Power Management Integrated Circuits

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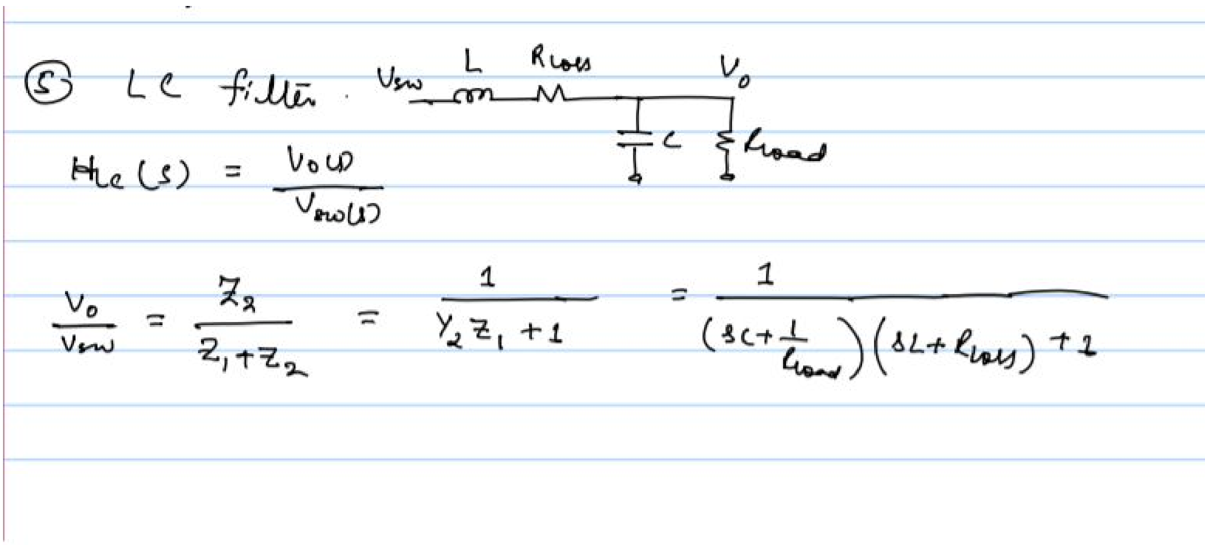
Lecture - 47

Stabilityy Analysis of Voltage-Mode Buck Converter - Part 2



5. LC filter :

The circuit of the LC filter is in the above image. The input and output of the LC filter are V_{SW} and V_O respectively. R_{LOSS} is the combined resistance of the switches and inductor resistance. The transfer function of the LC filter will be :

$$H_{LC}(s) = \frac{V_O(s)}{V_{SW}(s)}$$

$$= \frac{1}{s^2LC + R_{loss}Cs + \frac{L}{R_{load}}s + \frac{R_{loss}}{R_{load}} + 1}$$

$$= \frac{1/LC}{s^2 + s\left(\frac{R_{loss}}{L} + \frac{1}{R_{load}C}\right) + \left(\frac{R_{loss}}{R_{load}} + 1\right)\frac{1}{LC}}$$

$R_{loss} << R_{load}$

$$H_{LC}(s) = \frac{1/LC}{s^2 + s\left(\frac{R_{loss}}{L} + \frac{1}{R_{load}C}\right) + \frac{1}{LC}}$$

The final transfer function of the LC filter will be(derivation is shown in the above images) :

$$H_{LC}(s) = \frac{1/LC}{s^2+s(\frac{R_{LOSS}}{L}+\frac{1}{R_{LOAD}C})+\frac{1}{LC}(1+\frac{R_{LOSS}}{R_{LOAD}})}$$

Assuming $R_{LOSS} << R_{LOAD}$ then we can approximate above transfer function as :

$$H_{LC}(s) = \frac{1/LC}{s^2+s(\frac{R_{LOSS}}{L}+\frac{1}{R_{LOAD}C})+\frac{1}{LC}}$$

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

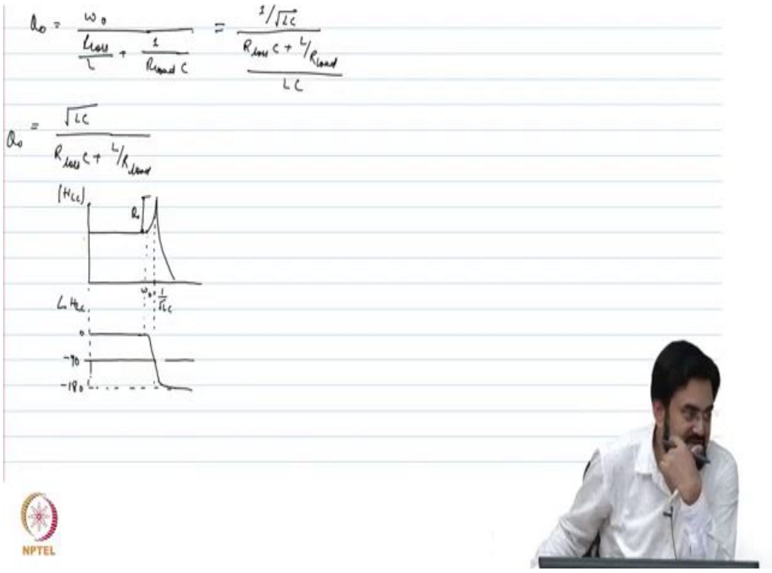
$$\frac{\omega_0}{Q_0} = \frac{R_{loss}}{L} + \frac{1}{R_{load}\,C}$$

$$Q_0 = \frac{\omega_0}{\frac{R_{loss}}{L} + \frac{1}{R_{load}\,C}} = \frac{1/\sqrt{LC}}{\frac{C R_{loss} + L/R_{load}}{LC}}$$

The above equation is the standard second-order low pass filter.

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

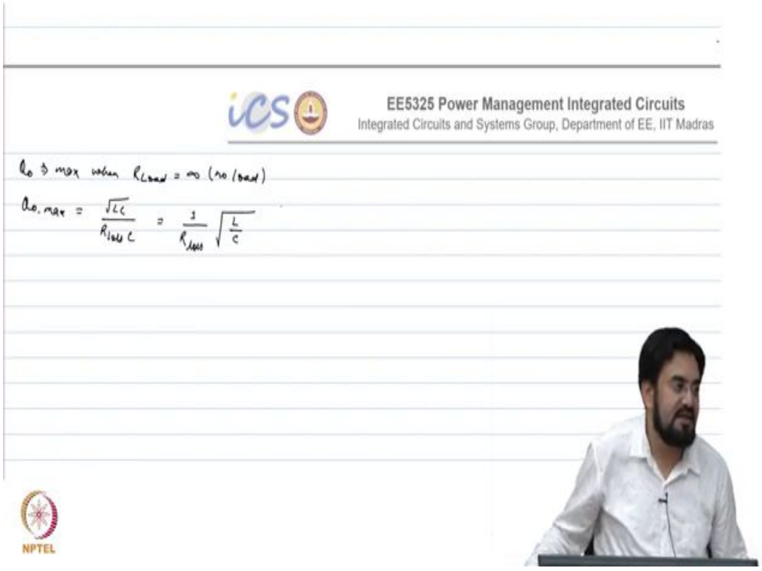
$$Q_0 = \frac{\omega_0}{\frac{R_{LOSS}}{L} + \frac{1}{R_{LOAD}C}}$$



After substituting the value of ω_0 in Q_0, we get:

$$Q_0 = \frac{\sqrt{LC}}{R_{LOSS}C + \frac{L}{R_{LOAD}}}$$

The magnitude plot of H_{LC} will look like a second-order low pass filter and will have a peaking of magnitude Q_0 at ω_0. The phase will fall very sharply from 0° to 180° around ω_0 because of a very high Q. So the worst-case condition for phase shift will be for maximum Q. Q is maximum at no-load condition.

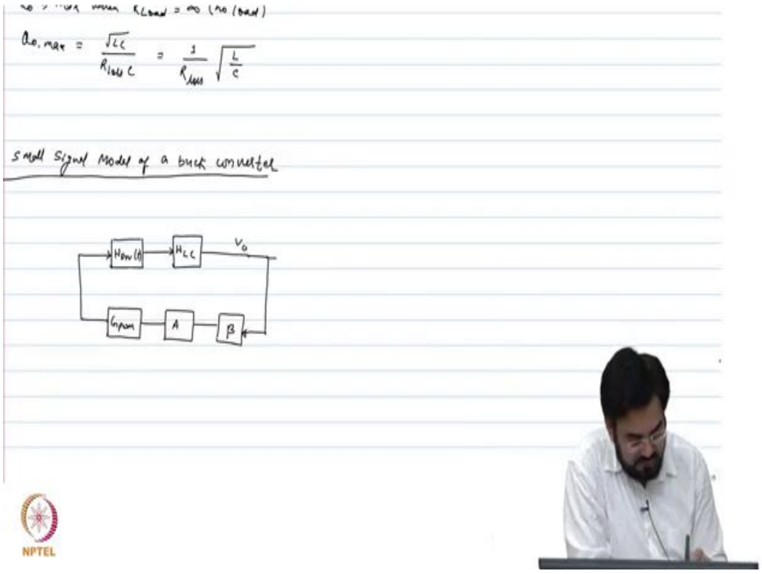


At no load, R_{LOAD} will be infinity. So, we can assume that the R_{LOAD} term in the denominator of Q_0 zero.

$$Q_{0-max} = \frac{\sqrt{LC}}{R_{LOSS}C} = \frac{1}{R_{LOSS}}\sqrt{\frac{L}{C}}$$

There is a trade-off here in between efficiency and stability because if we are looking for higher efficiency, we have to reduce R_{LOSS} but then Q_{0-max} will increase and phase shift will be very sharp at ω_0.

We know that as long as the product of L and C remains constant then ripple voltage will not change. Q_{0-max} will reduce when we choose smaller L and larger C and we will also get a better transient response. Poles will be near $j\omega$ when we have a high value of Q in a second-order system and we will get more ringing in the output.



The small-signal model of the buck converter is shown in the above image.