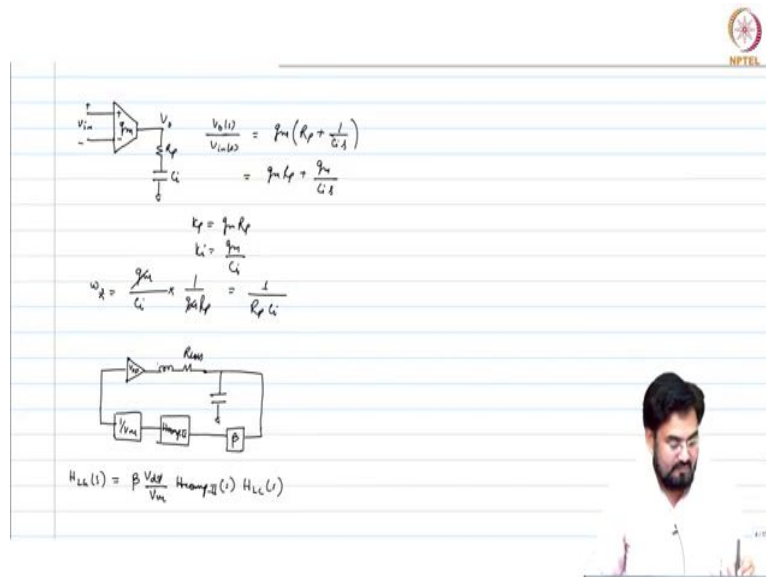


Power Management Integrated Circuits
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Lecture – 52

Type – II Compensator using Gm-C Architecture – Part 1



Small signal model of a buck converter with type-II compensation is shown in the above image. The loop gain transfer function will be :

$$H_{LG}(s) = \beta \frac{V_{dd}}{V_m} H_{COMP-II}(s) H_{LC}(s)$$

After substituting the value of $H_{COMP-II}(s)$ and $H_{LC}(s)$, the loop gain transfer function will be :

$$H_{LG}(s) = \beta \left(\frac{V_{dd}}{V_m} \right) \frac{K_i}{s} \left(1 + \frac{k_p}{k_i} s \right) \left(\frac{1/LC}{s^2 + s \left(\frac{R_{LOSS}}{L} \right) + \frac{1}{LC}} \right)$$



$$= \beta \frac{V_{out}}{V_{in}} \frac{k_2}{s} \left(1 + \frac{k_1}{k_2} s\right) \left(\frac{1/LC}{s^2 + \frac{R_{ESR}}{L} s + 1/LC}\right)$$

$$Q_o = \frac{1}{R_{ESR}} \sqrt{\frac{L}{C}}$$

$$\frac{\omega_o}{Q_o} = \frac{\beta \sqrt{LC}}{\frac{1}{V_{out}} \sqrt{L/C}} = \frac{R_{ESR}}{L}$$

L.G(s) has 3-poles and 1-zero.
 so system is unstable.
 We need one more zero.
 Add ESR with output cap C.
 $H_{CL}(s) = \frac{1}{LC} (1 + R_{ESR} s)$



We have three poles and one zero in the loop gain transfer function which means the system is unstable and type II compensation will not work. We have to introduce one more zero inside the ugb to make the system stable. We can introduce zero by putting a resistance in series with an output capacitor of the buck converter. R_{ESR} is now connected in series with the output capacitor. ESR stands for equivalent series resistance. ESR is parasitic resistance that comes generally with the capacitor but here we are intentionally putting a higher value resistance.



$$\frac{1}{V_{out}} \sqrt{L/C} \quad L$$

L.G(s) has 3-poles and 1-zero.
 so system is unstable.
 We need one more zero.
 Add ESR with output cap C.

$$H_{CL}(s) = \frac{\frac{1}{LC} (1 + R_{ESR} s)}{s^2 + \frac{R_{ESR}}{L} s + 1/LC}$$


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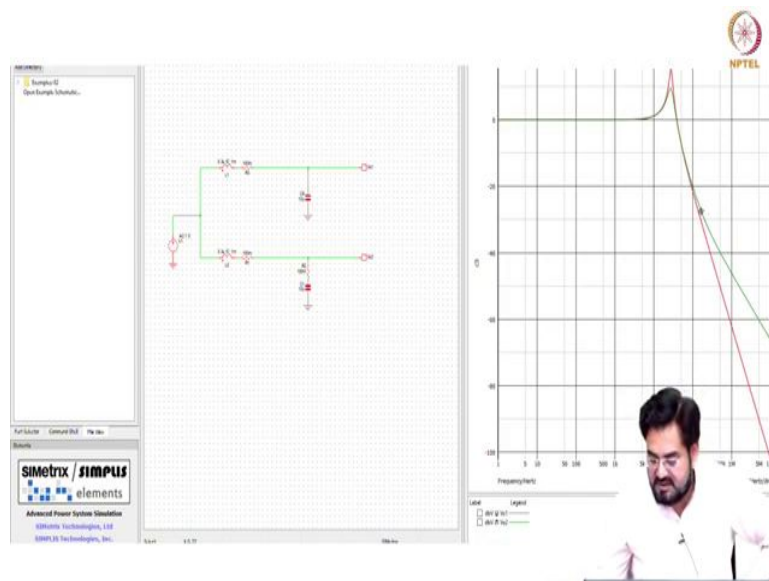
$H_{LC}(s)$ will change after adding R_{ESR} . The transfer function of the LC filter will be :

$$H_{LC}(s) = \left(\frac{1/LC(1+R_{esr}Cs)}{s^2+s(\frac{R_{LOSS}+R_{esr}}{L})+\frac{1}{LC}} \right)$$

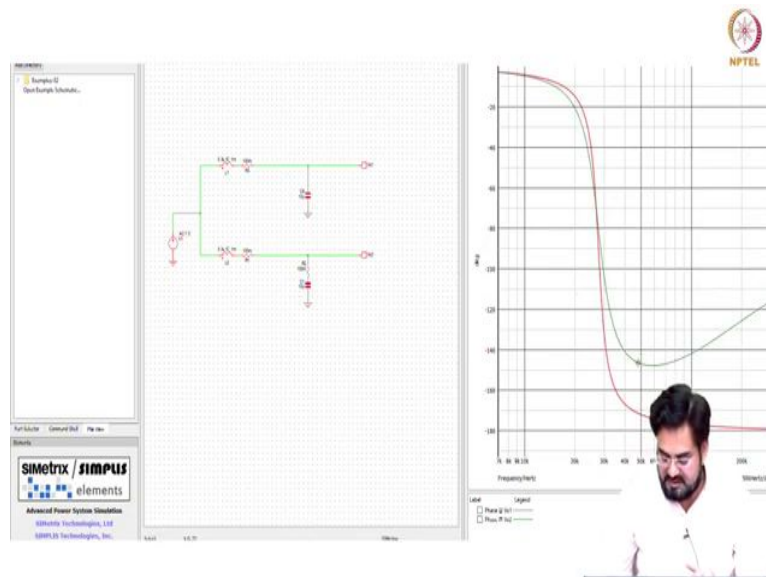
Zero due to ESR will be :

$$\omega_{Z-esr} = \frac{1}{R_{esr} C}$$

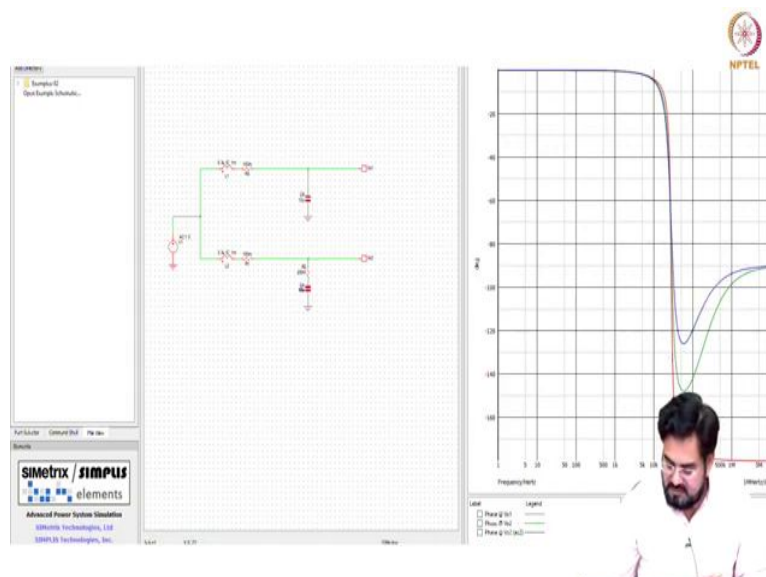
The resonance frequency is not changing but the frequency response and Q factor changed.



The above image shows the frequency response of the LC filter with (green) and without (red) R_{esr} . Q factor reduced after adding R_{esr} and the magnitude response at the end is -20dB/dec instead of -40dB/de because we have introduced one zero.



The above image shows the phase response of the LC filter with (green) and without (red) R_{esr} . We can see the phase around resonance is dropping very quickly without zero and there is a boost in phase after introducing the zero. The phase margin will improve after adding the zero.



If we move zero at a lower frequency then the phase boost will increase and increase the phase margin. The result can be seen in the above image in the blue color plot. Zero frequency will decrease on increasing the R_{esr} . So by simply changing the R_{esr} we can keep increasing the phase margin. Zero added by the compensator will not define phase margin and will cancel the pole of the integrator. Zero in the LC filter will define the phase margin. If

we put zero at the resonance frequency then we will get 45° phase margin but if we have a high Q system and zero is slightly off then the phase margin will degrade. So we usually put the zero slightly before the LC poles.





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$$\omega_{z, \text{err}} = \frac{1}{R_{\text{err}} C} \text{ should appear before resonance } \left(\frac{1}{\sqrt{LC}} \right)$$

$$L = 3.3 \mu\text{H}, C = 10 \mu\text{F}$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3.3 \times 10 \times 10^{-12}}} = \frac{10^6}{\sqrt{33}} = 0.174 \text{ Mrad/sec}$$

$$\frac{1}{R_{\text{err}} C} = 0.174 \times 10^6 \text{ rad/sec}$$

$$R_{\text{err}} = \frac{1}{C \times 0.174 \times 10^6} = 0.57 \Omega$$


The above image shows the example where ESR zero is equal to the resonance frequency.