Power Management Integrated Circuits
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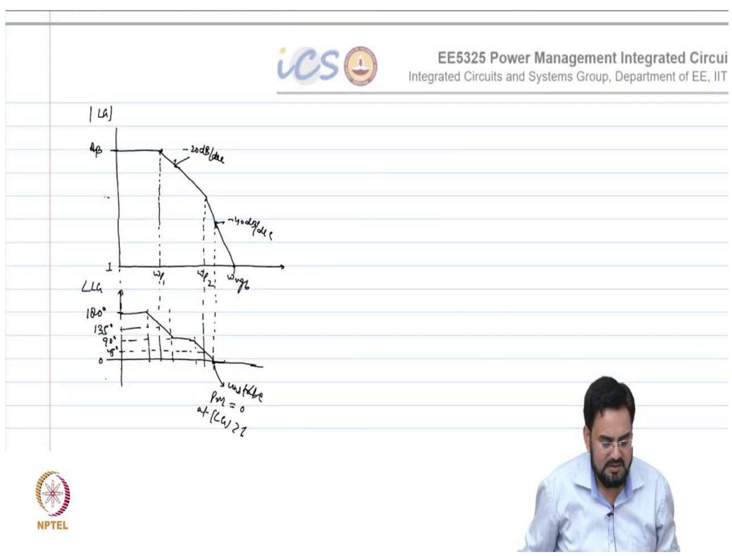
Lecture – 14
Closed-Loop Response of Second-Order Systems

Now, consider the second order system. So, assume that A(s) has two real poles.

$$A(s) = \frac{1}{(1+\frac{s}{\omega_{p1}})(1+\frac{s}{\omega_{p2}})}$$

Assume ω_{p2} is at much higher frequency compared to ω_{p1}. The bode plot for loop gain is shown in below figure.



So, you are hitting gain 1 at higher frequency but your phase has already crossed 0°. Which means your system is unstable because phase margin is 0° at loop gain greater than 1. As long as this 0° phase happens before your ω_{ugb} your system is unstable. If this 0° phase would have happened after ω_{ugb}, then you could have achieved some phase margin and that is what we try to do. Which means your second order system has a tendency to become unstable.

Now let us assume $\omega_{p2} = \omega_{ugb}$ then phase margin is 45°. And in order to have a very good stable system the phase margin required is above 60°. This is a practical requirement. Theoretically you can say system is a stable even if your phase margin is more than 0°. If your phase margin is 10° then system is not unstable because it will not keep oscillating, the oscillations will die out. But, it will have a lot of ringing and it will take very long time to settle.

So, when you are designing; the practical requirement for a good stable system will always be a phase margin of 60° or higher. So, 45° is a very badly designed system even though it is stable. Practically phase margin of below 45°, we consider it as an unstable system even though theoretically it is not. Even if you design your system with a 55° phase margin, it is still ok. Sometimes when you design a system your phase margin may vary across the corners. At the worst corner if you have a 55° then it is acceptable but never hit 45° or so, it will have a lot of ringing.

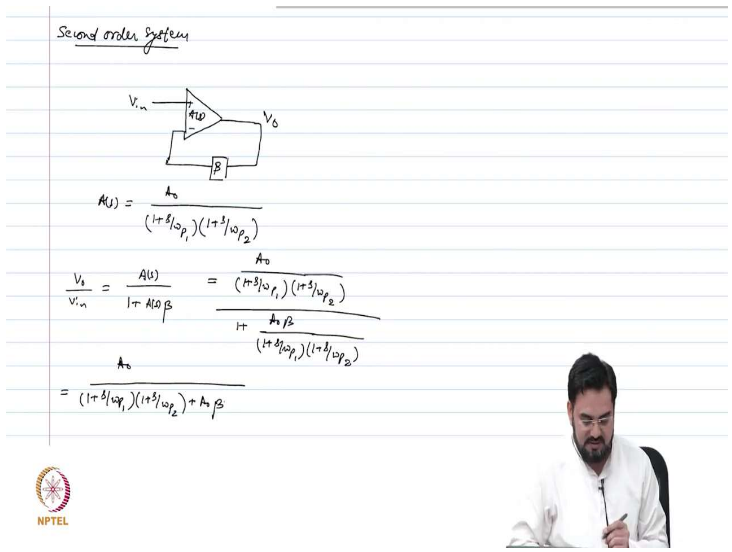
Which means even if you place your second pole at ω_{ugb}, the maximum phase margin you can get is 45° which is practically not good, and we cannot call it as a good stable system. So, your second pole should always be outside your ω_{ugb}. And that is what we do when we say compensation we try to push the second pole outside your ω_{ugb}.

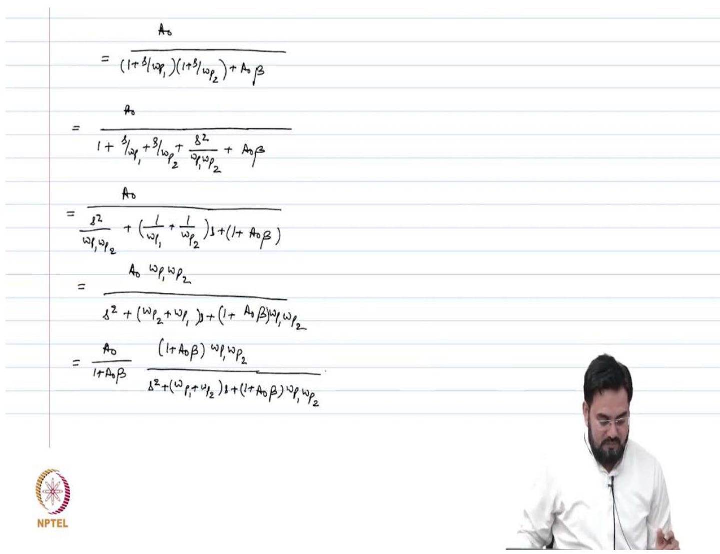
This is the thumb rule: you cannot have 2 or more than 2 poles within your ω_{ugb} assuming that there are no zeros. Whenever you are designing your system, make sure you have only 1 pole and the second pole should be outside your unity gain bandwidth.

In order to achieve this one way, you can reduce the pole frequency of the first pole so that the second pole will automatically outside ω_{ugb}. If I reduce the frequency of my first pole, then my ω_{ugb} will also shift inside and your second pole will automatically go outside ω_{ugb}. So, reduce your bandwidth. That is one type of compensation. Other, you can place a zero and cancel that second pole. So, both dominant pole compensation and pole-zero cancellation are two main techniques which are used.

Now we will look into the closed loop response of the second order system and will relate this loop gain parameters to phase margin. That is the main motive to analyze the loop gain because we want to find the phase margin. And we will try to relate the closed loop system with the open loop and see what is the relationship between your damping factor (ζ) and phase margin.

Negative feedback system is shown in below figure and follow the derivation shown in below figures.





And we know the standard transfer function for second order system. By comparing with the standard transfer function of second order system, we get the natural frequency (ω_n) and damping factor (ζ) expressions as shown in below figure.