


**DC Power Transmission Systems**  
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**Lecture – 38**  
**Steady state analysis of a general LCC: Part 2**

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Steady State Analysis of a General Converter

2 subintervals

①  $\frac{\alpha}{\omega_0} < t < \frac{\alpha}{\omega_0} + t_1 \rightarrow$  conduction of  $m+1$  valves

②  $\frac{\alpha}{\omega_0} + t_1 < t < \frac{\alpha}{\omega_0} + T_1 \rightarrow$  conduction of  $m$  valves

$T_1 = \frac{I}{P}, \quad 0 < t_1 < T_1$


1<sup>st</sup> subinterval  $\rightarrow$  state vector is  $x_1$

2<sup>nd</sup> subinterval  $\rightarrow$  state vector is  $x_2$

$x_1: (m+1) \times 1$  | State variables in  $x_2$  are in  $x_1$

$x_2: n \times 1$  | There is one state variable in  $x_1$  which is not in  $x_2$

outgoing valve current



So, if I try to do what is known as a generalized steady state analysis so, we started with this Steady state analysis of a general converter. So, there are two sub intervals in general of course, there are a number of intervals in each cycle. So, the number of intervals is equal to the pulse number. So, for the circuit that we have studied till now I mean there is only one circuit that we have studied that is 6 pulse circuit. So, there is only 6 intervals in one cycle. And in general of course, I mean there is one more converter which is used in practice of

course, the practical converter is not 6 pulse converter, it is a 12 pulse converter. So, it will be  $p$  is equal to 12; so, 12 intervals per second.

So, in each interval there are two subintervals irrespective of the pulse number ok. So, the two sub intervals I mean this was already mentioned in the last class. So, I just repeat these two points; two sub intervals are there. The first sub interval is from  $\alpha$  by  $\omega t$  to  $\alpha$  by  $\omega t + \frac{\pi}{p}$ . So, this corresponds to conduction of  $m + 1$  valves and the second sub intervals starts from  $\alpha$  by  $\omega t + \frac{\pi}{p}$  and it goes up to  $\alpha$  by  $\omega t + \frac{\pi}{p} + T$  with the subscript 1.


So, this corresponds to conduction of  $m$  valves. So,  $T_1$  by definition is the period of the ac side wave form  $T$ . So, if it is 50 Hertz,  $T$  is 20 milliseconds divided by  $p$   $T$  by  $p$  gives  $t_1$  and  $t_1$  can be somewhere between 0 and uppercase  $T$ . So, what we do is we try to take the state variables in the first subinterval as  $x_1$ . So, in the first subinterval so, I presume you are familiar with the word state variable state vector. So, state vector is not unique, but the size of the state vector is unique that is the point. So, in the first sub interval a state vector is say denoted by  $x_1$  and in the second subinterval state vector is  $x_2$

So, it so, happens that more number of valves actually conduct in the first sub interval. So, the size of  $x_1$  is greater than size of  $x_2$  and it is greater by 1 ok. So, if I take  $x_1$   $x_1$  is represented by a column matrix or a column vector so, I say that the size of  $x_1$  is  $n + 1$  cross one where  $n$  is some positive integer or some non negative integer. Then  $x_2$  is of size  $n$  cross 1. So, I already mentioned that the state variables in  $x_2$  are already there in  $x_1$ . So, state variables in  $x_2$ ;  $x_2$  is a state vector. So, there are  $n$  number of state variables in this state vector  $x_2$ .

So, the state variables in  $x_2$  are in  $x_1$ ; that means, there is one state variable in  $x_1$  which is not there in  $x_2$  ok. So, there is one state variable in  $x_1$  which is not in  $x_2$ . So, this one state variable which is in  $x_1$  and not in  $x_2$  is nothing, but the current in the outgoing valve. So, the outgoing valve current is the one which is not in  $x_2$ , but in  $x_1$  ok. So, what we will do is we

will try to just analyze ah these two subintervals. So, in general I can write these equations governing the circuit in the two sub intervals as linear time invariant equations.

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$$\frac{dx_1}{dt} = A_1 x_1 + u_1(t), \frac{\alpha}{\omega_0} \leq t \leq \frac{\alpha}{\omega_0} + T_1 \quad \text{--- (1)}$$

$$\frac{dx_2}{dt} = A_2 x_2 + u_2(t), \frac{\alpha}{\omega_0} + T_1 \leq t \leq \frac{\alpha}{\omega_0} + T_1 - T_2 \quad \text{--- (2)}$$

$A_1: (n+1) \times (n+1), u_1: (n+1) \times 1, A_2: n \times n, u_2: n \times 1$


$$\frac{dx}{dt} = Ax + u(t) \rightarrow \text{LTI System}$$

Solution  $\rightarrow x(t)$

$x(t_0) \leftarrow$  Initial condition

For  $t \geq t_0$ , solution is

$$x(t) = e^{A(t-t_0)} x(t_0) + e^{At} \int_{t_0}^t e^{-A\tau} u(\tau) d\tau$$



So, I think you are familiar with the representation of a linear time invariant equation in the form of equations differential equations the two a set of first order differential equations ok. So, a if I write only first order differential equations, there are ah n plus one first order differential equations n plus 1 scalar first order differential. So, I write it I write a vector differential equation. So, it will be  $\frac{dx}{dt}$  is equal to a constant matrix  $A$  times  $x$  plus  $u$ . So,  $u$  is a forcing function or the input. So, in our case we have some sources on the ac side dc side voltage sources, current sources. So, that can come in your own ok.

So, this is the equation for the circuit in the first sub interval. So, the first subinterval is applicable from  $\alpha$  by  $\omega_0$  less than or equal to  $t$  less than or equal to  $\alpha$  by  $\omega_0$

$o + t$ . Similarly if I take the second sub interval, the second sub interval is also I mean a circuit and it is governed by a linear time invariant equation sorry  $\frac{dx}{dt}$  is  $A \times 2 + u$ . So, in general this  $u_1$  and  $u_2$  are functions of time say if I just now said that there they can be the ac side voltages, they are functions of time because they are sinusoidal functions of time.

So, this is applicable from  $\alpha \text{ by } \omega o + t$  to  $\alpha \text{ by } \omega o + \text{upper case } T$  with the subscript 1 ok. So, if I take  $A_1$  I said  $A_1$  is a matrix. So, what is the size of this matrix  $A_1$ ?

Student:  $n + 1 \times n + 1$  cross 1.

Yeah it is  $n + 1 \times n + 1$ . If I take  $u_1$  what is the size of  $u_1$ ?

Student:  $n + 1 \times 1$

$n + 1$ .

Student: cross 1.

Cross 1. It is a column vector then  $A_2$  is also a square matrix. So, it is  $n \times n$  and  $u_2$  is  $n \times 1$ . Now, how do we solve this equation? Now this is a linear time invariant equation solution is well known. See I am assuming that you know the solution of this equation. Suppose  $x$  is a vector column vector; suppose I have a linear time invariant equation in the form of first order vector differential equation  $\frac{dx}{dt}$  is equal to  $Ax + u$  of  $t$ . So, this is a linear time invariant system and it has one more speciality it has one more speciality; it is not just linear see time invariant means  $a$  is not a function of time ok

So, if  $a$  is not a function of time I mean its not only linear it is time invariant also. So, the solution of this is. So, how do you get the solution of this say we usually start with some value of  $x$  solution means the expression for  $x$  as a function of the independent variable time is a

independent variable. So, let me say what is the solution first of all solution means  $x$  as a function of time ok. So, what we normally know I mean many problems is the value of  $x$  at some instance say it  $t_0$ . So, we call this as the initial value or the initial condition.

So, we write the solution in terms of this initial condition. So, the word initial means we can get the solution only for  $t$  greater than  $t_0$  that is what we mean ok. So, most of our problems that we come across we are not interested in the solution that is say solution for  $t$  less than  $t_0$  only for  $t$  greater than or equal to  $t_0$   $t$  equal to  $t_0$ , we already know. So, for  $t$  greater than  $t_0$  what is the solution? So, for  $t$  greater than or equal to  $t_0$  the solution is  $x$  of  $t$  so, what are the solution? Are you familiar with the solution of this linear time invariant system?


Student: (Refer Time: 11:109).

Yeah, I can write this as exponential of  $A$  into  $t$  minus  $t_0$  into.

Student: (Refer Time: 11:25)

$x$  of  $t_0$  plus exponential  $A$  integral, now we integrate from  $t_0$  upper limit is  $t$  exponential minus  $A$  tau  $u$  of tau  $d$  tau. So, this is the solution of this equation  $dx$  by  $dt$  equal to  $Ax + u$  of  $t$  ok. Now, can we get the solution of equation 1 and 2. So, since I know the general solution of a linear time invariant system. So, 1 and 2 are linear time invariant systems are which are applicable for the two sub intervals. So, if you look at the equation that for which we have the solution instead of  $x$ , I have  $x_1$  instead of  $A$ ,  $A_1$  instead of  $u$ ,  $u_1$  that is all.

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$$x_1(t) = e^{A_1(t-\alpha/\omega_0)} x_1(\frac{\alpha}{\omega_0}) + e^{A_1 t} \int_{\alpha/\omega_0}^t e^{-A_1 \tau} u_1(\tau) d\tau, \quad \frac{\alpha}{\omega_0} \leq t \leq \frac{\alpha}{\omega_0} + T_1 \quad (1)$$

$$x_2(t) = e^{A_2(t-\frac{\alpha}{\omega_0}-t_1)} x_2(\frac{\alpha}{\omega_0} + t_1) + e^{A_2 t} \int_{\frac{\alpha}{\omega_0} + t_1}^t e^{-A_2 \tau} u_2(\tau) d\tau, \quad \frac{\alpha}{\omega_0} + t_1 \leq t \leq \frac{\alpha}{\omega_0} + T_1 \quad (2)$$

Without loss of generality, let the outgoing value current be the last element of  $x_1$

$$C x_1(\frac{\alpha}{\omega_0} + t_1) = 0 \quad (3)$$


where  $C: 1 \times (n+1)$ ,  $C = [0, \dots, 1]$  where  $O_1$  is  $1 \times n$  zero matrix.

$$x_2(\frac{\alpha}{\omega_0} + t_1) = D x_1(\frac{\alpha}{\omega_0} + t_1) \quad (4)$$

where  $D: n \times (n+1)$ ,  $D = [I, 0^T]$  where  $I_1$  is  $n \times n$  identity matrix

$$x_1(\frac{\alpha}{\omega_0}) = F x_2(\frac{\alpha}{\omega_0} + T_1) + G \quad (5)$$

where  $F: (n+1) \times n$ ,  $G: (n+1) \times 1$



So, I have the solution ah. So, I write it as solution is  $x_1$  of  $t$  is equal to exponential  $A_1$ . Now I do not have  $t$  naught now what is there in place of  $t$  naught?

Student: Sir, alpha by omega o.

Alpha by.

Student: Omega o.

Omega o is there in place of  $t$  naught ok. So, the solution is exponential of  $A_1$  into  $t$  minus alpha by omega o into  $x_1$  of alpha by omega o. So, I assume that the initial condition is the value of  $x_1$  at alpha by omega o say that is the instant of starting of the first sub interval. Then

the second term see the first term is due to the initial condition the second term is due to the input. So, exponential  $A_1 t$  integral from  $\alpha$  by  $\omega_0$  to  $t$  exponential minus  $A_1 \tau$ ;  $\tau$  is the dummy variable into  $u_1$  of  $\tau d \tau$ . And this is applicable only from  $\alpha$  by  $\omega_0$  to what see we are writing the solution for the first differential equation which is applicable for the first sub interval.

So, first sub interval is only upto  $\alpha$  by  $\omega_0$  plus  $t_1$ . Similarly for the second sub interval the solution is  $x_2$  of  $t$  is exponential  $A_2$ . Now, the initial condition is at what time? So, it is corresponding to the initial instant of second sub interval that is  $\alpha$  by.

Student:  $\omega_0$  plus  $t_1$ .

$\omega_0$  plus  $t_1$  into  $x_2$  of  $\alpha$  by  $\omega_0$  plus  $t_1$  plus exponential  $A_2 t$  integral  $\alpha$  by  $\omega_0$  plus  $t_1$  to  $t$  exponential minus  $A_2 \tau$   $u_2$  of  $\tau d \tau$ . So, this is applicable from  $\alpha$  by  $\omega_0$  plus  $t_1$  to  $\alpha$  by  $\omega_0$  plus upper case  $t$  with the subscript 1. So, let me call the solution in the first interval I mean the equation given in the solution as equation 1 and this which is giving the solution in sub interval 2 as equation 2. Now, I said that there is one state variable in  $x_1$  which is not in  $x_2$  and that state variable is the outgoing valve current. Now, there is no loss of generality if I assume that this out going valve current is the last element of  $x_1$ .

Student: Yeah.

There is no loss of generality, I can keep it anyway ok. So, without loss of generality, let the outgoing valve current be the last element of  $x_1$  and this is not there in  $x_2$ . So, it is only in  $x_1$ . So, let it be the last element of  $x_1$ . So, what is this value of outgoing valve current which is nothing, but the value of last element of  $x_1$  at the end of the first sub interval that is  $\alpha$  by  $\omega_0$  plus  $t_1$ . What is the value of this last element of  $x_1$  at  $\alpha$  by  $\omega_0$  plus  $t_1$ ? 0.

Student: 0

.So, because the current becomes 0 at the end of first sub interval. So, how can I write this in terms of  $x_1$ ? See  $x_1$  contains many elements I am just saying the last element is 0 at the end of first sub interval. So, can I write it in terms of  $x_1$ ? So, I will not try to split  $x_1$  and  $x_2$  into individual state variables. So, I will use  $x_1$  itself without splitting  $x_1$  into individual state variables, can I write this condition in the form of an equation?

Student: 0 0 0 0 0 1 into (Refer Time: 17:17).

Yes, pre multiplied by row matrix. So, I will write this as  $C$  into  $x_1$  evaluated at  $\alpha$  by  $\omega t + 1$  is 0 where  $C$  is I will where  $C$  is what is the size of  $c$ ? Obviously, it is a row matrix. So, one row columns.

Student:  $n + 1$ .

$n + 1$  because  $x_1$  is having  $n + 1$  rows, then what are the elements of  $C$ ? So, I can write  $C$  as so, I will write this as; so, just to differentiate between 0 and a matrix which is a 0 matrix I will just put a subscript 1  $O_1$ ;  $O$  with the subscript 1 in and 1 where  $O_1$  is a 0 row matrix.

Student: Which one?

So, where  $O_1$  what is the size of  $O_1$ ?  $1 \times n$  matrix. So, is that ok? So, let me call this equation as equation 3. Now suppose I take  $x_2$  all elements of  $x_2$  are also in  $x_1$ ; all elements of  $x_2$  are in  $x_1$ . Now, if I retain the same order of these elements in  $x_1$  and  $x_2$  with there is no again, there is no loss of generality if I maintain the same order. Then can I relate  $x_2$  at the beginning of second interval and  $x_1$  the end of first sub interval? Can I relate  $x_2$  at the beginning of the second sub interval?

See  $x_2$  is defined for only second sub interval and  $x_1$  is defined for sub first sub interval. So, can I relate  $x_2$  at the beginning of first sub interval and the value of  $x_1$  at the end of sorry I will I will repeat  $x_2$  at the beginning of the second sub interval and  $x_1$  at the end of first sub



interval. So, what essentially what I am trying to say is can I relate  $x_2$  at see this beginning of second sub interval is same as the.

Student: (Refer Time: 19:38)

End of first sub interval. So, it is nothing, but  $\alpha$  by  $\omega_0$  plus  $t_1$  and  $x_1$  at  $\alpha$  by  $\omega_0$  plus  $t_1$ . So, how these two are related? Yes.

Student: Whether the elements present in the  $x_1$  is equal to  $\alpha$  by  $\omega_0$ .

So, what we will do is we will take up special cases and try to find what is  $x_1$ , what is  $x_2$ , what is  $A_1$ , what is  $A_2$ , what is  $u_1$ , what is  $u_2$  for all the special cases. We will consider some special cases ok. So, now I am assuming that you have a circuit. If you have a circuit, there is there is no unique way of selecting the state variables of the circuit, but the number is fixed ok. What I am only saying is one let one of the state variables in  $x_1$  be the outgoing valve current that is the only restriction. I am not putting any restriction right now on the other state variables.

So, there is no unique way of saying what is the set of state variables, but I only put one restriction. One of the outgoing valve current, let it be an element of  $x_1$  that is all we will come to that. So, we will try to find answers for what can be possible  $x_1$  possible  $x_2$   $A_1$   $A_2$   $u_1$   $u_2$ . So, how are these two are related?

Student: Sir some pre multiplied by.

Ok.

Student: Pre multiplied by.

I will say that pre multiplied by some matrix  $D$  where first of all what is the size of  $D$ ?

Student: It is  $n + 1$  cross.

See  $x_2$  is having  $n$  elements. So, the number of rows in  $D$  should be  $n$ .

Student: (Refer Time: 21:05).

$x_2$ , what is the size of  $x_2$ ? See just go back to the previous page, what is the size of  $x_2$ ?  
Look at the size of  $x_2$ ; oh I did not I mean I mentioned in the last class. Yeah  $x_2$  is  $n$  cross  $1$   
 $x_1$  is  $n + 1$  cross  $1$ . So,  $d$  is  $n$  cross.

Student:  $n + 1$ .

$n + 1$  and what should be  $D$ ; what should be  $D$ ?

Student:  $n + 1$  (Refer Time: 21:50).

Yeah what are the elements of  $D$ ? What are the possible values for elements of  $D$ ?

Student: Diagonal  $1$ .

Diagonal  $1$ .

Student: Rest all  $0$ .

Rest all  $0$ . So, can I say that it is equal to an identity matrix, I will call this identity matrix as  $I$   
 $1$ .

Student: Into  $D$   $1$

Then I already have a 0 matrix  $O_1$ . So, can I write this as

Student:  $O_1$ .

$O_1$  transpose?

Student: Yeah.

$O_1$  is of size  $1 \times n$   $O_1$  transpose is of size  $n \times 1$  ok. But what is the size of  $I_1$ ?  $I_1$  is the identity matrix.

Student:  $n \times n$ .

$n \times n$ . So, it is  $n \times n$  identity matrix. So, let me call this equation as equation number 4. If I take the first sub interval, then it starts at  $\alpha$  by  $\omega$  the value of  $x_1$  at  $\alpha$  by  $\omega$  and if I take  $x_2$  at  $\alpha$  by  $\omega$  plus  $t_1$  the end of the second sub interval. Now, at this stage it seems I do not know whether it seems that they I mean the two can be related.

Student: Yeah.

It seems now what I will do is I will not try to prove this, but I will just make a statement and verify later. So, right now it seems that is all I mean we cannot prove it as it is with the current information. So, I will make a statement without proof, we will verify that it is correct later. So, I will say that this is  $F$  times  $x_2$  at  $\alpha$  by  $\omega$  plus  $T_1$  plus  $G$  so, where  $F$  and  $G$  have appropriate sizes. So, what is their size of  $F$ ? Size of  $F$  actually comes from the size of  $x_1$  and  $x_2$ , the number of rows should be the number of elements in  $x_1$  that is  $n + 1$  size of  $x_2$  is  $n$ .

So, it is  $n + 1$  cross  $n$  and  $G$  ok. What is  $G$ , what is the size of  $G$  that should be obvious right; what is the size of  $G$ ?

Student:  $n + 1$  x  $n$ .


Say when I am adding two elements and saying, it is equal to a vector both elements should be vectors column vectors. So, size of  $G$  is same as size of  $x + 1$ .

Student:  $n + 1$ .

$n + 1$  cross  $n$ .

So, let me call this equation 5 which I have just mentioned without proof. We will verify it later we will take this for granted 5 is just taken for granted which is mentioned without proof. We will verify that 5 is actually true. Now, we have so far 5 equations. Now, I take equation 3; take equation 3 and get an expression for  $x + 1$  evaluated by evaluated at  $\alpha$  by  $\omega$   $o$  plus  $t + 1$  using equation 1, can I do that? See the first equation gives the expression for  $x + 1$  from  $\alpha$  by  $\omega$   $o$  to  $\alpha$  by  $\omega$   $o$  plus  $t + 1$ . So, I can get the expression for  $\alpha$  by  $\omega$   $o$  plus  $t + 1$  using equation 1 ok. So, that let me do that first.

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Without loss of generality, let the outgoing value current be the last element of  $x_1$ .

$$C x_1 \left( \frac{\alpha}{\omega_0} + t \right) = 0 \quad (3)$$

where  $C: 1 \times (n+1)$ ,  $c = [0, \dots, 1]$  where  $0$  is  $1 \times n$  zero matrix.

$$x_2 \left( \frac{\alpha}{\omega_0} + t \right) = D x_1 \left( \frac{\alpha}{\omega_0} + t \right) \quad (4)$$


where  $D: n \times (n+1)$ ,  $D = [\bar{I}_1, 0^T]$  where  $\bar{I}_1$  is  $n \times n$  identity matrix

$$x_1 \left( \frac{\alpha}{\omega_0} \right) = F x_2 \left( \frac{\alpha}{\omega_0} + T_1 \right) + G \quad (5)$$

where  $F: (n+1) \times n$ ,  $G: (n+1) \times 1$

From (1) and (3),

$$C e^{A_1 t} x_1 \left( \frac{\alpha}{\omega_0} \right) + C e^{A_1 \left( \frac{\alpha}{\omega_0} + t \right)} \int_{\frac{\alpha}{\omega_0}}^{\frac{\alpha}{\omega_0} + t} e^{-A_1 \tau} u_1(\tau) d\tau = 0 \quad (6)$$



So, from 1 and 3, what I do is I take the equation one just take the solution. So, it is  $x_1$  at  $\alpha$  by  $\omega_0$  plus  $t_0$  means wherever there is  $t$ , I have to replace that  $\alpha$  by  $\omega_0$  plus  $t_1$  that is all ok. So, the right hand side if you look at the right hand side the first term is exponential  $A_1$  into  $t$  minus  $\alpha$  by  $\omega_0$ . So, replace  $t$  by  $\alpha$  by  $\omega_0$  plus  $t_1$ . So, what? So, the result is exponential of  $A_1$ .

Student:  $t_1$ .

$t_1$ ;  $A_1 t_1$  that is the first term into  $x_1$  evaluated at  $\alpha$  by  $\omega_0$ . There is one more term plus there is a  $I$  mean there is a factor  $C$  there plus  $C$  into exponential  $A_1$ . So, in place of  $t$ , I have  $t$  minus sorry in place of  $t$ ; I have  $ah$


Student: Alpha by omega o.

Yeah alpha by omega o. See if the letters are too small to read, let me know. Are you able to see read that ok? Alpha by omega o plus t 1 into integral the lower limit is alpha by omega o, the upper limit is alpha by omega o plus t 1 exponential minus A 1 tau u 1 of tau D tau. So, this is equal to 0 that is equation 3; that is what is equation 3 ok. So, let me give a number for this, let me call this equation number 6. Now, take equation 5. If you look at the right hand side of the equation 5, there is x 2 evaluated at alpha by omega o plus uppercase t with the subscript 1. Now, I can get an expression for this value of x 2 using equation.

Student: 2.

2 because this is the last instant of the second sub interval that is why I can use equation 2 to get the expression for this x 2 at alpha by omega o plus t 1 in equation 2.

(Refer Slide Time: 28:14)



From ⑤ and ②,


$$x_1\left(\frac{s}{\omega_0}\right) = F e^{A_2(\tau_1 - t_1)} x_2\left(\frac{s}{\omega_0} + t_1\right) + F e^{A_2(\alpha/\omega_0 + T_1)} \int_{\frac{\alpha}{\omega_0} + t_1}^{\frac{\alpha}{\omega_0} + T_1} e^{-A_2 \tau} u_2(\tau) d\tau + G - ⑦$$

From ④ and ⑦,

$$x_1\left(\frac{s}{\omega_0}\right) = F e^{A_2(\tau_1 - t_1)} D x_1\left(\frac{s}{\omega_0} + t_1\right) + F e^{A_2(\alpha/\omega_0 + T_1)} \int_{\frac{\alpha}{\omega_0} + t_1}^{\frac{\alpha}{\omega_0} + T_1} e^{-A_2 \tau} u_2(\tau) d\tau + G - ⑧$$

From ① and ⑧,

$$x_1\left(\frac{s}{\omega_0}\right) = F e^{A_2(\tau_1 - t_1)} D e^{A_1 t_1} x_1\left(\frac{s}{\omega_0}\right) + F e^{A_2(\tau_1 - t_1)} D e^{A_1(\alpha/\omega_0 + t_1)} \int_{\frac{\alpha}{\omega_0}}^{\frac{\alpha}{\omega_0} + t_1} e^{-A_1 \tau} u_1(\tau) d\tau + F e^{A_2(\alpha/\omega_0 + T_1)} \int_{\frac{\alpha}{\omega_0} + t_1}^{\frac{\alpha}{\omega_0} + T_1} e^{-A_2 \tau} u_2(\tau) d\tau + G - ⑨$$

$$x_1\left(\frac{s}{\omega_0}\right) = \left[ I_2 - F e^{A_2(\tau_1 - t_1)} D e^{A_1 t_1} \right]^{-1} \left[ F e^{A_2(\tau_1 - t_1)} D e^{A_1(\alpha/\omega_0 + t_1)} \int_{\frac{\alpha}{\omega_0}}^{\frac{\alpha}{\omega_0} + t_1} e^{-A_1 \tau} u_1(\tau) d\tau + F e^{A_2(\alpha/\omega_0 + T_1)} \int_{\frac{\alpha}{\omega_0} + t_1}^{\frac{\alpha}{\omega_0} + T_1} e^{-A_2 \tau} u_2(\tau) d\tau + G \right] \quad \text{--- (10)}$$


So, from equations 5 and 2; so, from 5 and 2 so, 5 says  $x_1$  at  $\alpha$  by  $\omega_0$  is equal to  $F$  into  $x_2$ . So,  $x_2$  if you look at equation 2 exponential  $A_2$  into so, wherever again, there is time replace it by  $\alpha$  by  $\omega_0$  plus  $t_1$ . So, if I do that what do I get? Exponential  $A_2$  into what?

Student: Capital T (Refer Time: 28:58).

Yeah uppercase T 1 minus lower case  $t_1$  ok. So, this multiplied with  $x_2$  at  $\alpha$  by  $\omega_0$  plus  $t_1$  plus plus there is one more term  $F$  into exponential  $A_2$  at  $\alpha$  by  $\omega_0$  plus  $T_1$  into integral  $\alpha$  by  $\omega_0$  plus  $t_1$  to  $\alpha$  by  $\omega_0$  plus  $T_1$ . So, integral of exponential minus  $A_2 \tau$   $u_2$  of  $\tau$   $D \tau$  plus; plus what?

Student: Plus G.

Plus G; plus G ok. So, let me call this equation number 7.

Now, what I will do is I will use this equation 7 and there is one equation 4. So, I will just replace  $x_2$  at  $\alpha$  by  $\omega_0 + t_1$  by  $D$  into  $x_1$  at  $\alpha$  by  $\omega_0 + t_1$  using equation 4. See what I am trying to do is I am using equations 4 and 7 at the same time and get rid of  $x_2$ . See there is one  $x_2$  in equation seven in the first term on the right hand side.

So, I will write that in terms of  $x_1$ . So, from 4 and 7,  $x_1$  at  $\alpha$  by  $\omega_0$  is equal to  $F$  exponential  $A_2$  into  $T_1$  minus  $t_1$  into  $x_2$ . So,  $x_2$  is now  $x_2$  at  $\alpha$  by  $\omega_0 + t_1$  from equation four is  $D$  into  $x_1$  of  $\alpha$  by  $\omega_0 + t_1$ .

Student: Which is capital T 1?

Which one? No I am just using equation 4 here; 4 and 7 right; now is that ok? This is up to now its if there is any error let me know, then plus  $F$  exponential  $A_2$   $\alpha$  by  $\omega_0 + t_1$  integral  $\alpha$  by  $\omega_0 + t_1$  to  $\alpha$  by  $\omega_0 + t_1$  exponential minus  $A_2$   $\tau_u$  2 of  $\tau$   $D$   $\tau$  plus  $G$  ok. I mean it appears doing I am doing too many manipulations, but I will ask you why I am doing this I mean once it becomes a bit clear ok. So, let me give some name number for this also equation number 8.

Now, what I will do is again use equation 1 and solve I mean get an expression for  $x_1$  at  $\alpha$  by  $\omega_0 + t_1$  in the first term on the right hand side. So, from equation 8 and 1; so, from 8 and 1 so, what I get is  $x_1$  at  $\alpha$  by  $\omega_0$  see finally, our intention is to get solution. Please note finally, our intention is to get solution, but the question is what are the quantities that are already known and what are the quantities that we need to determine; obviously, we need to determine  $x_1$  as a function of time  $x_2$  as a function of time.

But are there some quantities which are unknown that is the question ok. We will answer that question ok. Let me just do one more step and try to see. So, it is  $F$  exponential  $A_2$   $T_1$  minus



$t^{-1} D$ , now substitute for  $x_1$  using equation 1. So, what I get is exponential  $A^{-1}$  what? If I use the first equation  $A^{-1}$

Student:  $t^{-1}$ .

$t^{-1}$  into  $x_1$  at  $\alpha$  by  $\omega_0$  that is the first term plus  $F$  exponential  $A^{-2} t^{-1} \text{minus } t^{-1} D$  and the second term is exponential  $A^{-1}$  into  $\alpha$  by  $\omega_0$  plus  $t^{-1}$  into integral  $\alpha$  by  $\omega_0$  to  $\alpha$  by  $\omega_0$  plus  $t^{-1}$  exponential minus  $A^{-1} \tau u_1$  of  $\tau D \tau$ . So, this is nothing, but the first term in equation 8 upto this is first term in equation 8. So, the second term in equation 8 is written as it is plus  $F$  exponential  $A^{-2} \alpha$  by  $\omega_0$  plus  $t^{-1}$  integral  $\alpha$  by  $\omega_0$  plus  $t^{-1} \alpha$  by  $\omega_0$  plus  $t^{-1}$  exponential minus  $A^{-2} \tau u_2$  of  $\tau D \tau$  plus  $G$ .

So, I will give one number for this 9. Now in equation 9, you see that there is  $x_1$  at  $\alpha$  by  $\omega_0$  on the left hand side and there is also  $x_1$  at  $\omega_0$  in the first term on the right hand side.


So, I will just get an expression for  $x_1$  of  $\alpha$  by  $\omega_0$ . So, I will say that  $x_1$  of  $\alpha$ . So, bring the first term on the right hand side to the left hand side; first term on the right hand side of equation nine to the left hand side and then I get a factor. So, if I pre multiply by the inverse of that what I get is  $x_1$  at  $\alpha$  by  $\omega_0$  is equal to.

Student:  $I$  minus.

I already used  $I_1$  for some other identity matrix, now I use  $I_2$ . It I mean will show that both are not same they are of different sizes. So,  $I_2$  minus  $F$  exponential  $A^{-2} T^{-1} \text{minus } t^{-1} D$  exponential  $A^{-1} t^{-1}$ . Now, first of all you notice that  $F$  into exponential of  $A^{-2} T^{-1} \text{minus } t^{-1} D$  exponential of  $A^{-1} t^{-1}$  is a square matrix. This coefficient of  $x_1$  see in equation nine the first term equation 9 first term the coefficient  $x_1$  of  $\alpha$  by  $\omega_0$  is a square matrix that is a square matrix ok. So, the square matrix inverse into what, the rest of the terms on the right hand side.

So, the rest of the terms or the second term, the third term there are three more terms ok. So, let me write all those steps F exponential A 2 t 1 minus t 1 D exponential A 1 alpha by omega o plus t 1 alpha by omega o alpha by omega o plus t 1 exponential minus A 1 tau u of tau. So, sorry u 1 of tau D tau plus F exponential A 2 alpha by omega o plus T 1 integral alpha by omega o plus t 1 alpha by omega o plus T 1 exponential minus A 2 tau u 2 of tau D tau plus G, is this? Ok.

(Refer Slide Time: 39:01)




From (8) and (9),

$$x_1\left(\frac{\alpha}{\omega_0}\right) = F e^{A_2(\tau_1-t_1)} D e^{A_1 t_1} x_1\left(\frac{\alpha}{\omega_0}\right) + F e^{A_2(\tau_1-t_1)} D e^{A_1(\alpha/\omega_0+t_1)} \int_{\alpha/\omega_0}^{\alpha/\omega_0+t_1} e^{-A_1 \tau} u_1(\tau) d\tau + G \quad (8)$$

$$+ F e^{A_2(\alpha/\omega_0+\tau_1)} \int_{\alpha/\omega_0}^{\alpha/\omega_0+\tau_1} e^{-A_2 \tau} u_2(\tau) d\tau + G \quad (9)$$

$$x_1\left(\frac{\alpha}{\omega_0}\right) = \left[ I_2 - F e^{A_2(\tau_1-t_1)} D e^{A_1 t_1} \right]^{-1} \left[ F e^{A_2(\tau_1-t_1)} D e^{A_1(\alpha/\omega_0+t_1)} \int_{\alpha/\omega_0}^{\alpha/\omega_0+t_1} e^{-A_1 \tau} u_1(\tau) d\tau + F e^{A_2(\alpha/\omega_0+\tau_1)} \int_{\alpha/\omega_0}^{\alpha/\omega_0+\tau_1} e^{-A_2 \tau} u_2(\tau) d\tau + G \right] \quad (10)$$

$I_2$  is  $(n+1) \times (n+1)$  identity matrix



So, we call this as a equation 10 ok. So, I will just do one more step and then see why we do all these manipulations. By this time if you are able to guess very good, otherwise let us discuss. Now, we have used an identity matrix  $I_2$ , what is the size  $I_2$  is an identity matrix;  $I_1$  was  $n$  cross  $n$  identity matrix. This  $I_2$  is what is the size of  $I_2$ ?

Student:  $n + 1$ .

$N + 1$  cross  $n + 1$ .

Student:  $\tau_1$  varies.

Equation 10.

Student: Size of  $I_2$ .

Size of  $I_2$ ; see left hand side is having  $n + 1$  elements  $\times 1$ .

Student: (Refer Time: 39:50) also you said that this is a square matrix

Yes.

Student:  $F e^{\text{power } A^2}$ . Yeah that is from equation 9.

Student: Yes.

See left hand side is  $\times 1$ , the first term on the right hand side also has  $\times 1$ .

Student: Ok.

So; that means, this coefficient should be square matrix see the other possibility is scalar.

Student: Yes.

But it is not a scalar.

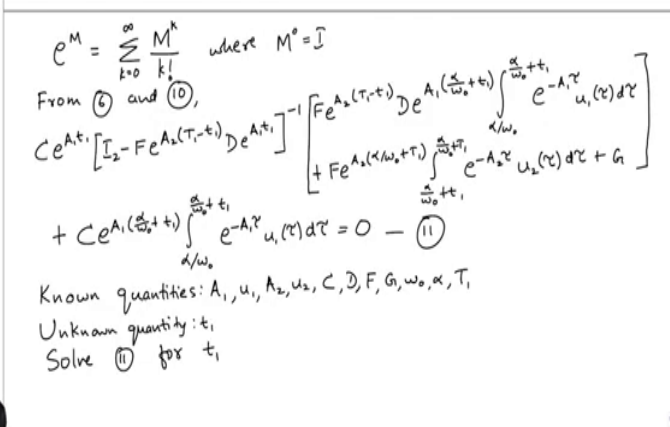

Student: Yes.

Say let me first.

Student: Got it.

Make a few points ah; see I am assuming many things. For example, I presume all of you know what is matrix exponential I mean I did not even make a mention of that I presume that you know; you are familiar with matrix exponential ok.

(Refer Slide Time: 40:44)



The whiteboard contains the following text and equations:

$$e^M = \sum_{k=0}^{\infty} \frac{M^k}{k!} \text{ where } M^0 = I$$

From (6) and (10),

$$C e^{A_1 t_1} \left[ I_2 - F e^{A_2 (T_1 - t_1)} D e^{A_1 t_1} \right]^{-1} \left[ F e^{A_2 (T_1 - t_1)} D e^{A_1 \left( \frac{\alpha}{\omega_0} + t_1 \right)} \int_{\frac{\alpha}{\omega_0}}^{\frac{\alpha}{\omega_0} + t_1} e^{-A_1 \tau} u_1(\tau) d\tau \right. \\ \left. + F e^{A_2 (\alpha/\omega_0 + T_1)} \int_{\frac{\alpha}{\omega_0}}^{\frac{\alpha}{\omega_0} + T_1} e^{-A_2 \tau} u_2(\tau) d\tau + G \right]$$
$$+ C e^{A_1 \left( \frac{\alpha}{\omega_0} + t_1 \right)} \int_{\frac{\alpha}{\omega_0}}^{\frac{\alpha}{\omega_0} + t_1} e^{-A_1 \tau} u_1(\tau) d\tau = 0 \quad (11)$$

Known quantities:  $A_1, u_1, A_2, u_2, C, D, F, G, \omega_0, \alpha, T_1$   
Unknown quantity:  $t_1$   
Solve (11) for  $t_1$

Let me just see suppose I have a matrix M ok. So, if M is a square matrix exponential M as long as M is square matrix that is the restriction M is square matrix. What is exponential M?

By definition, it is given by the series. So, it is similar to the scalar exponential  $M$ , it is  $\sum M^k / k!$ .

Student:  $k$  factorial.

$k$  factorial and  $k$  take values from?

Student: 0 to.

0 to?

Student: Infinity.

Infinity. And you know what is  $M$  raised to  $k$ .

Student: Yes.

$M$  raised to 1 is  $M$ ,  $M$  raised to 2 is  $M$  into  $M$ ,  $M$  raised to 3 is  $M$  into  $M$  into  $M$  what is  $M$  raised to 0?

Student: Identity matrix.

Identity matrix by definition  $M$  raised to 0 is identity matrix. So, this is the definition of matrix exponential. So, that so if  $M$  is as a matrix square matrix then exponential of  $M$  is also.

Student: Square matrix.

Square matrix because you look at the terms. See  $M$  raised to 0 is identity matrix of the same size  $M$  raised to 1 is just  $M$ ;  $M$  raised to 2 is again a square matrix of the same size as  $M$ .

Student: Yes.

So exponential of the matrix is also a square matrix ok, yeah. Now let me finally, write one more equation. So, what I will do is I will use this equation 10 and substitute this in equation 6. See equation 6. If you notice the first term has  $x_1$  at  $\alpha$  by  $\omega_0$ . So, now, I have an expression for  $x_1$  at  $\alpha$  by  $\omega_0$  from equation 10. See equation 10 is essentially giving the expression for  $x_1$  at  $\alpha$  by  $\omega_0$ .

So, I substitute this expression for  $x_1$  at  $\alpha$  by  $\omega_0$  in 6. So, from 6 and 10 so, the first term of equation 6 is  $C \exp(-A_1 t) C \exp(-A_1 t)$  into  $x_1$  at  $\alpha$  by  $\omega_0$ . So, I write this as  $I_2 \cos(\omega_0 t - \phi) \exp(-A_1 t)$  inverse into  $F \exp(-A_2 T) \cos(\omega_0 t - \phi) \exp(-A_1 t)$  plus  $t$  into  $\int_{\alpha}^{\omega_0} \omega_0 \exp(-A_1 \tau) u_1(\tau) D \tau$ .

Plus  $F_2$  sorry  $F \exp(-A_2)$  into  $\alpha$  by  $\omega_0$  plus  $T$  into  $\int_{\alpha}^{\omega_0} \omega_0 \exp(-A_1 \tau) u_1(\tau) D \tau$  plus  $G$ . So, this is the first term of equation 6. If you look at equation 6. So, this is the first term of equation 6, the second term written as it is. So, the second term of equation 6 is on the left hand side  $C \exp(-A_1 \alpha) \exp(-A_1 \omega_0) \exp(-A_1 t)$  plus  $t$  into  $\int_{\alpha}^{\omega_0} \omega_0 \exp(-A_1 \tau) u_1(\tau) D \tau$ . This is equal to 0. Now, why I did all these manipulations?

Now, I ask the question what is the purpose of doing all these manipulations? Myself my intention is to get the solution yeah, I did not want to spell out the answer that is why I did maybe I thought, you will guess or find out why I did all these ok. See let me give a hint. What are the known quantities; what are the known quantities? Let us start with the differential equation. First if you take a first order differential equation, what is known there?  $A_1$ ;  $A_1$  is known  $u_1$  is known see there  $u_1$  is known because you know what is voltage source or current source  $A$ . Then  $A_2$  is known;  $u_2$  is known then?

Student: C is known.

C is known, then what about D?

Student: D is also.

D is also known. Then I mean now I will say that see when it comes to the equation relating  $x_1$  at  $\alpha$  by  $\omega_0$  and  $x_2$  at  $\alpha$  by  $\omega_0$  plus upper case T 1. There are 2 matrices F and G ok. So, that t equation was just assumed. So, I will make one more statement and prove it later that F and G are also known ok. So, F and G are known. Now, there are a few other quantities that come. For example,  $\omega_0$  it is known. See if it is 50 Hertz, it is  $100\pi$ ; if it is 60 Hertz, it is  $120\pi$  that is also known ok. Then what else we know? There are few more terms  $\alpha$  is there. Now, I assume that  $\alpha$  is also known; for a given value of  $\alpha$ , I am doing the analysis. Then there is an upper case.

Student: T1 T 1.

T 1 that is also known. All these are known quantities; see upper case T 1 is nothing, but the total period divided by  $p$ ; total period is known, pulse number is known. Now there is one more quantity which is lower case t with a subscript 1.

Student: Which is not known?

That is not known that is the answer. So, the unknown quantity is lower case t with the subscript 1. Now, why do we need this lowercase t 1? See if you look at the solution say solution is given by equations 1 and 2, it is given by equations 1 and 2 now t 1 is deciding when the first int sub interval stops. So, I should know when it stops. So, when is equation 1 applicable upto what time it is applicable I should know ok. So, I should know, but ah I mean as of now I do not know I have to solve for it. So, how to solve for it? So, all the manipulations done so far is for solving for t 1.

So, if you look at the last equation so, let me give a number for that. So, the last equation was this equation 11. So, in equation 11, it is a long equation, but everything other than  $t_1$  is known. So, solve for this unknown quantity  $t_1$  using equation 11. So, equation 11 is not a vector matrix equation, it is scalar equation. So, it is just one non-linear equation, it has to be solved for  $t_1$ . So, solve equation 11 for  $t_1$ . So, how do you solve? Yeah use any numerical method Newton's method, Newton Raphson method is one of the I mean obvious choice for a method ok.

Of course calculator it is tough; calculator will not allow matrix exponential at 6 ok. So, it is not ruled out calculator is ruled out. So, one has to use ah computer I mean try to write a short program for any numerical method, Newton Raphson is all of you are familiar with Newton Raphson. I guess it is used for power flow. So, one can use for example, Newton Raphson method and solve for  $t_1$ .

Now our objective is not solving  $t_1$  alone, our objective is to get the solution. So, then what if  $t_1$  is known then can I get the solution directly?

See go to the solution. What is the solution equation 1 and 2? Oh sorry equation 1 and 2 are solutions, but they are solutions provided everything on the right hand side is known.

Student: We do not know M; M.

M? Where is M?

Student: Number of switches that are (Refer Time: 50:26).

No M is nowhere coming in the.

Student: They are not in the equation.



No we will look at the size of  $M$  for different cases.  $M$  is right now not known. It depends on whether it is 2 and three valve conduction mode 3 and 4 valve conduction mode.

Student: Yes.

That is not known. Now that is itself is also not known ok. Now what we normally do is first assume something. Suppose it is 2 and 3 valve conduction mode. Try to find an answer if you get an answer, it is 2 and 3 valve conduction. If you do not get an answer, it is not 2 and 3. Try something else see whether it is 3 and 4 the again if you get a solution that is the answer otherwise, it is not the answer. So, again it is only by trial and error, we can make out whether it is 2 and 3 or 3 and 4.

Student: But from the 12 terms.

That I mean that is even more complicated. Let us first concentrate on the simple case ok. So, so if 1 and 2 are solutions what is required?


Student: Initial values (Refer Time: 51:21)

Yeah, to use this equation one we should know all things on the right hand side. So,  $x_A 1$  is known  $A 1$  is known, but what about  $x_1$  at  $\alpha$  by  $\omega$  o? See that is the only thing that is required  $u_1$  is also given. So,  $x_1$  at  $\alpha$  by  $\omega$  has to be determined. So, how to find that? Which equation will give me  $x_1$  at  $\alpha$  by  $\omega$  o??

Student: 10;10.

Equation number 10 10 because if you look at 10, 10 has something which is I mean having all known quantities on the right hand side so, but for using ten you need  $t_1$  and  $t_1$  is obtained from 11 ok.


(Refer Slide Time: 52:06)



From (6) and (10),

$$C e^{A_1 t_1} \left[ I_2 - F e^{A_2 (T_1 - t_1)} D e^{A_1 t_1} \right]^{-1} \left[ F e^{A_2 (T_1 - t_1)} D e^{A_1 \left( \frac{\alpha}{\omega_0} + t_1 \right)} \int_{\frac{\alpha}{\omega_0}}^{\frac{\alpha}{\omega_0} + t_1} e^{-A_1 \tau} u_1(\tau) d\tau \right. \\ \left. + F e^{A_2 (\alpha/\omega_0 + T_1)} \int_{\frac{\alpha}{\omega_0}}^{\frac{\alpha}{\omega_0} + t_1} e^{-A_2 \tau} u_2(\tau) d\tau + G \right] \\ + C e^{A_1 \left( \frac{\alpha}{\omega_0} + t_1 \right)} \int_{\frac{\alpha}{\omega_0}}^{\frac{\alpha}{\omega_0} + t_1} e^{-A_1 \tau} u_1(\tau) d\tau = 0 \quad (11)$$

Known quantities:  $A_1, u_1, A_2, u_2, C, D, F, G, \omega_0, \alpha, T_1$   
 Unknown quantity:  $t_1$   
 Solve (11) for  $t_1$   
 Obtain  $x_1 \left( \frac{\alpha}{\omega_0} \right)$  using (10)  $\rightarrow$  Use (1) for obtaining solution in 1<sup>st</sup> subinterval  
 Compute  $x_2 \left( \frac{\alpha}{\omega_0} + t_1 \right)$  using (4)  $\rightarrow$  Use (2) for obtaining solution in 2<sup>nd</sup> subinterval



Once you know  $t_1$ , then obtain  $x_1$  at  $\alpha$  by  $\omega_0$  using (10). Then from this you can just use equation (1) for solution in first sub interval for obtaining the solution in first sub interval, is that ok? If I just know  $x_1$  at  $\alpha$  by  $\omega_0$  once I know this can I just use equation (1) and solve for  $x_1$  in the first sub interval. Then in the second sub interval I want  $x_2$ . So, what is unknown on the right hand side?

Student:  $x_2$ .

$x_2$  at  $\alpha$  by  $\omega_0$  plus  $t_1$ . So, how to find that? That is the only unknown quantity on the right hand side of equation (2). So, how to get  $x_2$ ?

Student: 4 4.

So, it is nothing, but equation.

Student: 4.

4.

Student: 4.

Because right hand side we know  $x_1$  at the end of first sub interval. So, get the value of  $x_2$  at  $\alpha + \omega_0 + t_1$  using 4.

Student: (Refer Time: 53:29) at  $x_1$  at  $\alpha + \omega_0$ .

Now, that  $x_1$  is obtained from the previous step. See when I say solution, solution means.

Student: (Refer Time: 53:34).

Getting the expression from the starting to ending point of the first sub interval ok. So, then compute  $x_2$  at  $\alpha + \omega_0 + t_1$  using 4.

Student: (Refer Time: 54:01).

Then use equation 2.

Student: (Refer Time: 54:03).

For obtaining solution in second sub interval obtaining the solution in the second sub interval fine. So, it requires initially the solution of a non-linear algebraic equation, equation 11 to

solve for this unknown quantity  $t_1$ . From that everything else follows. So, how exactly to use this in the special cases; we will see that in the next class.