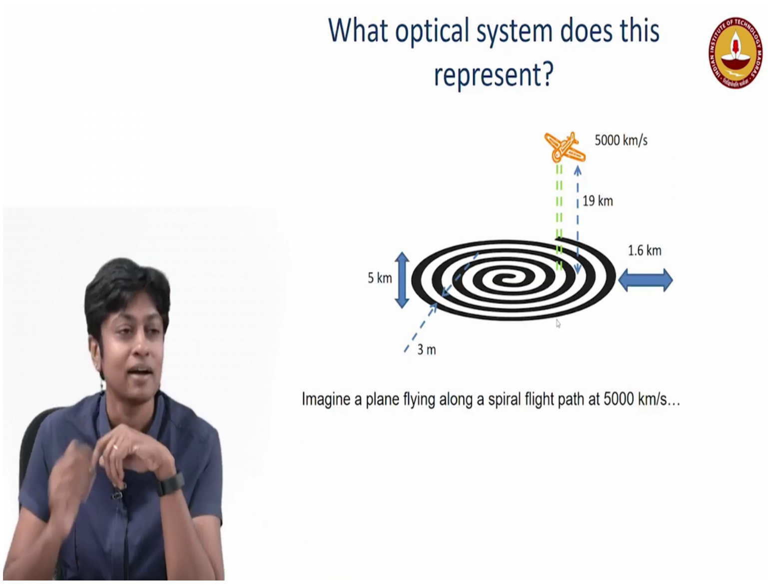
So, I am in my analogy something is moving, and in my analogy, I am calling that an aeroplane, it is moving with the velocity of 5000 kilometers per second. I have given numbers for the height it is above the ground, and these blue arrows 1.6 kilometers horizontal and 5 kilometers vertical indicate the accuracy, with which it has to maintain this height and keeping in mind it is moving at this velocity. Can you tell me what you think this optical system is, it is a system, not an element, not a device. It is a fully system.

(Refer Slide Time: 01:43)



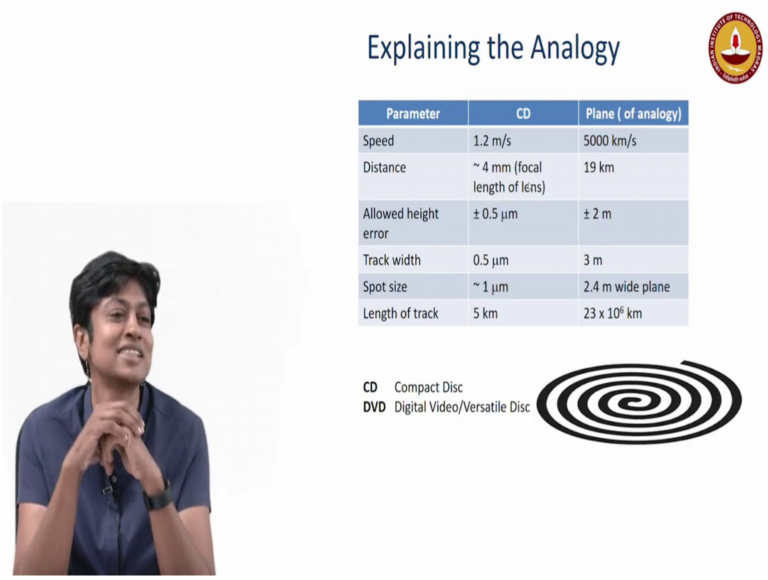
I know you have at some point of time used it, I should add that this system is already a little bit old technology, we have moved on to newer technology. So, it is not something that you would be using currently, but I am very sure you have used it, you might even own it. So, it is not I am not talking about a technology that is far away from everyday life, and it is not some complicated, sophisticated technology.

(Refer Slide Time: 02:17)



Give you one more clue, the path that is moving, the path along which the object is moving is not a straight or a linear path, but has this spiral nature as shown in this figure here, maybe this figure gives you a clue as to what I am talking about you know.

(Refer Slide Time: 02:40)



So, I am talking about a CD system; a CD player, a DVD player. I am sure you have one on your laptop, right.

Student: I didn’t checked there mam, that’s why.

So, how do I read? So, the question is what how is a CD or DVD similar to this analogy of an aeroplane moving at this velocity? How does one read or get information from a CD or a DVD player? Now, the system has several parts to it, one it has the disk which contains the data that is to be read and of course, there is the, what we call the pickup head which reads the data and I could go a step further and say I can also write with my player. So, I could have a head that is used for writing on a CD.

Let us stick with the first part. We have a disc which has information on it, and we want to read information. How is that information read? The pickup head has a source of light in it, and it sends the light onto the disc; the light reflects of the disc, and somehow in that reflection, information has been transferred. So, that reflected light now carries the information. How does it translate to this analogy? Well, the disc is not stationary, the disc is spinning, and it is moving at a certain velocity there are tracks.

So, you if you look so, often people get the clue to get the answer by looking at this because this gives them the idea of a CD there are tracks. And, as you are reading information from one track, you, the beam of light should not accidentally move on to the next track because then you would be getting the wrong information.

So, the disc is spinning at a certain velocity and at that velocity the beam of light has to accurately stay on this track. Now, I am using the word CD or DVD interchangeably over here, but there is a difference between the two of them; what is the obvious difference between a CD and the DVD? So, the way in which data stored is one difference, but if I had to say the prime difference is the amount of data that is stored, right a DVD has a lot more information.

Now, that might be partly because the coding techniques have involved it have evolved, but it is also to do with the optics and the technology the hardware of the system. So, the tracks in a DVD are spaced much closer. So, the tolerance what the error in how much the beam can move is much tighter and that is where the analogy comes, if you ramp up these numbers and here in the second column you have the numbers for the CD, if you ramp up the numbers you can get to the numbers of the analogy of the plane.

So, it is moving at a linear velocity of 1.2 meters per second; there is a lens that focuses this beam of light to a spot that is incident on the track that is being read. All this time this disc is spinning, but the spot has to stay correctly on that track and if there is an error and there will be some error that error is of course, only allow given up to the spacing between the tracks.

If the focus changes, the focal position changes; the beam is focusing on this layer, but if the focal position changes, you go a little too high or below, then the reflected light does not go back with enough power. So, again you are going to lose data. So, if you ramp up these numbers of the CD then you get to the analogy of the plane, and if I actually unspiral CD that track of the CD, a CD would have 5-kilometer length of data and it is much more for the DVD, ok; so, this is how the analogy comes about.

Now, many times when I ask this question to students, they do not come up and say the answer is this and sometimes it is because as you had mentioned that maybe the analogy of the plane is not clear, but for many students, they do not think of this as an optical device.

So, when I say give me the name of an optical system, you do not think of a CD player as an optical system, right. And, this is I do not know whether I should say unfortunately or not, but this is the role optics place, it plays a very important role in many different systems and systems and devices that we use today, but it is always playing part of the role it is very rarely that you have a purely optical system in today's world.

Now, our world would be completely different today if we took optics out of it, because if I took optics out of it I would remove the entire fiber-optic network, and that would change everything, right. Again, people use the internet, everyone is on the world-wide-web, but very few people are thinking because of optics this is possible, right.

So, this is really one of the reasons why this course is so important, because optics is there everywhere subtly sitting there in many devices. Some of them may be purely entertainment, and you might some people might say unimportant; we can argue about that, but it is there in many systems that are very important crucial lifesaving, ok.

And, I will come I like the analogy of the CD. I like talking about the CD player because I am in an optical engineer, I will pull up the CD player and say this is an optical system, but I know it is not purely an optical system. You would not have a CD or a DVD player, if you did not have good developments in mechanical engineering. The whole system had to be shrunk down made small; a CD player do you know how many motors it has in it?

Student: 3.

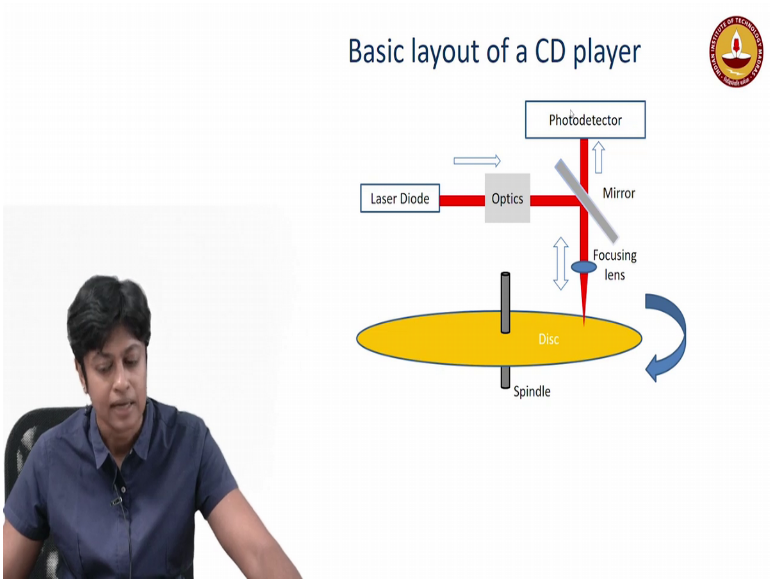
It has 3 motors, yes, because you have 1 that slides out the tray onto which you put the device it has one that spins the CD, and it has one that moves the head up and down. So, you have 3 motors; now, those motors have to be extremely small; they cannot send out a lot of noise they cannot send out a lot of heat. So, there is a lot of development that went into designing these very small, very efficient motors. So, there is developments in that technology there is developments in mechanical engineering,

The CD itself you have to be able to write on to the CD, it has to last through the time you need to have CDs that are writable CDs. So, there is a lot of material technology that has gone into it is a whole lot of electronics, and we are talking about using light. Now, the original lasers are well big devices, now we use semiconductor lasers. So, that entire technology had to develop to build semiconductor lasers and to make them cheap when manufacturing in large numbers.

So, there are many different fields that have gone into it, and a CD has this ability that if part of the CD is damaged or scratched, it does not immediately mean you have lost all your data. So, there is a lot of redundancy in the way the data is put there. So, it is a lot of work that is gone in the algorithms that convert data into data that is stored on the CD. So, all these fields were necessary to make this compact device.

Now, while today's world, we may not be using CDs or DVDs all that much anymore, a lot of that technology continues in various other optical systems that are relevant today. So, it is a very nice system to study from an engineer, and it does not matter whether you are coming from a chemical engineering background or mechanical engineering background and electrical engineering background or you are coming from it as a physicist, right. So, that is why I like that system.

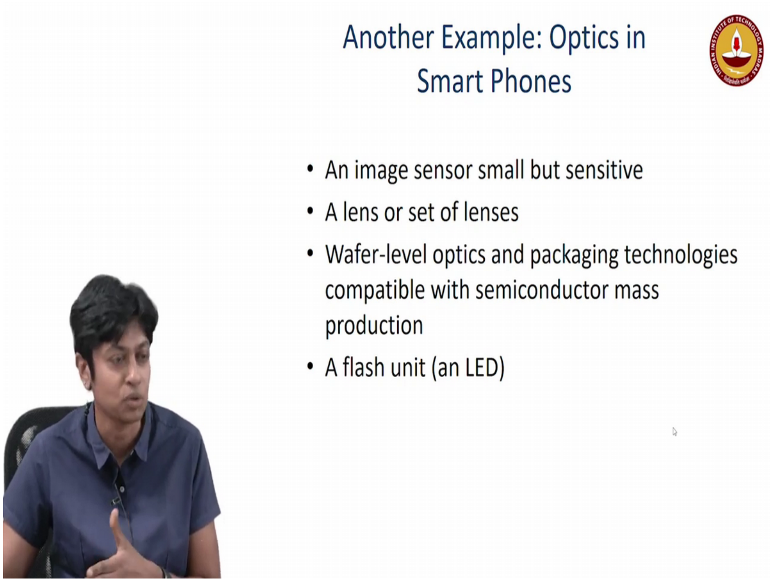
(Refer Slide Time: 11:27)



But having said that, I can come to a more modern system this is; however, just an image of the CD player just to make it clear to you. So, this is what I was talking about you have to have a compact light source, the laser diode you need optics that redirects maybe reshape this light sends it on to the disk which is spinning and then a photo the reflected light.

And later on in this course, you will see how information is transferred from the disc to the reflected light, and it is not just reflection there is something else that is happening there. And of course, that is detected by a photodetector, and all the processing happens finally, on an electrical signal, not an optical signal, and those arrows just show you the path of light the onward and then the return path of light, ok.

(Refer Slide Time: 12:19)



Now, coming to something that is more relevant in today's world the smartphone, every-body has a smartphone if not a couple of smartphones right Where are the optics in a smartphone?

Student: Camera.

Camera, ok. In fact, many of the new phones have 2 cameras, 3 cameras, a front camera 2 reverse cameras, what else.

Student: Screen LCD screen.

The LCD scream yes, screen yes, oh you mean yes, there is a sensor well that might not always be optical. The, so, you are talking about the sensors when you raise the phone to answer a call, it switches off deactivates the screen that need not always be optical that may not be optical. In fact, it may be a capacitive sensor that is not typically optical.

You have the flashlight, the flash that is used, and of course, what is also important is you the camera has all it is lenses. So, they have to be fairly good quality; you can get very good images from a smartphone camera. And, you need this is not the wafer-level optics and packaging there is a lot of optics that have gone into the machines and systems that are used to fabricate and assemble the smartphone. So, it is not on the phone directly, but without that, you would not be able to have your phone manufactured.

So, there is a lot of optics that go into a smartphone, and again if you typically ask somebody, is this an optical device. The first answer might be no; I would not think of this as an optical I think of it maybe it is a communication device, you would not think of it as an optical device.

But, today's world, the phone is used nearly for everything other than having an audio phone call, right and if you took all the optics out of your phone. I am sure most people today would not be happy with that device, they would say give me back my phone with all those other utilities in it, and most of those utilities are using optics in some way, ok.

So, I am giving these 2 examples, the DVD player and the smartphone, just to show you the importance of optics, right. Now, if you are watching this course or sitting in this class, it means you already had some interest or some idea that there is a need to learn optics. So, maybe I do not need to convince you, right.

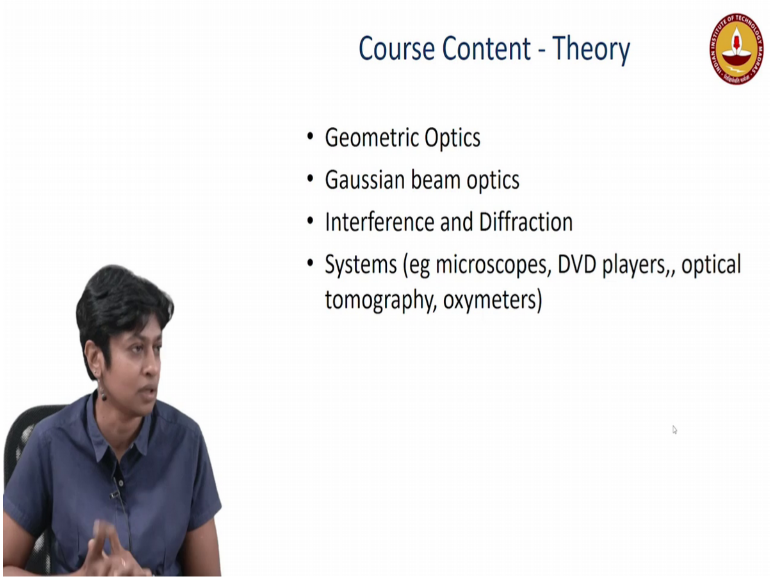
But, as we go long in the course, you might ask yourself what is the need to learn all of this or what is the need to learn it in this detail. And, you can see in the 2 examples that I have given if you knew only optics, you would not be able to come up with these systems. In fact, if you knew anyone subject alone, no matter how well you knew it, you would not be able to develop these systems. Both these examples are very interdisciplinary in nature, and that is where you will find optics, you will find optics in systems and in experiments which are very interdisciplinary in nature.

So, why do you need to learn optics, you might say well I am an electrical engineer, I am a mechanical engineer, I will work with somebody who has this skill let that person to the optics I will do my bit and we will do the work together. How do you communicate with somebody who offers different skill? Learning the content of this course will allow you at least to understand what that person is saying or for you to communicate your

Needs in your field, but in a better way you your expectations will be more realistic because you understand what is possible in optics.

So, the communication would become better, and hopefully, that means, the collaboration will become better, so, that is why it is important to learn optics.

(Refer Slide Time: 16:17)



And, what do we do in this course this course actually has only a few lecture hours a week and a lab associated with it. So, we do not have a lot of time to do theory. So, we restrict ourselves to some very basic theory, which is more than adequate to explain a lot of the systems and to explain a lot of the optics in many applications that we use today.

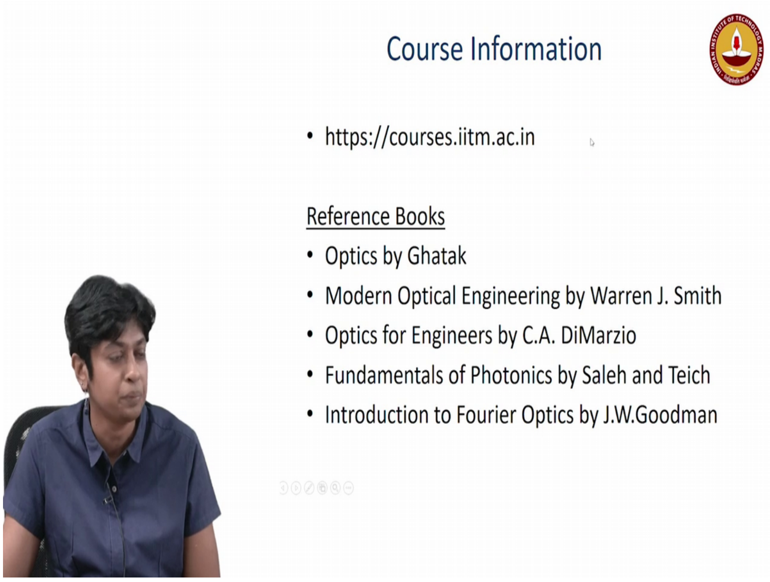
We will start with geometric, or it is also known as ray optics we will then go to Gaussian beam optics. So, the Gaussian beam is a kind of beam that typically emerges from a laser, and we will see how this is different from the concept of geometric optics. We will move from geometric or ray optics to the wave picture. We know that light can be observed or studied in different ways, thought of as having different behavior.

So, in the ray picture, we do not consider the wave nature of the light at all, we will move to the wave nature of the light when we study interference and diffraction. And then we will spend the end of the course looking at different systems, and I have put down a list here, it is not a comprehensive, we may do more than this we may do some extra things, ok.

(Refer Slide Time: 17:48)

In the lab part of the course, we will spend a good part of the course learning optical design software, it is called OSLO; Optics Software for Layout and Optimization. So, you will learn how to design lenses and mirrors to carry out a certain function. We will then do simulations using the software Matlab or Python for carrying out interference and diffraction experiments. Then we will do some actual hands-on experiments again in interference and diffraction, ok. So, this is the content of the lab.

(Refer Slide Time: 18:29)

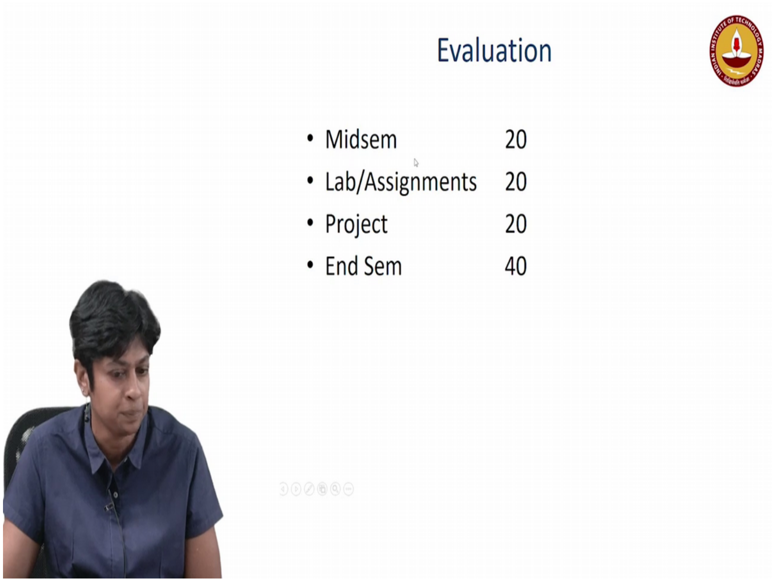


All the information required for this course will be uploaded in models so, you should be registered on that. I am running 2 courses in parallel here. So, if you are in the UG course, send me an email, and I will ensure you are registered in the other course because I will upload all the information in one course.

And, we do not have any one book specifically for this course so, as and when necessary I will upload material, the lecture notes will also be available, but the books that we will be using are optics by I will be using as reference a primarily the first three books, which are all optics or optical engineering books. The latter two are meant more for reference and just for one or two topics.

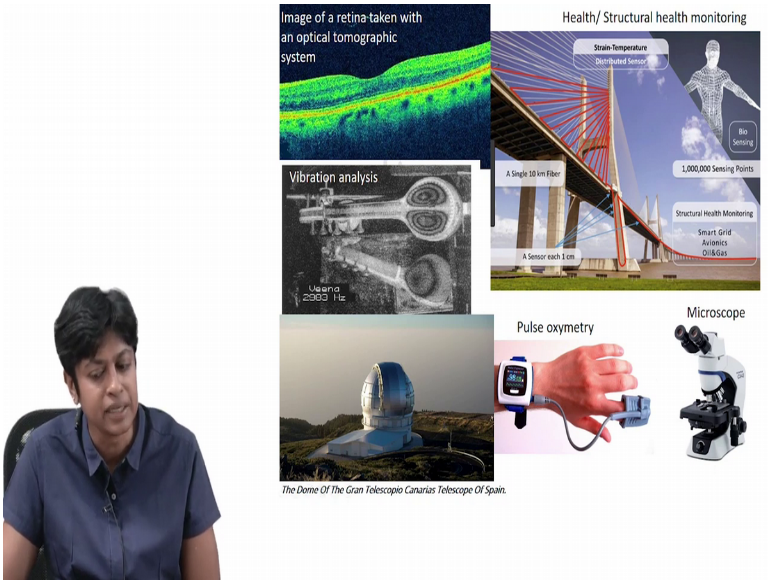
You are not restricted to using these books; any basic book on optics will cover a good part of the first part of the course or any book on lens design, ok. Now, as and when required, I will also give you suitable reference material during the course, ok.

(Refer Slide Time: 19:41)



The evaluation scheme for this course is we will have one mid-semester exam for 20 marks. There is a lab, and there will be assignments, and together that will be 20 marks. You will have a mini project which I will talk about in later in greater detail later on. So, that will also be 20 marks and then an end-semester exam for 40 marks, ok.

(Refer Slide Time: 20:09)



Already answered this question why study optics as a separate subject, but I just wanted to show you some more applications. So, this slide has some images of various applications, this top picture has image of a retina taken with an optical tomographic system.

So, if you go into any fairly big hospital today, at least in a big city or town, they will have what is called an OCT system, it is a non-destructive way of studying your eye, which is a good thing you do not want to destroy your eye. And you can see you can very nicely see the different layers of the retina. It is used as a diagnostic tool. So, if there is something wrong in that retina doctor can see where something is wrong and then maybe also figure out what is wrong.

Below is a picture of a veena, and it has been set into vibration so, some string of the veena has been plucked. And, what you see are interference patterns so, this is the way of nature of light being used. And these interference patterns would change depending upon which string of the veena was plucked.

Now, this result is was pulled out of a study that was done on Indian musical instruments, there are lot of studies that have been done on western musical instruments and how they vibrate, what the frequency content is and so on. So, this was a similar study being carried out for Indian musical instruments; in fact, this 1 was done by somebody in the physics department here.

The point to notice is the veena is a fairly large instrument about so big and this entire instrument is being studied at one go, and this technique this vibration analysis technique is actually used to study very large surfaces and to get information about these surfaces simultaneously. So, a good example is people use it to study an aeroplane wing, you know they have to do very rigorous tests on such devices or I should not call it a device on such parts of the aeroplane because you need to continually monitor whether there is any damage. So, you have to monitor before something happens, well before something happens.

Now, how do you monitor such a large device, if you have some an system that scans, it would take a very long time to scan an entire aeroplane wing or surface of the entire aeroplane. You want to quickly look at it and then narrow down on the point where there seems to be an issue, and if there was an issue, if there was a problem, if there was a defect this fringe pattern would have some strange behavior, and then you can narrow down and study that in greater detail.

So, it would make it quickly tells them a place where there is a potential problem, and the aero plane business as you know they do not want the planes down for a long period of time it needs to be up and running for as long as possible. So, the downtimes even just for checking, has to be shortened. So, having a method by which to check a large area quickly makes it more economical for them, ok.

I have also put this telescope here because that is one of the standard applications people always think of, as is the microscope. The bridge over here is put because you can lay fiber optic cables which have gratings in them. You send light down this and the gratings will reflect one wavelength, and the wavelength that is reflected changes depending on the stress, and strain that the grating sees which depends on the stress and strain that the bridge is seeing.

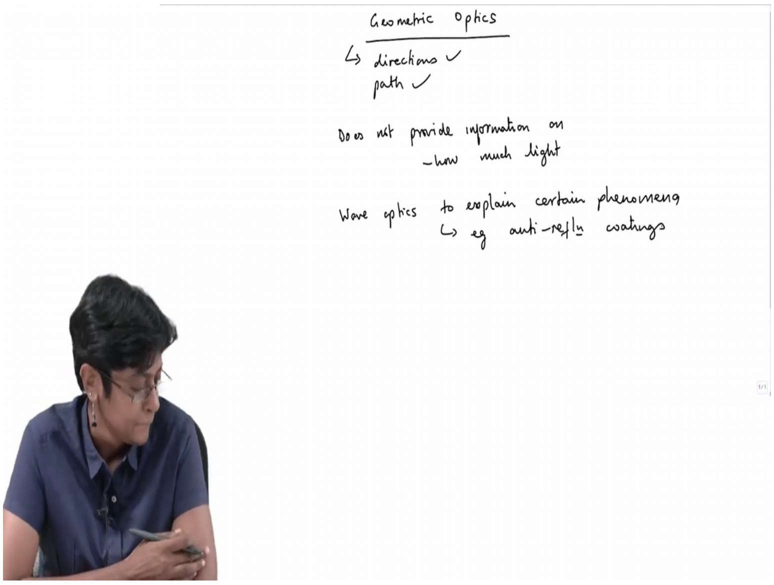
So, again you are monitoring a very large system you can monitor the building, you could monitor a bridge, a railway track you can monitor these very long very large structures and quickly find out if there is something wrong and fix it before a disaster happens and this is called structural health monitoring, ok. So, there is a lot of research going on in that.

This particular example uses fiber optics, but for the most part of this course, we will actually discuss free-space optics. The example in the middle down here is a pulse oximeter. So, it is clipped onto a person's hand you might have seen it if you have visited someone in a hospital or if you watch movies there is always someone lying in bed with this clip on their hand, and it measures the pulse rate as well as the oxygen level in the blood. Again, you might not look at it think this is an optical device, but it is an optical device, and we will cover that in the model in this course.

(Refer Slide Time: 25:21)

So, this is just a snapshot of a few of the places optics gets used I have listed out here a more comprehensive list. Again, just to emphasize the importance of optics and how much it affects us, our impacts us in our everyday life, although we might be unaware of it ok. Any questions till now? No, ok. So, I am going to switch to the next part we will start looking at geometric optics.

(Refer Slide Time: 25:51)



Now, geometric optics was well understood even 400 years ago, People knew that if they used a certain surface or a transparent material of a certain shape, they could change the way light traveled. What was not understood was why that happened, that understanding came much later right, but mirrors have been found in archaeological digs from many centuries ago.

So, people had shaped and fashioned mirrors and used them we know microscopes and telescopes were invented again centuries ago and have been used. So, the lenses were made, the why of these elements came much later. People understood the first understanding came with certain laws they again they did not understand why these laws were true, but they found out these laws were true, and they were able to use these laws and, therefore, fashion these devices.

So, I am going to start by listing out some of the postulates of geometric optics ok, no proofs for them; proofs were are come arrived at later on through other means, but when these were used there were no proofs for them. The good thing or the nice thing about geometric optics is that it works for a lot of devices it is a simple way of tracing light that is basically what we are doing, we are going to trace light that is the first step.

The second step we are going to say is what element should I have such that light travels a certain part and carries out a certain function for me. And, keeping it very simple keeping the math's very minimum I am able to actually develop fairly sophisticated systems.

And so, we use geometric optics even though we do not need their full understanding for it because it works.

So, what is geometric optics not explain it can explain things like how light bends. So, I and I know if I put a mirror, I know where the reflected light will go, I know the direction it is going to take. So, it will tell me things like directions, or it will tell me the path so, this is it tells me this. What it does not tell me so, it does not provide information on how much light takes goes in a certain direction.

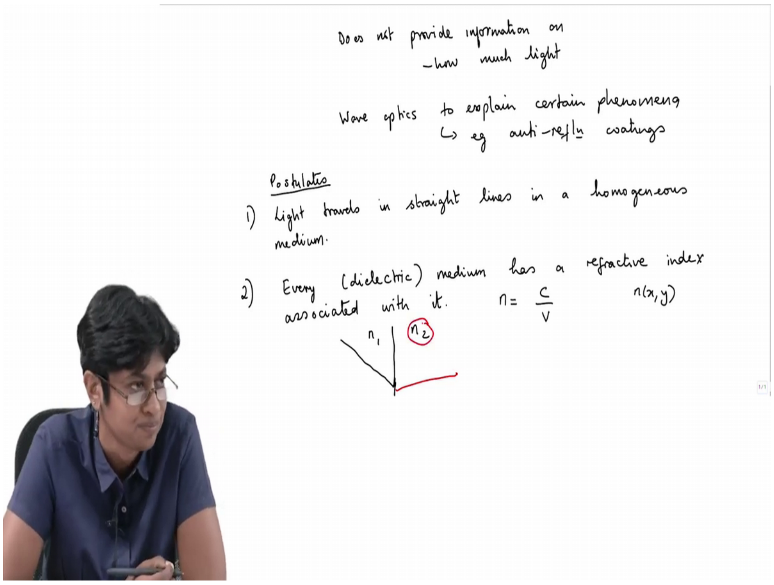
So, for example, I could be standing in front of a glass window, and I am looking out, now that means, somehow light is traveling through this glass window, but I can also see a reflection of myself maybe not a very good reflection, but I see a reflection; that means, some light is being reflected.

Geometric optics I could not use geometric optics and say from where I am standing I expect to see a reflection here, I understand how light is traveling from outside to inside, but I do not know from geometric optics. What is the amount of light that goes into the reflected light, how much gets transmitted I cannot tell anything about the quantities.

We have applications where we have thin layers of film on glasses. So, if you wear glasses when you buy your glasses, they may ask you do you want a coating, what coating do you want on it. Now that coating is called an anti-reflection coating, it is to make sure that most of the light that is incident on the glass travels through the glass and reaches your eye if a lot of it reflects it is not reaching your eye, and it is that much less light coming to you. So, you do not want to lose that light. So, you put an anti-reflection coating, cannot explain my anti-reflection coating behavior by geometric optics at all; you cannot say why reflection does not happen.

Geometric optics it says reflection happens; it can never explain why reflection does not occur, ok. So, you definitely need wave optics to explain certain phenomena ok, an example is the anti-reflection coatings there are many, of course, ok.

(Refer Slide Time: 30:48)



So, let us start with some postulates. The first postulate is what people observed long ago is that light travels in straight lines. Now, it is not correct to end the sentence here because does light always travel in straight lines no matter, what when would it deviate from the path?

Student: (Refer Time: 31:28) replace the mirror or when you have a different refractive index.

So, you say if there is a different refractive index. So, I will make it more general, and I will say light travels in straight lines in a homogeneous medium in a medium of constant refractive index. We will come to the point refractive index.

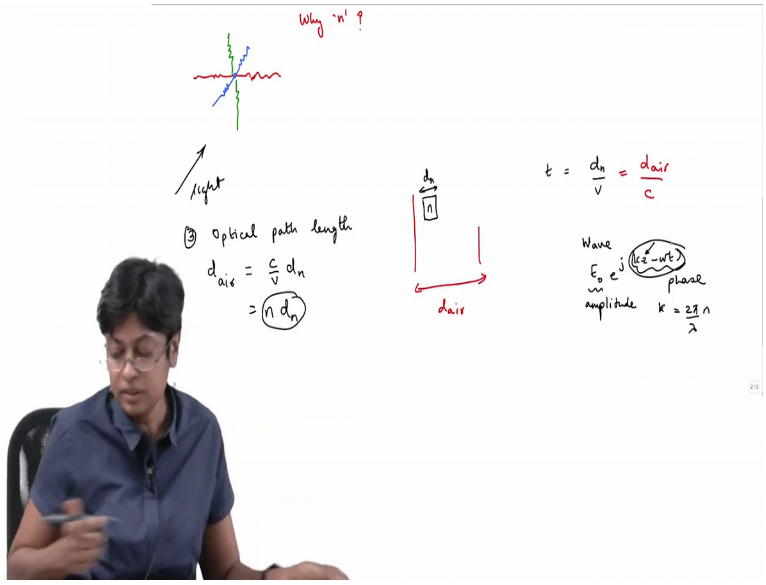
So, light travels in straight lines in a homogeneous medium, and since you brought up the point of refractive index. Let us put that down as our second postulate every, and I will keep it simple for this course. I am going to say every dielectric medium has a parameter known as the refractive index associated with it, ok. It is defined usually it is given the where letter n, and it is defined as the velocity of light in free space divided by the velocity of light in that particular medium that it is traveling.

The way I have written it here it means n is a constant in a some medium and the light is traveling with velocity v in that medium, but of course, n could also be a function of space I could have n of x, y and; that means, that the refractive index is varying in the medium; medium is not homogeneous. If I look back at the first postulate, this tells me that if there is an interface, light travels in a straight line in the first medium.

So, let us say I have n_1 and n_2 light travels in a straight line here, but when it reaches the second medium, something happens it need not be a straight line anymore; something happens, and it bends. I am drawing some arbitrary direction now, and in that medium, it will travel in a straight line again as long as this has constant refractive index sent, right.

How exactly it bends, we will look at ok, ok. That; now, you could also ask where does this idea of refractive index come from and of a simple way of looking at it is to think of every medium having atoms and molecules

(Refer Slide Time: 34:15)



And you can imagine that or you can rather think of the electron cloud around these atoms and molecules right and you can imagine this cloud has is connected through a set of springs, ok. So, why do you have a refractive index? Right.

Now, when the light is incident on this, the electron cloud gets set into oscillation um. The analogy the reason we put those springs in there is to say it need not get set into oscillation with the same frequency in every direction, which is why you have materials, which will have different refractive indices in different directions, right.

This sets it into oscillation, and then it reradiates, there is no absorption nothing is happening it just gets reradiated at the same frequency, but this whole process has slowed down the light, and that is why there is this concept of refractive index. And you can again take this analogy of the springs and say if the spring constants are stronger, it will slow it down even further, and that is why some materials slow down like more than others, right.

In addition, you will have materials where there is some atomic resonance with the frequency that is incident, and it may absorb, and that is what gives us color because certain wavelengths absorb, and then what remains and come out combines and give us a certain color, right.

So, this it is a very simple picture can explain can be used to help understand a lot of the everyday phenomena that we are seeing, but there is just a little background as to why the refractive index comes. We are not going to go into detail here, we are instead going to use this idea of refractive index and say when light travels what is important is an optical path length

So, we are used to worrying about the distance traveled by something, but when light travels, it turns out it is not just the physical distance that it is essential, but some other parameter. To give you an example, let us look at a case where you have a block of letting us say a glass of refractive index n and thickness d_n and you have a beam of light that is incident on the glass and beyond the glass.

Now, in the time t, it takes to travel through this glass with velocity v the same beam of light would have traveled further in the air because it got slowed down in the glass. So, it would have traveled a distance d in air with velocity c. So, this optical path length is

$$d_{air} = \frac{c}{v} d_n = n d_n$$

Now, in ray optics, we leave it at that there is a parameter called optical path length it is the refractive index into the physical length traveled, but when you move on to wave optics you will understand this gets used because it changes the phase of the wave. And, you know that in wave optics say how does a wave travels well a lot of a lot of control of how the wave travels lies in the phase of the wave and you can see the optical path length is playing around with the phase of the wave, ok

Student: Waves are the (Refer Time: 38:33).

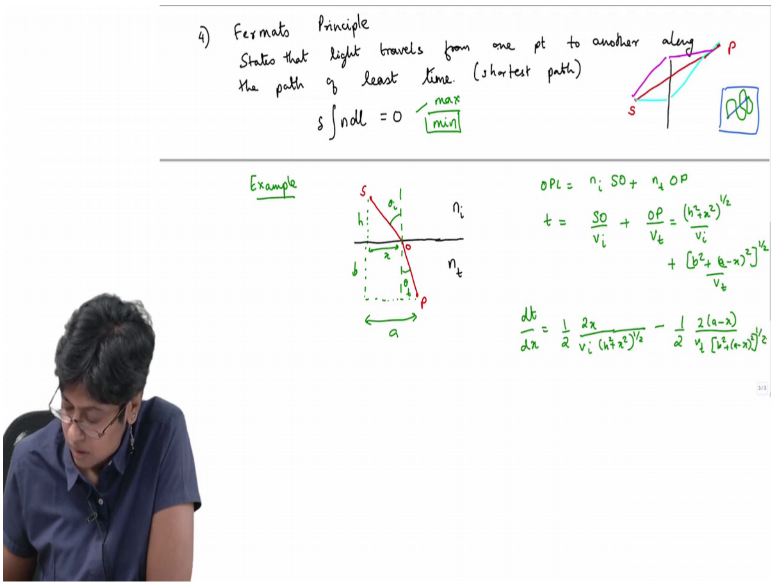
So, in electromagnetics if I write out a wave plane wave equation, I will have $E = E_0 e^{i(k_z z - \omega t)}$, where E_0 is the amplitude and $k_z - \omega t$ term is the phase and you can see this controls the if I if it travels through distance z the phase is changing. So, the phase is telling you how the wave is propagating and in that k actually n is appearing; k is nothing, but the wave number which is $\frac{2\pi}{\lambda}$ and in a medium it is $\frac{\lambda}{n}$, right.

So, this path length is actually playing a role here in the phase of the wave, ok. While I said in geometric optics we cannot explain, we do not explain how much why how a certain quantity of light is reflected in a certain quantity t is refracted.

In wave optics I can explain exactly why that happens if go to Maxwell's equations and you can take those equations, you can apply boundary conditions, and you can explain why the quantities are there; you can predict, you can calculate exactly how much light is going to get reflected or refracted based on the boundary conditions.

So, geometric optics, we just say this is the direction it is going to go, and you will you are not going into that depth, ok.

(Refer Slide Time: 40:21)



So, this brings me to the fourth postulate and it is a law or a principle called Fermat's Principle, ok. Fermat's Principle states that light travels from one point to another along the path of least time, or you could think, you could say maybe I could also say therefore, that is the shortest path,.

We are now when we say path we mean the optical path length. So, we are saying the rate of change oh sorry, would not write the rate of change of the optical path length is 0, and when I write it like this you could ask well this could be either a minimum or a maximum, ok.

Now ok, before I go down that let me again explain what I mean here, light travels from one point to another point along the path of least time. So, let us say I had two media and I have a source here and you know light has reached this point here. So, you have a source here and you know the light has reached this point.

Now, you could ask what is the path it took that is what this principle is answering, it could travel like this sorry, it could travel like this it could travel like this and it could travel in a infinite number of other paths, but, it travels along a particular path. How do we decide that path?

Now, I know if it is 1 medium and I was going from here to here we know it is traveling in the straight line, we said light travels in straight light. So, this all these postulates are linked and that is the shortest path because if I did this clearly a longer path that is not the path that light takes.

People did not understand why light travel like this, but they saw that it does it travels from 1 point to another on the path of least time, but when you write out the equation as I have done this could refer to a maximum path or a minimum. And, it turns out in our everyday occurrences in our everyday phenomena it is always the minimum, but when light bends around a planet due to gravity it actually takes the maximum path. So, for much principle is valid even there when light bends due to gravity.

Now, that is not a phenomena that we are encountering every day. So, it is not something that people arrived at naturally, that realization came much later. But, it is an interesting point to note that sometimes does take the maximum path if light is bending due to gravity, ok.

Now, how can I use Fermat's principle let us start with a very simple example, ok. Again, I will draw what I had drawn over here let us have an interface, the moment I say interface it means that I have one medium here and another medium here. I have a point of light source here and what I want to do is I am saying it has traveled like this.

So, from s it has traveled to p through this point o, the normal to the surface is drawn here and in optics we always indicate angles as the angle between the ray and the normal to the surface. So, this is the incident angle, this is the transmitted angle and let us to help us calculate let us give the height here this distance let us say is x, this distance is b, this distance is a ok, I think I have all my points. The path length is n_iSO + n_tOP.

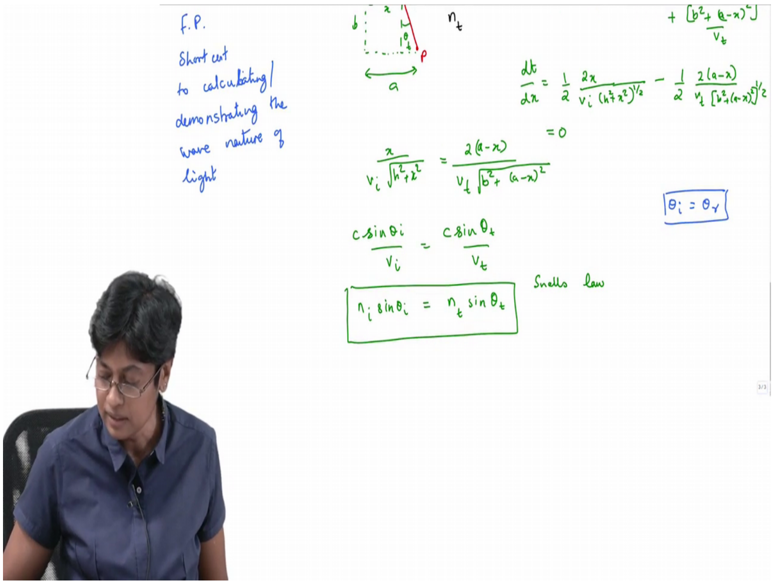
I want to use Fermat's principle; so, I want to take the path of least time. So, I am going to write out the time when I write out the time then I have to take the physical distance into account. So, I have SO/V_I + OP/Vt , where V_I and Vt are the velocity in those respective media. I want to minimize the time; so, I am going to take dt by and now I have to decide what variable to minimize it with, we are trying to find out the path it is taking.

So, if it had traveled through some other path in this figure that I have drawn right, that variable x would have changed that x would change. So, I am going to minimize this with respect to x because if it if the location of o was different x would be different and o would be in a different place if it had taken a different path,.

So, if I carry out this; so, let me write out

$$t = \frac{\left(h^2 + x^2\right)^{1/2}}{Vi} + \frac{\left(b^2 + (a-x)^2\right)^{1/2}}{Vt}$$

(Refer Slide Time: 47:43)



And we want this part to be minimized so, we are setting

$$\frac{dt}{dx}=\frac{1}{2Vi}\frac{2x}{\left(h^2+x^2\right)^{1/2}}+\frac{\left(b^2+(a-x)^2\right)^{1/2}}{2}=0$$

. So, if I do that what we will left with

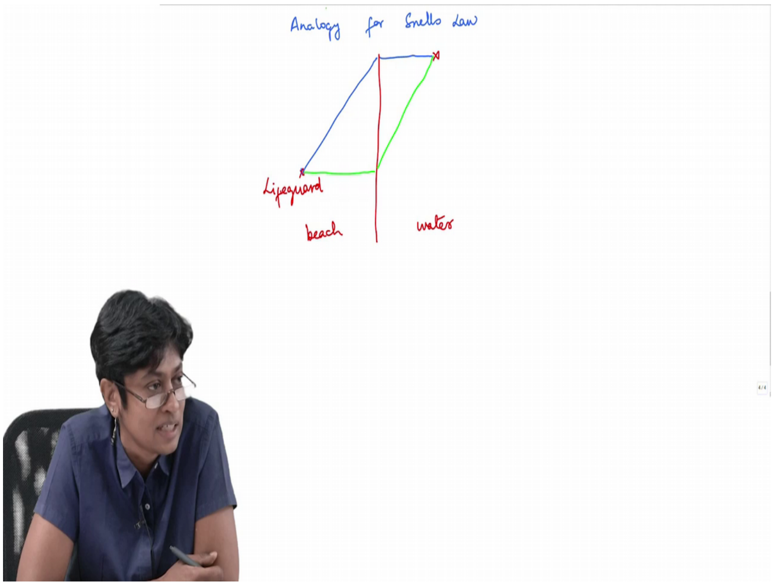
$$\frac{x}{vi\left(h^2+x^2\right)^{1/2}}=\frac{2(a-x)}{v_t\left(b^2+(a-x)^2\right)^{1/2}}$$

So, this is nothing, but $c\frac{\sin\theta_i}{V_i}$ = $c\frac{\sin\theta_i}{V_i}$ and if I multiply this by velocity of light in free space I can now write this in terms of the re-fractive index of each of these media. i.e $n_i \sin\theta_i = n_t \sin\theta_t$ And, what have we arrived at very well known what you have been doing from school Snell's law of, but Snell's law, Snell's law of refraction, right. $\frac{\sin\theta_i}{V_i}=\frac{\sin\theta_t}{V_t}$ If we take Fermat's principle as true and we do this derivation we arrive at Snell's law. Now, you can ask why is Fermat's principle true and as I said before if you go back to Maxwell's equations and boundary conditions you can prove Snell's law again using those equations and that math and you will understand why it is true.

So, you can consider Fermat's principle as a shortcut to calculating or demonstrating the wave nature without having to go through Maxwell's equations, you are able to arrive at something. So, it is a it is a shortcut.

You can use Fermat's principle to show the law of reflection which is theta i is equal to theta r ok, we will not do that here right and I will end this now with a nice analogy.

(Refer Slide Time: 50:44)



So, if you think of so, this and it is an analogy for this law, ok. Imagine you have a swimmer sorry a lifeguard and the lifeguard is on the beach medium one and there is someone drowning over here, ok; so, this is water. Now, of course, the aim of the lifeguard is to reach this person who is drowning as fast as possible. Now, what does lifeguard do, he has to choose a path the path involves both running and swimming.

Now, the lifeguard may be a faster runner than a swimmer. So, as it makes sense to choose the longest path or this let me say rather than this sorry, should he choose the longest path running and then swim the shortest path, but he is he is he still has to swim that much distance. But he is running much more now or say no we would not do this let him run this distance and swim this distance, but then he is swimming a much longer path.

What path should he choose and what Fermat's principle is letting us arrive at is what is the optimum path considering that in one medium there is one velocity and another medium there is another velocity.

If I optimize and say well he is a very fast runner let him run more, but you make him run really a lot more. Even if you say he is the fastest swimmer you have the same problem occurs there you are making him swim a lot more. So, what is the optimum path and Snell's law basically is helping us arrive at that optimum path. The amazing thing is that is how light sorry, that is how light travels it finds that optimum path in any situation and it travels, ok. So, it is a nice analogy to Snell's law, ok.

So, that is all for this class and we will continue in the next class with a little more on geometric optics.