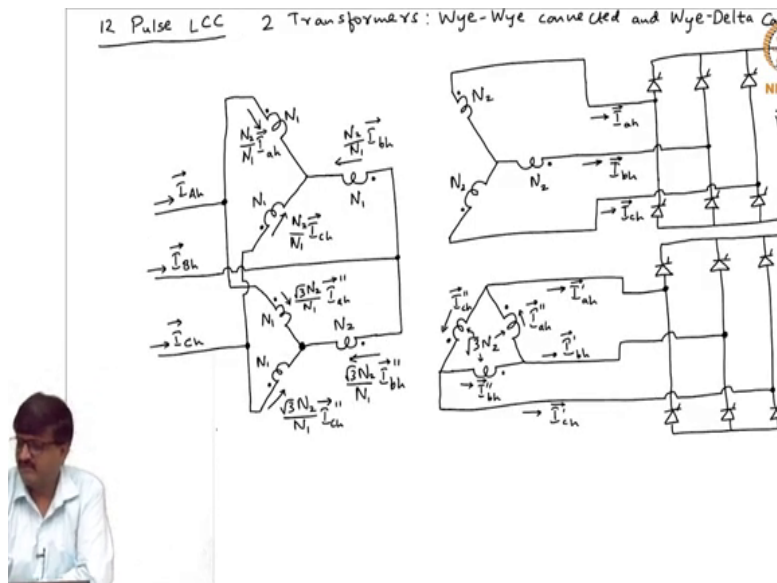


**DC Power Transmission Systems**  
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**Lecture – 47**  
**12 pulse LCC: Part 2**

(Refer Slide Time: 00:16)



Let us look at the currents in the delta winding.  $I_{A'h}$   $I_{B'h}$   $I_{C'h}$  double prime. Can I express these currents in the delta winding in terms of the currents  $I_{a'h}$   $I_{b'h}$   $I_{c'h}$  prime? So, why we try to do that is finally, we will get expressions for the currents that are fed to the entire 12 pulse LCC on the AC side. See what I am talking about are these currents, I want expressions for this. We will show that there are some harmonics that get eliminated in these currents which are actually present in these currents. So,

there are these currents which are fed to the individual 6 pulse LCC. There are some harmonics that get eliminated in the currents that are fed to the entire area 12 pulse LCC.

So, that is something which we are going to establish. So, to find the currents which are fed to the 12 pulse LCC, I know what is the current that is flowing for example, here. Let me just erase this. What is the current that flows here? Can I write an expression for this current in terms of a current which is already in the figure?

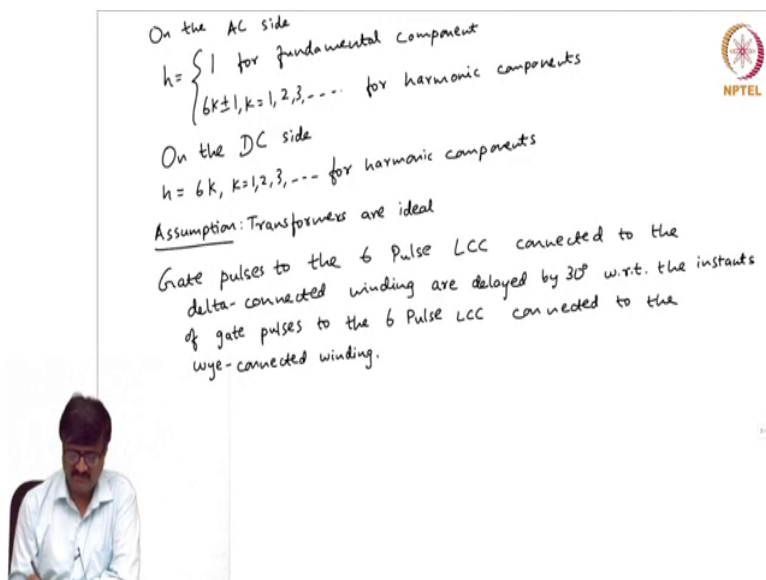
Student: (Refer Time: 01:36).

Yeah. I am talking about the current here.

Student: (Refer Time: 01:43) is same as.

I a h into by N 1, say it is in terms of this current I a h ok.

(Refer Slide Time: 02:01)



On the AC side  
 $h = \begin{cases} 1 & \text{for fundamental component} \\ 6k \pm 1, k=1, 2, 3, \dots & \text{for harmonic components} \end{cases}$

On the DC side  
 $h = 6k, k=1, 2, 3, \dots$  for harmonic components

Assumption: Transformers are ideal

Gate pulses to the 6 Pulse LCC connected to the delta-connected winding are delayed by  $30^\circ$  w.r.t. the instants of gate pulses to the 6 Pulse LCC connected to the wye-connected winding.

NPTTEL

So, I can write this as, see please note we have made some assumptions if you recall. I also said the transformers are ideal. Please note our transformers are ideal transformers no leakage inductance also ok. So, the 2 currents are related and, let us assume that these 2 transformers are obtained from 3 single phase transformers. So, if you take this wye-wye transformer, it is obtained from 3 single phase transformers. You should take this wye delta transformer; it is obtained from 3 single phase transformers. So, this current is  $N_2$  by  $N_1$  into.

Student: I a h.

I a h. So, similarly I can get the currents that are flowing here as well as here. So, here I show the currents as entering the dot. So, you are familiar with the dot convention I suppose. In the other winding the currents are leaving the dot ok. So, I will not get into those details of dot convention here, I presume you are familiar with it. So, this is  $N_2$  by  $N_1$  I b h and this is  $N_2$

by  $N_1 I_c h$ . Now what is the current that is flowing in? Suppose I take this winding what is the current here? So, instead of  $N_2 I$  have  $\sqrt{3} N_2$  on the other side. So, it is  $\sqrt{3} N_2$  by  $N_1$  into.

Student:  $I_a h$ .

$I_a h$ .

Student: double prime.

Double prime ok. So, similarly I can show this is  $N_2$ . So, the current that is flowing in this winding is  $\sqrt{3} N_2$  by  $N_1 I_b h$  double prime and the current through this winding is  $\sqrt{3} N_2$  by  $N_1 I_c h$  double prime. Now if I take the currents that are entering the 12 pulse LCC, that is the current  $I$  with a subscript uppercase  $A$   $h$ . So, if I take these 3 currents it can be written as sum of 2 currents, one is suppose I take  $I_a h$  it is  $N_2$  by  $N_1 I_a h$  plus  $\sqrt{3} N_2$  by  $N_1 I_a h$  double prime ok.

So, let me write expressions for these 3 currents which are entering the 12 pulse LCC. Please note the 12 pulse LCC has a AC side which has only 3 terminals ok, though there are 3 terminals for 2 6 pulse LCC. Individually the entire 12 pulse LCC has only 3 terminals on the AC side. So, let me write expressions for this  $I_a h$   $I_b h$   $I_c h$  which.

(Refer Slide Time: 05:07)

$$\vec{I}_{Ah}'' = \frac{N_2}{N_1} \vec{I}_{Ah}' + \frac{\sqrt{3} N_2}{N_1} \vec{I}_{Ah}''$$

$$\vec{I}_{Bh}'' = \frac{N_2}{N_1} \vec{I}_{Bh}' + \frac{\sqrt{3} N_2}{N_1} \vec{I}_{Bh}''$$

$$\vec{I}_{Ch}'' = \frac{N_2}{N_1} \vec{I}_{Ch}' + \frac{\sqrt{3} N_2}{N_1} \vec{I}_{Ch}''$$

$$\vec{I}_{Ah}' = \frac{1}{3} (\vec{I}_{Ah}' - \vec{I}_{Bh}') \quad \left[ \begin{matrix} \vec{I}_{Ah}' + \vec{I}_{Bh}' + \vec{I}_{Ch}' = 0 \\ \vec{I}_{Ah}' - \vec{I}_{Bh}' = \vec{I}_{Ch}' \end{matrix} \right]$$

$$\vec{I}_{Bh}' = \frac{1}{3} (\vec{I}_{Ah}' + 2\vec{I}_{Bh}') = \frac{1}{3} (\vec{I}_{Bh}' - \vec{I}_{Ch}') \quad \left[ \begin{matrix} \vec{I}_{Ah}' + \vec{I}_{Bh}' + \vec{I}_{Ch}' = 0 \\ \vec{I}_{Ah}' - \vec{I}_{Bh}' = \vec{I}_{Ch}' \end{matrix} \right]$$

$$\vec{I}_{Ch}'' = -\vec{I}_{Ah}'' - \vec{I}_{Bh}'' = \frac{1}{3} (\vec{I}_{Ch}' - \vec{I}_{Ah}') \quad \left[ \begin{matrix} \vec{I}_{Ah}' + \vec{I}_{Bh}' + \vec{I}_{Ch}' = 0 \\ \vec{I}_{Ah}' - \vec{I}_{Bh}' = \vec{I}_{Ch}' \end{matrix} \right]$$

$$\vec{I}_{Ah}'' - \vec{I}_{Ch}'' = \vec{I}_{Ah}' \quad \checkmark$$

$$\vec{I}_{Bh}'' - \vec{I}_{Ah}'' = \vec{I}_{Bh}' \quad \checkmark$$

$$\vec{I}_{Ch}'' - \vec{I}_{Bh}'' = \vec{I}_{Ch}'$$

$$\vec{I}_{Ah}'' + \vec{I}_{Bh}'' + \vec{I}_{Ch}'' = 0 \quad \checkmark$$

So, if I take  $I_{Ah}$  by Kirchhoff's current law, it is equal to  $N_2$  by  $N_1 I_{Ah}$  plus.

Student: Root.

Root  $\frac{3 N_2}{N_1} I_{Ah}$ . Similarly,  $I_{Bh}$  is equal to  $N_2$  by  $N_1 I_{Bh}$  plus root  $\frac{3 N_2}{N_1} I_{Bh}$  and  $I_{Ch}$  is  $N_2$  by  $N_1 I_{Ch}$  plus root  $\frac{3 N_2}{N_1} I_{Ch}$ . Now this is ok, but I have expressions for  $I_{Ah}$   $I_{Bh}$   $I_{Ch}$ . I do not have expressions  $I_{Ah}$   $I_{Bh}$   $I_{Ch}$ .

(Refer Slide Time: 06:17)

$$\vec{I}_{a1} = I_1 \angle 0^\circ$$

$$\vec{I}_{b1} = I_1 \angle -120^\circ$$

$$\vec{I}_{c1} = I_1 \angle 120^\circ$$

$$\vec{I}_{a5} = I_5 \angle 0^\circ$$

$$\vec{I}_{b5} = I_5 \angle -120^\circ \times 5$$

$$\vec{I}_{c5} = I_5 \angle 120^\circ \times 5$$

For  $h=1$  or  $h=6k \pm 1, k=1, 2, 3, \dots$

$\vec{I}_{ah} = I_h \angle 0^\circ$	$\vec{I}'_{ah} = I_h \angle -30^\circ h$
$\vec{I}_{bh} = I_h \angle -120^\circ h$	$\vec{I}'_{bh} = I_h \angle -150^\circ h$
$\vec{I}_{ch} = I_h \angle 120^\circ h$	$\vec{I}'_{ch} = I_h \angle 90^\circ h$

$$\vec{I}'_{a1} = I_1 \angle -30^\circ$$

$$\vec{I}'_{b1} = I_1 \angle -30^\circ \times 5$$

$$\vec{I}'_{c1} = I_1 \angle -150^\circ$$

$$\vec{I}'_{a1} = I_1 \angle 90^\circ$$

NPTEL

See if you just go back to the previous page, the last 6 equations give equation I mean expressions for  $I_{ah} I_{bh} I_{ch}$  and also they give for this  $I_{ah}' I_{bh}' I_{ch}'$ , but for double prime currents I do not have the expression.

So, how to get the expressions for see only if I get this double primed current expressions, I can get the current expressions that are actually feeding the 12 pulse LCC ok. So, can I write this double prime currents  $I_{ah}'' I_{bh}'' I_{ch}''$  in terms of  $I_{ah}' I_{bh}' I_{ch}'$ ?

Student: Yes.

By like Kirchhoff's current law ok. So, how to do that? So, you take one of the terminals of the delta connection and apply suppose I take one of the terminals if you look at the circuit  $I_a$  double prime minus  $I_c$  double prime by Kirchhoff's current law is equal to what?

Student:  $I_a$  double prime.

$I_a$  double prime. So, I am referring to this circuit please note I am referring to this circuit. So,  $I_a$  double prime minus  $I_c$  double prime is  $I_a$  double prime. Similarly,  $I_b$  double prime minus  $I_a$  double prime is equal to what equal to what?

Student:  $I_b$  double prime.

$I_b$  double prime and  $I_c$  double prime minus  $I_b$  double prime is equal to  $I_c$  double prime. So, these 3 equations are obtained by applying Kirchhoff's current law to the 3 terminals of the delta connected winding, but from this can I solve for  $I_a$  double prime  $I_b$  double prime  $I_c$  double prime? I have 3 equations 3 unknowns, can I solve?

Student: (Refer Time: 08:27).

Yeah there are only 2 equations, why? Why there are only 2 equations?

Student: First one is the linear combinations.

Yeah how, so, if you take.

Student: First one.

So, subtract the do you get that? See, one is a linear combination of the other if you can get one of the equations by linear combinations of other equations, but how do you say that I can get say for example, third equation from first two?

Student: You add the.

Yeah if you add for example, if I take if you just add fine. I left hand side is fine what about right hand side?

Student: a plus b plus c is 0, right.

How? See go to the circuit, how is it 0? See a plus b you are saying that  $I_a$  prime  $I_b$  h prime and  $I_c$  h prime is 0.

Student: Yes.

How? How is it 0? I mean if you know how it is 0, 0 maybe ok, but how?

Student: There is no (Refer Time: 09:33).

See I know only Kirchhoff's current law applied to a node, these are not currents flowing out of the node or flowing into the node.

Student: We already have the equation.

Yeah we have the equation look at the equations. So, I have the equations right.

Student: Yeah.

From equation I can say that  $I_a$  h prime plus  $I_b$  h prime plus  $I_c$  h prime is equal to 0.

Student: 0, yeah.



So; that means, I cannot solve for  $I_a$ ,  $I_b$  and  $I_c$  in terms of  $I_a$ ,  $I_b$  and  $I_c$  using these equations.

Student: Yes.

Because there are only 2 independent equations; there are no 3 independent equations. So, I need one more, what is that?

Student: Sum 0, (Refer Time: 10:13).

You can try solving you can try to solve for these double prime currents in terms of single prime, you will see that there is a problem. What will be the problem? I mean when it is actually less number of equations and more number of unknowns ok. So, you cannot get a solution that we want. So, I need one more equation which is independent of these 3 that the sum of these primed currents, double prime currents is equal to 0 if you apply.

Student: KCL.

KCL to this.

Student: Node yes.

Node ok? So, from this node, I get that  $I_a$ ,  $I_b$  and  $I_c$  double prime add to 0 right.

Student: Yes.

So, that is obvious. So, what I can say is say there is a common factor  $\sqrt{3} N_2$  by  $N_1$ . So, if I ignore that common factor it is not 0. So, I can cancel out the common factor as long as it is not 0 it is not 0. So,  $I_a$  double prime plus  $I_b$  double prime plus  $I_c$  double prime is

equal to 0 by applying the Kirchhoff's current law to the neutral of the wye winding of the wye delta transformer by KCL applied to the neutral of the wye winding of the wye delta transformer.

So, now there are no I mean it is not 4 equations, I have to take say any 3 equations. So, any 2 among these 3 say suppose I take the first 2 and then this one. So, if I solve these 3 equations for  $I_a$ ,  $I_b$  and  $I_c$ , I get the expression in terms of single phase quantities. So, if you solve what do you get?  $I_a$  minus.

Student:  $I_b$ .

$I_b$  that is all.

Student: By 2.

By 2.

Student: By 3.

By 3, please check it is 1 by 3 into  $I_a$  minus  $I_b$ .

Say, it is just solving 3 linear equations, 3 linear equations the 3 unknown quantities are the double primed currents in terms of single phase currents you have to solve that is all. So, that is  $I_a$ . Then did you get  $I_b$ ?  $I_a$  plus 2  $I_b$ .

Student: By 3.

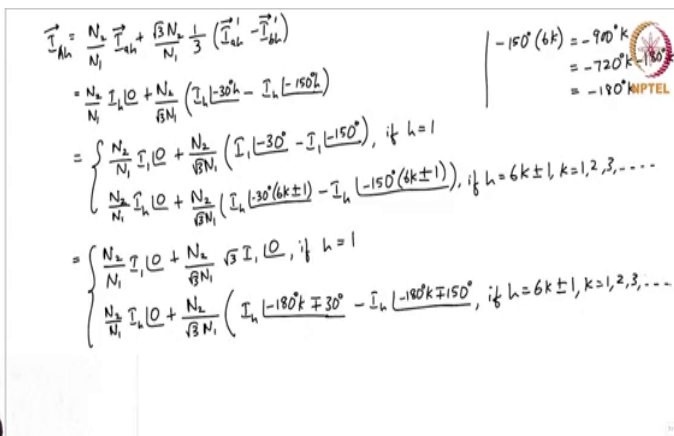
By 3, ok fine then  $I_c$ ; see once you know the 2 double prime quantity, so, it is equal to 1 by 3. So, it is say I should take the what I am trying to say is this is minus  $I_a$

double prime minus  $I_b h$  double prime. Now there is one more thing here. If you come look at  $I_b h$  double prime, can I say that this is equal to  $1/3 I_b h$  prime minus  $I_c h$  prime?

Now, this is because  $I_a h$  prime plus  $I_b h$  prime plus  $I_c h$  prime from the expressions for these currents is equal to 0. So,  $I_a h$  plus  $I_b h$   $I_a h$  prime plus  $I_b h$  prime is minus  $I_c h$  prime. So, I can write this as  $1/3 I_b h$  prime minus  $I_c h$  prime. So, if I want  $I_c h$  double prime, so, take the negative of  $I_a h$  prime and take the negative of  $I_b h$  I mean take the negative of  $I_a h$  double prime take the negative  $I_b h$  double prime add them and use this equation again;  $I_a h$  prime plus  $I_b h$  prime plus  $I_c h$  prime or ok I need not even use that. I will directly get this. Its equal to  $1/3 I_c h$  prime minus  $I_a h$  prime because  $I_b h$  prime gets cancelled ok.

So, that is straightforward. Now what I need to do is substitute this expression for  $I_a h$  prime in this equation, substitute this expression for  $I_b h$  prime in this equation and substitute this expression for I mean sorry all these are double prime currents  $I_a h$  double prime  $I_b h$  double prime and  $I_c h$  double prime expressions have to be substituted here ok. So, I will just I need a very long page. So, I will go to the next page in order to substitute this.

(Refer Slide Time: 15:35)



$$\begin{aligned} \vec{I}_{Ah} &= \frac{N_2}{N_1} \vec{I}_{1h} + \frac{\sqrt{3} N_2}{N_1} \frac{1}{3} (\vec{I}'_{1h} - \vec{I}'_{2h}) \\ &= \frac{N_2}{N_1} I_{1h} \angle 0 + \frac{N_2}{\sqrt{3} N_1} (I_{1h} \angle -30^\circ - I_{1h} \angle -150^\circ) \\ &= \begin{cases} \frac{N_2}{N_1} I_{1h} \angle 0 + \frac{N_2}{\sqrt{3} N_1} (I_{1h} \angle -30^\circ - I_{1h} \angle -150^\circ), & \text{if } h=1 \\ \frac{N_2}{N_1} I_{1h} \angle 0 + \frac{N_2}{\sqrt{3} N_1} (I_{1h} \angle -30^\circ(6k \pm 1) - I_{1h} \angle -150^\circ(6k \pm 1)), & \text{if } h=6k \pm 1, k=1,2,3,\dots \\ \frac{N_2}{N_1} I_{1h} \angle 0 + \frac{N_2}{\sqrt{3} N_1} \sqrt{3} I_{1h} \angle 0, & \text{if } h=1 \\ \frac{N_2}{N_1} I_{1h} \angle 0 + \frac{N_2}{\sqrt{3} N_1} (I_{1h} \angle -180^\circ \mp 30^\circ - I_{1h} \angle -180^\circ \mp 150^\circ), & \text{if } h=6k \pm 1, k=1,2,3,\dots \end{cases} \end{aligned}$$

$-150^\circ(6k) = -900^\circ k$   
 $= -720^\circ k - 180^\circ$   
 $= -180^\circ$

So, if I take  $I_{Ah}$  it is equal to  $\frac{N_2}{N_1} I_{1h} \angle 0 + \frac{\sqrt{3} N_2}{N_1} \frac{1}{3} (I'_{1h} - I'_{2h})$  prime. Now instead of  $I_{Ah}$  double prime I write  $I_{1h} \angle 0 + I_{2h} \angle -120^\circ$ ;  $I_{1h} \angle 0 + I_{2h} \angle -120^\circ$  prime minus  $I_{2h} \angle -120^\circ$  prime.

So, now you see that I have the expression for this  $I_{Ah}$  which is the current on the AC side of the 12 pulse LCC. In terms of these known currents  $I_{1h}$  and  $I_{2h}$  prime, now I have the expressions for these currents. See you got the expressions for these currents;  $I_{1h}$  is  $I_{1h}$  at an angle 0,  $I_{2h}$  is  $I_{1h}$  at an angle minus 120 h and so on. So, I substitute these expressions now. So, this is equal to  $\frac{N_2}{N_1} I_{1h} \angle 0 + \frac{\sqrt{3} N_2}{N_1} \frac{1}{3} (I_{1h} \angle 0 - I_{1h} \angle -120^\circ)$

Student: 0.

$0 \text{ plus } N^2 \text{ by } N^1 \text{ into root } 3 \text{ by } 3 \text{ is } N^2 \text{ by root } 3 \text{ } N^1 \text{ I a h prime is I h at an angle?}$

Student: Minus 30.

Minus 30 degree into h minus I b h prime, we have the expression it is I h at an angle minus?

Student: (Refer Time: 17:11).

150 degree into h, right. Now, I need to simplify this further. So, for the sake of simplifying I cannot just keep a general expression I have to take special cases, otherwise we will not be able to simplify.

See only if I go to special cases; see what are the special cases? The special cases are fundamental and harmonics; h is 1 for fundamental and h is equal to  $6k \text{ plus or minus } 1$  for harmonics. So, let me split this into 2 special cases. So, the first special case is for h equal to 1. So, when h is equal to 1, what I get is  $N^2 \text{ by } N^1$ . So, h is equal to 1 means I a h becomes I 1, I1 at an angle  $0 \text{ plus } N^2 \text{ by root } 3 \text{ } N^1 \text{ I h}$  becomes again I 1 minus 30 degree into h becomes minus 30 degrees minus again I 1 minus 150 degrees.

So, this is the expression if h is equal to 1. So, h can also take the values of the harmonic component orders. So, in that case h is equal to  $6k \text{ plus minus } 1$ . Now, I will replace this h by  $6k \text{ plus minus } 1$  only in the phase angles not in the subscripts for the current magnitude it is not necessary. So, I will still write I h as I h itself, but when it comes to the second term  $N^2 \text{ by root } 3 \text{ } N^1 \text{ I h}$ , I substitute here h by  $6k \text{ plus minus } 1$  minus I h minus 150 degrees into  $6k \text{ plus minus } 1$ .

So, this is the expression applicable if h is equal to  $6k \text{ plus or minus } 1$  and k is equal to 1, 2, 3 and so on up to infinity. Now the intention of going to special cases is simplifying; see now we can simplify. Suppose h is equal to 1 can we simplify? What is I 1 at an angle minus 30 minus I 1 at an angle minus 150?

Student: I 1 (Refer Time: 19:51).

Is it I 1 at an angle 0, phase angle is correct;  $\sqrt{3} I 1$  at an angle 0. So, let me write one more step.  $N 2$  by  $N 1 I 1$  at an angle 0 plus  $N 2$  by  $\sqrt{3} N 1$  into  $\sqrt{3} I 1$  at an angle 0, if  $h$  is equal to 1 is that ok. Now you see that it easily gets simplified, that  $\sqrt{3}$  gets cancelled. So, it is nothing but 2 times  $N 2$  by  $N 1 I 1$  at an angle 0. Now if I want to simplify the other things ok. Let us see how to simplify this, for harmonics it is  $N 2$  by  $N 1 I h$  at an angle 0 plus  $N 2$  by  $\sqrt{3} N 1$  into  $I h$ .

So, is there any simplification for this minus 30 degree ok, let us first multiply this 30 degree  $6 k$ , any simplification for minus 30 degree into  $6 k$ ?

Student: (Refer Time: 21:08). Minus 30.

Minus 30 degree into  $6 k$ .

Student: Minus  $180k$ .

Minus  $180 k$  minus  $180 k$ . Now let me first write one more step; minus 180 degree into  $k$ .

Student: Minus 30.

Minus 30 into plus minus 1 will give minus plus 1.

Student: 30.

Sorry minus plus 1 into 30 minus plus 30 degrees.

Now, one should note what is this order plus minus. If you look at the expression for  $h$  here, the expression for  $h$  is  $6k \pm 1$ , so; that means, when you take plus sign in the expression for  $h$  you get plus sign here, if you take minus sign you get minus sign here also.

Similarly, when you take plus sign in the expression for  $h$  you get minus sign here that is the point. See the order of plus and minus is important. See, it actually means what is the sign you get when you get plus sign in the when you take a plus sign in the expression for  $h$  and minus sign in expression for  $h$ ; minus  $150^\circ$  into  $6k$ . So, what do you say?  $150$  into  $6$  is of course  $900k$ , any simplification possible?

Suppose I take minus  $150^\circ$  into  $6k$  ok, so, this is minus  $900$  degrees into  $k$ . So, can I write this as minus  $720^\circ$  into  $k$  minus  $180^\circ$  into  $k$ ? So, what is this  $720$  into  $k$ ?  $k$  is an integer,  $k$  is a positive integer. So, minus  $720$  is  $2$  times  $360$ .

So, that is  $0$ . So, this is equal to minus  $180^\circ$  degree into  $k$ . So, it is minus  $180^\circ$  degree into  $k$ . So, minus  $150$  into plus minus  $1$  is minus plus  $150$  degrees, if  $h$  is equal to  $6k \pm 1$ ,  $k$  is  $1, 2, 3$  so on.

(Refer Slide Time: 23:53)

$$\begin{aligned}
 &= \begin{cases} \frac{N_2}{N_1} I_1 \cos 0 + \frac{N_2}{\sqrt{3} N_1} (I_1 \cos(-30^\circ) - I_1 \cos(150^\circ)), & \text{if } h=1 \\ \frac{N_2}{N_1} I_h \cos 0 + \frac{N_2}{\sqrt{3} N_1} (I_h \cos(30^\circ(6k \pm 1)) - I_h \cos(150^\circ(6k \pm 1))), & \text{if } h=6k \pm 1, k=1,2,3, \dots \end{cases} \\
 &= \begin{cases} \frac{N_2}{N_1} I_1 \cos 0 + \frac{N_2}{\sqrt{3} N_1} \sqrt{3} I_1 \cos 0, & \text{if } h=1 \\ \frac{N_2}{N_1} I_h \cos 0 + \frac{N_2}{\sqrt{3} N_1} (I_h \cos(180^\circ k \mp 30^\circ) - I_h \cos(180^\circ k \mp 150^\circ)), & \text{if } h=6k \pm 1, k=1,2,3, \dots \end{cases} \\
 &= \begin{cases} 2 \frac{N_2}{N_1} I_1 \cos 0, & \text{if } h=1 \\ \frac{N_2}{N_1} I_h \cos 0 + \frac{N_2}{\sqrt{3} N_1} (I_h \cos(\mp 30^\circ) - I_h \cos(\mp 150^\circ)), & \text{if } h=6k \pm 1, k=2,4,6, \dots \\ \frac{N_2}{N_1} I_h \cos 0 + \frac{N_2}{\sqrt{3} N_1} (I_h \cos(\pm 150^\circ) - I_h \cos(\pm 30^\circ)) & \end{cases}
 \end{aligned}$$

$180^\circ \mp 30^\circ$   
 $= 150^\circ, 210^\circ$   
 $\downarrow$   
 $-150^\circ$

Now, this can be written as. So, the if you take the first case h equal to 1, so, the root 3 in the second term gets cancelled in the numerator and denominator. So, it is 2 times N 2 by N 1 I 1 at an angle 0, if h is equal to 1 ok. So, I forgot to close the bracket here ok. Now what is a simplification of the case for harmonics? Can I simplify it further? Can I simplify this further? Yes or no?

Student: Yes.

How? How, can I simplify this? Any clue?

Student: (Refer Time: 24:51). We one should note that whenever k is an even integer this becomes 0, 180 degree into k is 0 when our k is an even integer and whenever k is odd it is minus or plus 180; minus 180 or plus 180 are one and the same ok. So, there are actually 2



further special cases. So, I split this special case into 2 further special cases; one case being  $k$  odd another is  $k$  even. So, let us do that.

So, it is  $N^2$  by  $N^1$  I h at an angle  $0$  plus  $N^2$  by root  $3$   $N^1$  into I h. So, what I will do is I will first take the easier case one first ok. Now at this stage it may be difficult to say which one is easy. Odd value of  $k$  is easy or even value of  $k$  is easy?

Student: Even (Refer Time: 25:49).

Student: Even is easy.

Why, yeah even is easy?

Student: It goes away.

Yeah it goes away; let us take even first then. So, it is equal to just minus plus  $30$  degrees, then I h at an angle minus plus  $150$  degrees, if h is equal to  $6k$  plus minus  $1$  and  $k$  is  $2, 4, 6$  so on. Now let us take the odd values of  $k$ . So, it is  $N^2$  by  $N^1$  I h at an angle  $0$  plus  $N^2$  by root  $3$   $N^1$  I h at an angle what?

Student:  $180$  minus plus  $30$ .

So,  $180$  minus plus  $30$ .

Student: Minus.

So, can I simplify that?

Student: Yeah.

What is 180 minus 30?

Student: 150.

See what I have here is this. See I can take minus or plus 180, they are one and the same. So, when our  $k$  is odd it is 180 with a minus or plus sign minus plus 30. So, 180 minus 30 is?

Student: 150.

150, 180 plus 30 is?

Student: 210 minus 150.

210. So, this 210 is nothing but?

Student: Minus 150.

Minus 150. So, can I say that it is plus minus 150?

Student: Yes.

Minus I h similarly.

Student: Plus minus.

$k$  is odd. So, 180 minus plus 150 is?

Student: 30 degree.

Plus minus 30.

Student: Yes.

Plus minus 30. So, I will erase these things.

(Refer Slide Time: 27:55)

$$= \begin{cases} 2 \frac{N_2}{N_1} I_1 \cos \theta, & \text{if } h=1 \\ \frac{N_2}{N_1} I_1 \cos \theta + \frac{N_2}{\sqrt{3} N_1} (I_h \cos 30^\circ - I_h \cos 150^\circ), & \text{if } h=6k \pm 1, k=2, 4, 6, \dots \\ \frac{N_2}{N_1} I_1 \cos \theta + \frac{N_2}{\sqrt{3} N_1} (I_h \cos 150^\circ - I_h \cos 30^\circ), & \text{if } h=6k \pm 1, k=1, 3, 5, \dots \end{cases}$$


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$$= \begin{cases} 2 \frac{N_2}{N_1} I_1 \cos \theta, & \text{if } h=1 \\ \frac{N_2}{N_1} I_1 \cos \theta + \frac{N_2}{\sqrt{3} N_1} \sqrt{3} I_h \cos \theta, & \text{if } h=6k \pm 1, k=2, 4, 6, \dots \\ \frac{N_2}{N_1} I_1 \cos \theta + \frac{N_2}{\sqrt{3} N_1} \sqrt{3} I_h \cos \theta, & \text{if } h=6k \pm 1, k=1, 3, 5, \dots \end{cases}$$

$$\begin{bmatrix} I_h \cos 30^\circ - I_h \cos 150^\circ = \sqrt{3} I_h \cos \theta \\ I_h \cos 30^\circ - I_h \cos 150^\circ = \sqrt{3} I_h \cos \theta \end{bmatrix} \quad \begin{bmatrix} I_h \cos 150^\circ - I_h \cos 30^\circ = \sqrt{3} I_h \cos \theta \\ I_h \cos 150^\circ - I_h \cos 30^\circ = \sqrt{3} I_h \cos \theta \end{bmatrix}$$

So, this is applicable if h is equal to 6 k plus minus 1, k is equal to 1.

Student: 3, 5.

3 5 so on. Let us take all these cases. So, the first case is as it is.  $2 \frac{N_2}{N_1} I_1 \cos \theta$  at an angle 0, if h is equal to 1, then the second case  $\frac{N_2}{N_1} I_1 \cos \theta + \frac{N_2}{\sqrt{3} N_1} \sqrt{3} I_h \cos \theta$ .

Now, there are 2 cases here. See one is  $I h$  at an angle minus 30, minus  $I h$  at an angle minus 150. I have to do this. The other thing is see when I take the top sign in the first term, I have to take the top sign in the second term also, because top sign is minus means it actually means I am taking the expression for  $h$  as  $6k + 1$ .

Student: Yes.

The bottom sign corresponds to  $h$  equal to  $6k - 1$ . So, I also need  $I h$  at an angle 30 minus  $I h$  at an angle 150. So, what is the first  $I h$  at an angle minus 30, minus  $I h$  at an angle minus 150?

Student:  $\sqrt{3} I h$ .

$\sqrt{3} I h \cos 30$ .

Student: 0.

Minus 0.

Student: 0.

Right. What about this case,  $\sqrt{3} I h$ ?

Student: 0.

0. So, whether it is the top sign or the bottom sign, what I get is  $\sqrt{3} I h$  at an angle 0, is that ok. In both cases I get  $\sqrt{3} I h$  at an angle 0 ok. So, this is applicable if  $h$  is equal to  $6k + 1$ . Let me just put this in bracket, this is just intermediate computation;  $6k + 1$ ,  $k$  is equal to 2 4 6 so on. Now, let us go to the other case,  $h$  is an odd integer.  $6k - 1$   $I h$  at an angle 0 plus  $6k - 1$  by  $\sqrt{3}$ .

Now again I have 2 cases:  $I h$  at an angle 150 degrees minus  $I h$  at an angle 30 degrees and the other case is  $I h$  at an angle minus 150 degrees minus  $I h$  at an angle minus 30 degrees. So, what is the first one?

Student: Minus 150 degrees is (Refer Time: 30:54).

We can say it directly?

Student: Because (Refer Time: 31:02).

Yeah fine ok. So, what do we get finally?

Student: (Refer Time: 31:16).

So, in both cases it is root 3.

Student: (Refer Time: 31:19).

180, ok. So, root 3  $I h$  at an angle 180. So, this is if  $h$  is equal to  $6k$  plus minus 1,  $k$  is 3, sorry  
1 3 5 so on.

(Refer Slide Time: 31:49)

$$\begin{aligned}
 & \left[ \begin{array}{l} I_h \angle -30^\circ - I_h \angle -150^\circ \\ I_h \angle 30^\circ - I_h \angle 150^\circ \end{array} = \sqrt{3} I_h \angle 0 \right] \quad \left[ \begin{array}{l} I_h \angle 150^\circ - I_h \angle -30^\circ \\ I_h \angle -150^\circ - I_h \angle 150^\circ \end{array} = \sqrt{3} I_h \angle 0 \right] \\
 & = \begin{cases} 2 \frac{N_2}{N_1} I_1 \angle 0, & \text{if } h=1 \\ 2 \frac{N_2}{N_1} I_1 \angle 0, & \text{if } h=6k \pm 1, k=2, 4, 6, \dots \\ 0, & \text{if } h=6k \pm 1, k=1, 3, 5, \dots \end{cases} \\
 \vec{I}_{Bh} = & \begin{cases} 2 \frac{N_2}{N_1} I_1 \angle -120^\circ, & \text{if } h=1 \\ 2 \frac{N_2}{N_1} I_1 \angle -120^\circ, & \text{if } h=6k \pm 1, k=2, 4, 6, \dots \\ 0, & \text{if } h=6k \pm 1, k=1, 3, 5, \dots \end{cases}
 \end{aligned}$$

So, we have come to the last step 2 times  $N_2$  by  $N_1$  into  $I_1$  at an angle 0 for fundamental, if  $h$  is equal to 1. Now, if  $h$  is equal to  $6k \pm 1$  and  $k$  is an even integer then what is the simplification? So, you see that root 3 in the second term the numerator and denominator gets cancelled. So, it is 2 times  $N_2$  by  $N_1$   $I_h$  at an angle 0, if  $h$  is equal to  $6k \pm 1$   $k$  is equal to 2 4 6 so on and the last case is 0, if  $h$  is equal to  $6k \pm 1$   $k$  is equal to 1 3 5 so on.

So, if you look at this expression this is the expression for what  $I_a$ , please note we started with this expression for  $I_a$ . One of the AC side currents of a 12 pulse LCC. So, what is this 0 for odd values of  $k$ ; that means, some of the harmonics are getting.

Student: Eliminated.

Not some infinite number of harmonics are getting eliminated. There are infinite harmonics half of them are getting eliminated ok. So, of course, one can verify what happens to  $I_b h$  and  $I_c h$  also. So, I will not go through the exercise for  $I_b h$  and  $I_c h$ , I will ask you to derive this. It is not obvious. Please note the expressions are not obvious. Even  $I_b h$  you cannot directly say what is the expression for  $I_b h$  by looking at the expression for  $I_a h$  ok.

If it is obvious then you should be able to guess, though some of them you can guess. You can I mean this one I guess I mean this one should be possible right. What do you expect?

Student: Minus 120 (Refer Time: 33:48).

So, it is 2 times  $N_2$  by  $N_1 I_1$  for  $h$  equal to 1 it is?

Student: Minus 120 degrees.

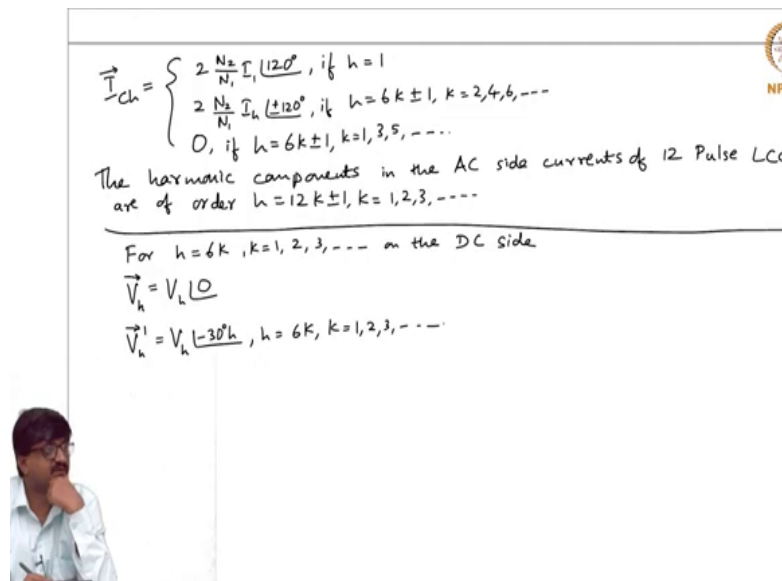
Minus 120 degrees if  $h$  is equal to 1. Now for harmonic with even values of  $k$ , what should be the expression? Can you guess? It appears as if say these are all balanced. So, the magnitude is obvious, but the phase angle is not.

Student: (Refer Time: 34:20).

So, it is the  $2 N_2$  by  $N_1 I_1 h$  minus ok. So, I will leave it to you to verify that this is minus plus 120 ok. So, if  $h$  is equal to  $6k + 1$  and  $k$  is equal 2 4 6. So, this minus 120 comes when you take  $6k + 1$  and plus 120 will come if you take  $h$  is equal to  $6k - 1$ . So, that is something which you have to verify and of course, when  $k$  is odd you get 0; that is something it is easy to guess.

So, if  $h$  is equal to  $6k + 1$  and  $k$  is 1 3 5 so on you get this.

(Refer Slide Time: 35:10)



$$\vec{I}_{ch} = \begin{cases} 2 \frac{N_2}{N_1} I_1 \angle 120^\circ, & \text{if } h=1 \\ 2 \frac{N_2}{N_1} I_h \angle \pm 120^\circ, & \text{if } h=6k \pm 1, k=2,4,6, \dots \\ 0, & \text{if } h=6k \pm 1, k=1,3,5, \dots \end{cases}$$

The harmonic components in the AC side currents of 12 Pulse LCC are of order  $h=12k \pm 1, k=1,2,3, \dots$

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For  $h=6k, k=1,2,3, \dots$  on the DC side

$$\vec{V}_h = V_h \angle 0$$

$$\vec{V}_h' = V_h \angle -30^\circ h, h=6k, k=1,2,3, \dots$$

So, even  $I_{ch}$  is not obvious the phase angle whenever the harmonic component does not get cancelled. So, I will leave it to you to verify. Thus this is  $2 \frac{N_2}{N_1} I_1$  at an angle plus 120 degrees if  $h$  is equal to 1 and  $2 \frac{N_2}{N_1} I_h$  at an angle what any guess?

Student: Plus minus 120.

Plus minus 120; again, please derive this if  $h$  is equal to  $6k \pm 1$   $k$  is equal to 2 4 6 so on. And it is 0 if  $h$  is equal to  $6k \pm 1, k$  is equal to 1 3 5 so on.

So, I am asking you to derive these expressions for  $I_{bh}$  and  $I_{ch}$  I have only derived for  $I_{ah}$ . Now if you look at this AC side currents of the 12 pulse LCC then what are the harmonic



components that are present?  $6k \pm 1$  and  $k$  equal to 2 4 6. So, I can say that its equivalent to.

Student: (Refer Time: 36:22).

harmonic for order  $h$  equal to  $12k \pm 1$  for all positive integer values of  $k$  ok. So, the harmonic components in the AC side currents of 12 pulse LCC are of order  $h$ . So, instead of saying  $6k \pm 1$   $k$  is equal to 2 4 6, I will straightaway say that it is  $12k \pm 1$ ,  $k$  is equal to 1 2 3 so on. Is that ok? Now these are as far as the harmonic components on the AC side is concerned. So, you see an advantage; the advantage is half the number of harmonics are getting eliminated.

So, what the harmonics 5th and 7th which were there in 6 pulse are no longer there. The harmonic components 15 sorry not 15, 17 and 19 are no longer there ok. So, half the numbers of harmonics are getting eliminated. So, the dominant harmonic now is not 5th it is 11th. So, this as far as AC side is concerned what about the DC side?

See DC side there is I mean let me take the simple case of constant current on the DC side, then there is no question of harmonics in the current, but what about voltage? Voltage has harmonics. Now if you take the 6 pulse LCC the voltage harmonics are of order what the DC side voltage?  $6k$  ok. Now what do we expect for 12 pulse LCC?

Student:  $12k$ .

$12k$ . So, it can be proved of course, that can be proved. Now there is no question of fundamental on the DC side what we have on the DC side is not fundamental what we, what is there on the DC side? It is the average value ok. So, what do we expect for the average value of the DC side voltage on the of the 12 pulse LCC? So, since they are connected in series on the DC side, look at the circuit; the minus sign terminal of the first 6 pulse LCC is connected to the plus sign terminal of the other 6 pulse LCC.

So, since the 2 voltages I mean these 2 voltages are appearing in series, the DC side or the average value of the entire 12 pulse LCC is sum of the averages of the two 6 pulse LCC's. So, that is obvious, but what happens to harmonics is the question. Now for harmonics on the DC side we take a different expression for  $h$ , see for AC side we took  $h$  equal to 1 for fundamental  $6k \pm 1$  for harmonics.

Now we will consider only harmonics there is no fundamental, there is a DC or average component. So, when it is having 1, I mean we are considering only harmonics  $h$  is equal to  $6k$ , for  $k$  is equal to 1 2 3. So, this is on the DC side.

Now, even in the figure we have written this use this notation  $V_h$  and  $V_h'$ . Now I said that we are using this notation for I mean what is this notation standing, for  $V_h$  or  $V_h'$ ? I think see on the DC side  $h$  is equal to  $6k$  for harmonic component. So,  $h$  is defined only for harmonic components.

So, I am only showing the harmonic components in the figure. When I say  $V_h$  and  $V_h'$ , it is not that average value,  $h$  is standing for harmonics ok. So, we are trying to analyze what happens to the harmonics. So,  $V_h$  and  $V_h'$  are actually the harmonic components.

So, can I get an expression for  $V_h$  and  $V_h'$ ? Suppose I take  $V_h$  as  $V_h$  at an angle  $\theta$ ,  $h$  is any harmonic component; 6th harmonic 12th harmonic 18th harmonic anything ok. So, for each I take say  $V_h$  as the complex  $V_h$  as  $V_h$  at an angle  $\theta$ , then what is  $V_h'$ , what is the magnitude?

Student: (Refer Time: 41:00).

It is having the same magnitude, what about phase angle?

Student: Minus 30 h.

Minus  $30^\circ$ , because the voltages that are applied to the second 6 pulse LCC are lagging the voltages applied to the first 6 pulse LCC by  $30^\circ$ . The gate pulses are also delayed by  $30^\circ$ . So, due to that this is minus  $30^\circ$  into  $h$ . So, here  $h$  is equal to  $6k$  and  $k$  is 1 2 3 so on. Now these are the voltages on the DC side of the 6 pulse LCC, there are two 6 pulses. Now if I take the total circuit its 12 pulse LCC.

So, what is the DC side voltage on the 12 of the 12 pulse LCC? See, if I take any harmonic component in the DC voltage of 12 pulse LCC, can I get that from the expressions for the voltages of 6 pulse LCC?

Student: 2 times.

Student: 2 times.

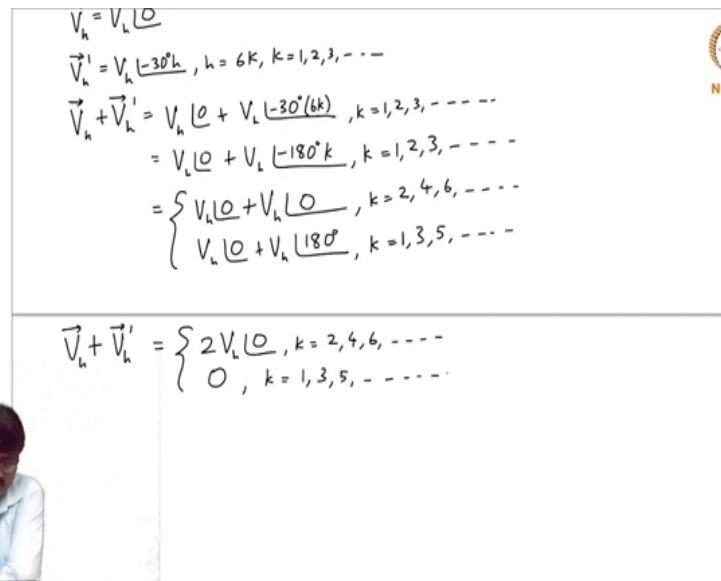
2 times. See if you look at the circuit I have written expression I mean I have used a notation  $V_h$  for the DC side voltage of the first 6 pulse LCC, I have used a notation  $V_h'$  for the on the DC side of the second 6 pulse LCC. Now, I want an expression for any harmonic component on the DC side of 12 pulse LCC in terms of these two.

Student: 12.

They are connected in series. So, it is  $V_h$  plus?

Student:  $V_h'$ .

(Refer Slide Time: 42:53)


$$\begin{aligned} V_h &= V_h \angle 0 \\ \vec{V}_h' &= V_h \angle -30^\circ h, \quad h = 6k, \quad k = 1, 2, 3, \dots \\ \vec{V}_h + \vec{V}_h' &= V_h \angle 0 + V_h \angle -30^\circ (6k), \quad k = 1, 2, 3, \dots \\ &= V_h \angle 0 + V_h \angle -180^\circ k, \quad k = 1, 2, 3, \dots \\ &= \begin{cases} V_h \angle 0 + V_h \angle 0, & k = 2, 4, 6, \dots \\ V_h \angle 0 + V_h \angle 180^\circ, & k = 1, 3, 5, \dots \end{cases} \end{aligned}$$

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$$\vec{V}_h + \vec{V}_h' = \begin{cases} 2V_h \angle 0, & k = 2, 4, 6, \dots \\ 0, & k = 1, 3, 5, \dots \end{cases}$$

$V_h$  prime. So, if I take the sum of  $V_h$  and  $V_h$  prime what do I get? So,  $V_h$  is  $V_h$  at an angle 0 and  $V_h$  prime is  $V_h$  at an angle minus 30 degree into  $h$ . Now we know that  $h$  is equal to?

Student:  $6k$ .

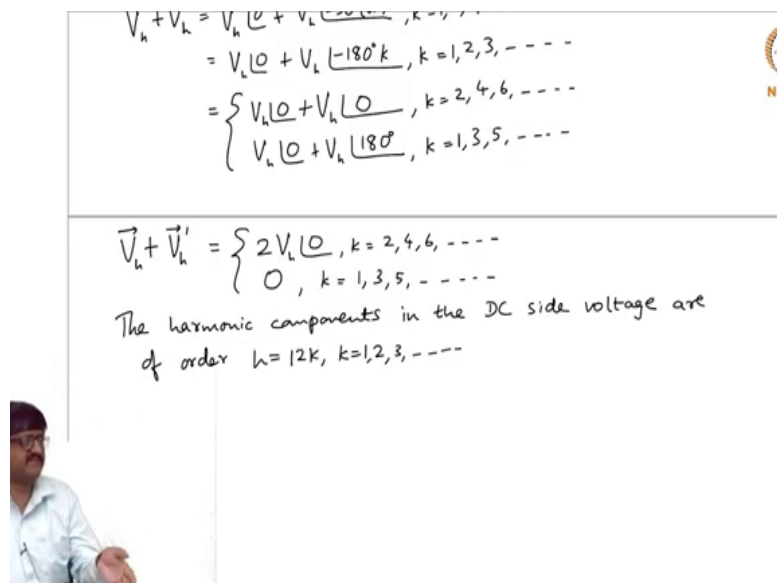
$6k$ . So,  $k$  is equal to 1 2 3 so on. So, this simplifies to  $V_h$  at an angle 0 plus  $V_h$  at an angle 30 into 6 is minus 180 degree into  $k$ ,  $k$  is equal to 1 2 3. So, how to do further simplification?

Student: Even and odd.

So, split it into 2 special cases, even and odd. So, it is if I take this easier case even case first, it is  $V_h$  at an angle 0 plus  $V_h$ . So, when  $k$  is an even integer this is 0. So, when  $k$  is equal to 2 4 6, it is 0 and it is  $V_h$  at an angle 0 plus  $V_h$  at an angle 180 degrees or minus 180 degrees, then  $k$  is equal to 1 3 5 so on. So, finally, this simplifies to  $2 V_h$  at an angle 0. So, what I am writing here is the expression for  $V_h$  plus  $V_h$  prime.

So, it is  $2 V_h$  at an angle 0 when  $k$  is 2 4 6 and whenever  $k$  is odd, the sum of these 2 voltages is 0. So, we say that whenever  $k$  is odd the harmonic components are eliminated and whenever  $k$  is even the harmonic components are just added ok. So, we can say that half the numbers of harmonics are eliminated.

(Refer Slide Time: 45:05)



$$\begin{aligned}
 V_h + V_h' &= V_h \angle 0 + V_h \angle -180^\circ k, k=1,2,3, \dots \\
 &= V_h \angle 0 + V_h \angle -180^\circ k, k=1,2,3, \dots \\
 &= \begin{cases} V_h \angle 0 + V_h \angle 0, & k=2,4,6, \dots \\ V_h \angle 0 + V_h \angle 180^\circ, & k=1,3,5, \dots \end{cases}
 \end{aligned}$$

$$\vec{V}_h + \vec{V}_h' = \begin{cases} 2V_h \angle 0, & k=2,4,6, \dots \\ 0, & k=1,3,5, \dots \end{cases}$$

The harmonic components in the DC side voltage are of order  $h=12k, k=1,2,3, \dots$

So, the harmonics that is present on the DC side voltage of 12 pulse LCC; so, the harmonic components on the or in the DC side voltage in the DC side voltage are of order  $h$  is equal to.

So, instead of saying  $h$  equal to 6,  $k$  equal to 2 4 6, what I will say is  $h$  is equal to 12,  $k$  is equal to 1 2 3 so on. So, earlier we had 6th harmonic 12th harmonic 18th harmonic 24th harmonic for 6 pulse.

For 12 pulse, it is 12th harmonic, the first harmonic is 12th harmonic then 24th 36th harmonic 48th so on ok. So, there is a harmonic elimination on the DC side also. So, we see that there is some advantage with 12 pulse.