

**Power Management Integrated Circuits**  
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**Lecture – 75**

**Stabilising a Voltage-Mode Hysteretic Converter using Resr, Relation between Fsw and the Hysteresis Window**

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Con is discharging    Con is charging  
 $I_L$      $I_{Lmax}$      $I_{Lmin}$   
 $V_o$   
 $V_{nom}$   
 $90^\circ$  phase shifted  
 $V_o$  is not bounded between  $V_{ref} \pm \frac{\Delta V_o}{2}$  due to  $90^\circ$  phase shift.  
 This makes it unstable.  
 In order to stabilize the output we want  $V_o$  in phase with  $I_L$ .  
 $\Delta I_L$      $V_o \rightarrow 90^\circ$  out of phase  
 $C_{out}$

How can we bring that in phase? Then it will make it unstable 180 degree phase shift total. Then if you make another 90 degrees then it will become a positive feedback. Already, I have a 180 degree phase shift due to negative feedback additional 90 degree coming from the output. I cannot add another 90 degrees there. I have to get rid of that additional 90 degree actually correct. So, how can we reduce that phase shift? Let me so, this is your C out let me mark it I will call it C out.

So, we know this current is going I correct. So, this voltage is 90 degree out of phase. So, you remember Resr what would happen now? What will the ripple be?

So, you are this is your V out. So, delta V out will be. Delta I L this is delta IL actually. Only the ripple component is going inside the capacitor. So, delta I L Resr plus what is the other component? This is charge Q, how much was Q? This is Tsw by 2 and this is.

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$$\Delta V_o = \Delta I_L R_{esr} + \frac{\Delta I_L}{8 C F_{sw}}$$

$\downarrow$  in phase                       $\downarrow$  90° out of phase

$$\Delta I_L R_{esr} \gg \frac{\Delta I_L}{8 C F_{sw}}$$

Usually if  $R_{esr} C > \frac{D \cdot T_{sw}}{2}$  &  $(1-D) \frac{T_{sw}}{2}$

then output is stable  $\rightarrow \Delta V_o$  is in phase with  $\Delta I_L$  and  $V$  is bounded

So, half into Tsw by 2 into delta IL by 2 correct and then q equal to CV C delta V naught Q by C ok. So, it makes it delta I L by 8 times of Fsw C Fsw correct. Now, if you break delta IL into V in minus V out by L and d times T s w then you will get Fsw square ah, but for simplicity I am just writing in terms of delta I L ok. So, this is in phase this is 90 degree out of phase. And I am inserting in phase which means delta IL Resr should be much much greater than delta IL over 8 C Fsw let us call it C out here ok, but how much more? Not dominating actually the only thing is that.

See you may have a little phase shift as long as your peaks are aligned you are fine ok. You may have some non-linearity, I mean if you keep increasing the this Rc actually it will become more linear and linear let us say I keep increasing the if I keep increasing the R then my delta I L Resr component will keep increasing. If I increase the C then it will keep reducing the other components.

So, both will have the same effect. So, it is a product of R and C which matters here. So, if I keep increasing the product of R and C your delta IL Resr component will start dominating over delta I L over 8 C out Fsw which is 90 degree out of phase ok. So, you have to make sure that your Resr product in such a way that your peaks are aligned ok. And usually if your Resr C out is greater than D times Tsw by 2 and 1 minus D times Tsw by 2 then output is stable which means delta V out is in phase with delta IL and V out is bounded ok. So, we

need to meet both the conditions  $D \times T_{sw} > 2$  and  $(1-D) \times T_{sw} > 2$  and if you want to cater to full range 0 to 100 percent what does it mean?

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The whiteboard contains the following content:

- Circuit Diagram:** A buck converter circuit with input voltage  $V_o$ , inductor  $L$ , and capacitor  $C$ . The ripple current  $\Delta I_L$  is shown across the inductor.
- Waveform:** A triangular ripple current waveform with peak-to-peak value  $\Delta I_L$  and period  $T_{sw}$ .
- Equation 1:** 
$$\Delta V_o = \Delta I_L R_{err} + \frac{\Delta I_L}{8 C_{nt} f_{sw}}$$

Annotations:  $\Delta I_L R_{err}$  is labeled "70° phase", and  $\frac{\Delta I_L}{8 C_{nt} f_{sw}}$  is labeled "90° out of phase".
- Equation 2:** 
$$\Delta V_o = \frac{\Delta I_L}{8 C_{nt} f_{sw}}$$
- Equation 3:** 
$$R = C \Delta V_o \Rightarrow \Delta V_o = \frac{R}{C}$$
- Text:** "Usually if  $R_{err} \cdot C_{nt} > D \cdot \frac{T_{sw}}{2}$  &  $(1-D) \frac{T_{sw}}{2}$  then output is stable  $\rightarrow \Delta V_o$  is in phase with  $\Delta I_L$  and  $V_o$  is bounded."
- Text:** "for  $D=0$  to  $1$   $R_{err} \cdot C_{nt} > \frac{T_{sw}}{2}$ "

For  $D$  equal to 0 to 1  $R_{err}$  should be greater than  $T_{sw} / 2$  and if you do your hand calculation, you will see your this is a 0 actually. So, this will add phase lead and that phase lead is I think 70 degrees or so ok.

So, out of 90 degrees, 70 degrees is canceled. So, the left phase shift left over will be 20 degrees or less that is keeping increasing  $R_{err}$  it will start approaching to 0 ok. So, there is a derivation for that. So, basically, you have to write the equation in terms of  $t$  and then you would have to differentiate it to find the maxima of the outputs. So, the maxima of this  $V_{out}$  the maximum point of this is same as the maximum point of the other component which is.

So, the peak of this is aligned with the peak of the other, I mean you have two components. So, both peaks are aligned. So, which means the maximum of that should be same as the other and you will get a condition for  $t$  and then you will see that if you substitute that then you will get  $T_{sw} / 2$  that the derivation is given in my thesis. So, you have to write everything in the time domain and just keep following the how you have I mean you have a current. So, what will be the voltage from there you can find and write the equation and then you have to differentiate to get the peak and valley ok.

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Figure 4.3. Ripple of a hysteretic converter.

Since ripple peak will occur during  $T_{off}$  (due to phase lag) and  $V_i = -V_o$ ,  $\Delta I_L(t) = \frac{1}{L} \int_{T_{off}} V_{off} dt$  during OFF time, eq. 4.9 can be re-written for peak as:

$$I_{L_{peak}} = \frac{1}{2} T_{off} - R_{ESR} C_o \quad (4.10)$$

Similarly ripple valley will occur during  $T_{on}$  and  $V_i = V_{in} - V_o$ ,  $\Delta I_L(t) = \frac{1}{L} \int_{T_{on}} V_{on} dt$  during ON time, eq. 4.9 can be re-written for valley as:

$$I_{L_{valley}} = \frac{1}{2} T_{on} - R_{ESR} C_o \quad (4.11)$$

In order to bound the output ripple between hysteretic window, both  $I_{L_{peak}}$  and  $I_{L_{valley}}$  should be zero i.e.  $i=0$  in eq. 4.8 will make the ripple voltage same as voltage across ESR which is in-phase with the inductor ripple current and bound

So, maxima and minima can be found and that will give you the condition for  $t_{peak}$  and you substitute for  $t$  value is this and you substitute in the original ripple equation this is what you get actually ok.  $t_{off}$  you get here and. So,  $t_{off}$  is nothing, but  $(1 - D) T_{sw}$  and for the other case. So, this is a peak and for valley you get  $t_{on}$  by  $2 t_{on}$  by  $2$  is nothing, but  $D T_{sw}$ .

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Figure 4.4. Offset in the output ripple due to lower  $R_{ESR} C_o$ .

#### 4.1.2 Transient Response of a Hysteretic DC-DC Switching Converter

In order to understand the transient behavior of a hysteretic converter, let's first analyze the transient response of an ideal switching converter and then see how it gets affected by various circuit parameters of a hysteretic converter. Fig. 4.5 shows the load transient response of an ideal converter [25] depicting the behavior of output voltage,  $V_o$ , and inductor current,  $i_L$ , under a sudden change in the output load current,  $I_o$ . Suppose that the output voltage is initially regulated at  $V_{ref}$  and current load step of  $\Delta I_o$  is applied at  $t = t_1$ . Since we have assumed an ideal converter, it responds quickly to the perturbation applied at the output and keeps the switch  $M_2$  ON until the output voltage is recovered. Due to the

So, if you have to meet the entire range  $0 \leq D \leq 1$ . So, and this is what I have shown if we want to meet only if you meet only one condition, when you are changing the duty cycle then you

may have a problem around 50 percent you should be ok, but when you go more than 50 percent one will limit other will not limit and I have shown that condition here.

See this is limited, but other side is not limited and when you have a D greater than 0.5 and then. So, if I try to meet let us say at 50 percent duty cycle then this may happen at higher or lower duty cycle. So, one will be limited, but other ones see if you want to limit both sides then you have to meet that condition  $T_s w$  by 2 ok.

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$\Delta I_L \text{ Resr} \gg \frac{\Delta I_L}{8 C F_{sw}}$  then we can ignore  $\frac{\Delta I_L}{8 C F_{sw}}$

$\Delta V_o = \Delta I_L \cdot \text{Resr}$

$\Delta I_L = \frac{V_{in} D \cdot (1-D) T_{sw}}{L}$

$\Delta V_o = \frac{V_{in} D \cdot (1-D) T_{sw}}{L} \times \text{Resr}$

$\Delta V_o = \Delta V_H$

$F_{sw} = \frac{1}{T_{sw}} = \frac{V_{in} D \cdot (1-D)}{\Delta V_H \cdot L} \times \text{Resr}$

Assume  $V_{in}, \Delta V_H, L$  &  $\text{Resr}$  are fixed but  $V_o$  is variable  $\rightarrow D$  is varying.

$\Delta V_o$  vs  $D$  graph:  $F_{sw}$  is fixed. The graph shows a parabolic curve of  $\Delta V_o$  vs  $D$  with a peak at  $D = 0.5$ .

$F_{sw}$  vs  $D$  graph:  $\Delta V_o$  is fixed. The graph shows a parabolic curve of  $F_{sw}$  vs  $D$  with a peak at  $D = 0.5$ .

So, let us take an example now. So, if this is the condition  $\Delta I_L \text{ Resr}$  is much much greater than  $\Delta I_L$  by  $8 C F_{sw}$  then we can ignore ok. So, what will happen your  $\Delta V_o$  will be  $\Delta I_L$  into  $\text{Resr}$ . What is  $\Delta I_L$ ?

$V_{in} \text{ minus } V_{naught} \cdot D \text{ times } 1 \text{ minus } D \cdot T_{sw} \text{ by } L$ . So, which means  $\Delta V_o$  equal to  $V_{in}$  what is  $\Delta V_o$  here in this?  $\Delta V_H$ . So, which means  $F_{sw}$  equal to  $1$  over  $T_{sw}$  which becomes  $V_{in} 1 \text{ minus } D$  over into  $\text{Resr}$ . So, this is your switching frequency which is depending on your hysteretic window  $\Delta V_H$  ok. Your inductor value, your  $V_{in}$  and duty cycle also  $\text{Resr}$ . So, it is a function of everything if you look and everything can be changed except your  $\Delta V_H$ . You can only control  $\Delta V_H$  rest of the thing is not your control because  $V_{in}$  may vary if somebody is looking for variable output then  $V_o$  also may vary. So, let us say just take the case of your if output is varying. So, assume  $V_{in}, \Delta V_H, L$  and

Resr are fixed, but V out is variable, this implies D is varying. Do you remember the ripple curve duty cycle versus delta V out. If you have a fixed input and output is varying. How does it look like?

So, if this is your D going from 0 to 1. See it was symmetric around 0.5. This is your delta V out. So, which means and this is for Fsw is fixed. Now, if I fix delta V out I am forcing here delta V out to be delta VH. So, if this is Fsw.

What would happen to the switching frequency? Can anybody tell?

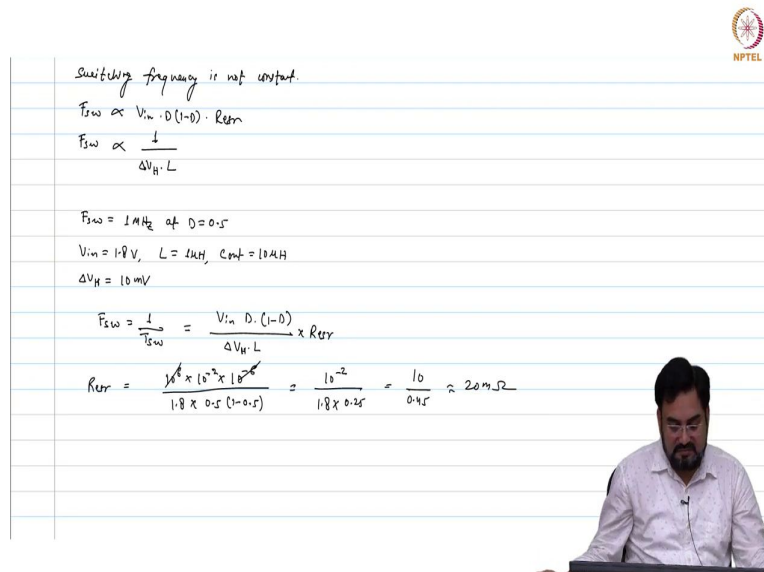
It will be parabolic, since it is proportional to D times into 1 minus D correct. So, if let us say, at 0 and 1 it will be 0 because it would not be switching at all let us take the case 0.1. So, 0.1 into 0.9 which is 0.09 pretty close to again 10 percent duty cycle and if you take the other case 90 percent duty cycle then again same ok. So, which means, whatever you had here it will be roughly at 0.5 it will be one tenth of that or it is a 0.09 actually.

So, it is slightly more than that, but roughly you can say that. So, the peak for the maximum frequency will be at 0.5 then it will start reducing. So, it will have the same profile. So, which means as long as I am operating around 0.5 my frequency would not vary, but the moment I go to the extreme duty cycles my frequency will look very bad.

So, this is only with the duty cycle now, assume your V in is also varying see if V in is increasing then, we know the ripple is in ripple increases with that. So, your switching frequency is proportional to V in here correct and inversely proportional to L and proportional to Resr ok. And from here you can intuitively look at if let us say let us say I increase the L.

Your ripple will reduce when ripple will reduce then the ripple which you seen by the comparator is less and, but it is you are limiting the ripple at delta V H plus minus delta V H by 2. So, the only way to increase that amplitude is to reduce the frequency. If you're increasing the L same thing happens with the Resr. If you are increasing the Resr then the ripple will increase, now it has to reduce, the feedback has to reduce because you have to limit within that window. So, it has to increase the switching frequency in that case ok.

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Switching frequency is not constant.

$$F_{sw} \propto V_{in} \cdot D(1-D) \cdot R_{esr}$$
$$F_{sw} \propto \frac{1}{\Delta V_H \cdot L}$$

$F_{sw} = 1 \text{ MHz}$  at  $D = 0.5$

$V_{in} = 1.8 \text{ V}$ ,  $L = 1 \mu\text{H}$ ,  $C_{out} = 10 \mu\text{F}$

$\Delta V_H = 10 \text{ mV}$

$$F_{sw} = \frac{1}{T_{sw}} = \frac{V_{in} \cdot D \cdot (1-D)}{\Delta V_H \cdot L} \times R_{esr}$$
$$R_{esr} = \frac{10^6 \times 10^{-6} \times 10^{-6}}{1.8 \times 0.5 \times (1-0.5)} = \frac{10^{-2}}{1.8 \times 0.25} = \frac{10}{0.45} \approx 20 \text{ m}\Omega$$

Switching frequency is not constant  $F_{sw}$  is proportional to  $V_{in}$ . then what else into  $D$  into  $1$  minus  $D$  to  $R_{esr}$  and its inversely proportional to  $1$  over  $\Delta V_H$  into  $L$ . So, once you have fixed the  $L$  and you know  $R_{esr}$  everything is fixed. Now, you can decide  $\Delta V_H$  at what frequency you want to operate, but it will vary  $R_{esr}$  is also fixed ok. So, one way is like you use a very small  $R_{esr}$  capacitor 1 milli ohm or so and you in series you had a very like a accurate resistor  $R_{esr}$  whatever you require ok.

So, let us say I want to design this let us see how I mean how much  $R_{esr}$  I will require. So, let us say  $F_{sw}$  equal to 1 megahertz at  $D$  equal to 0.5 correct.  $V_{in}$  equal to 1.8 volt I am choosing  $L$  equal to 1 microhenry and  $C$  equal to  $C_{out}$  equal to 10 micro. Any other parameter  $\Delta V_H$  equal to ripple let us say I want 10 millivolt ok. So, substitute here and we can find value of  $R_{esr}$  correct when 1 mega which is  $10^6$  into how much was  $10^6$  power minus 3 into  $10^6$  power minus 6 over.

Sorry  $R_{esr}$  that will  $10^6$  multiplied by this was 10 milli I think it was  $10^6$  power minus 3  $10^6$  power minus 2 into  $L$  is stable divided by 1.8 into 0.5  $1 - 0.5$ . How much is this? So, this cancels out. So, this is your  $10^6$  power minus 2 over 1.8 into this becomes 0.25. So, 10 divided by how much is this? 0.45 pretty close to 20 milli ohm.

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$$F_{sw} = \frac{I}{T_{sw}} = \frac{V_{in} \cdot D \cdot (1-D)}{A V_H L} \times R_{err}$$

$$R_{err} = \frac{10^6 \times 10^{-3} \times 10^{-6}}{1.8 \times 0.5 \times (1-0.5)} = \frac{10^{-2}}{1.8 \times 0.25} = \frac{10}{0.45} \approx 20 \text{ m}\Omega$$

$$R_{err} \cdot C_{out} > \frac{T_{sw}}{2}$$

$$\frac{T_{sw}}{2} = 500 \text{ ns}$$


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$$R_{err} \cdot C_{out} = 20 \times 10^{-3} \times 10^{-5}$$

$$= 2 \times 10^{-7} = 200 \text{ ns}$$

Correct. So, now, let us see if Resr meets that condition what we have and we know that Resr into C out should be greater than Tsw by 2.

So, how much is Resr? Tsw by 2 is 500 nanosecond and Resr in to C out is equal to 20 into 10 power minus 3 into to cap was 10 power minus 5 how much is this? 2 into 10 power minus 7, which is how much? 200 nanosecond which is less than that. So, this does not meet that condition which means you have to increase either C out or Resr. And then your frequency will change.

If I increase the C because my frequency is not depending on C here it may depend, but we are considering that that component is very small the ripple due to cap only. So, it may vary maybe 5 to 10 percent, but a frequency will be a strong function of Resr here ok. So, if you change the Resr then it will change the frequency, but if I change the capacitor then it will not affect your frequency much or you change the delta V H to get the frequency back.