**Power Management Integrated Circuits**
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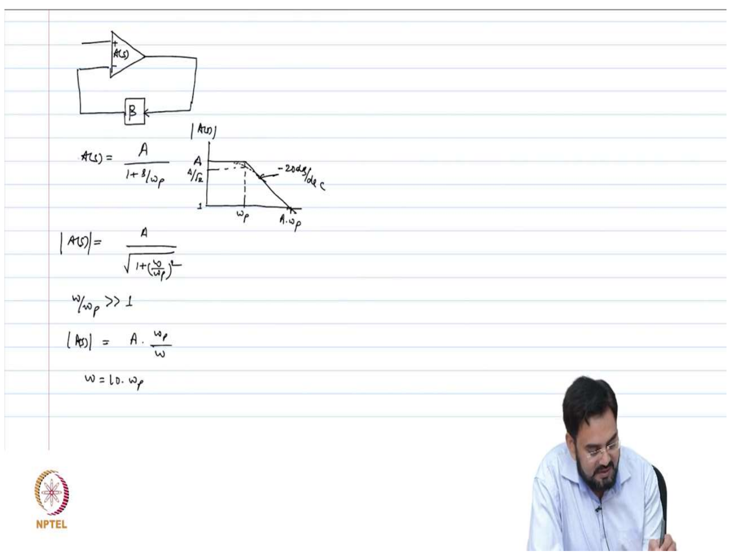
**Lecture – 13**
**First-Order Systems, Phase Margin**

Consider the negative feedback system and also consider A(s) as a first order system. Since it is a first order system; you have only 1 pole and your DC gain is A. But your gain is a function of frequency as shown in below figure.




For $\omega >> \omega_p$, magnitude of A(s) will become $A\,\frac{\omega_p}{\omega}$ . In the bode plot, at pole frequency ($\omega_p$) we show gain is A but in reality, the gain will be $\frac{A}{\sqrt{2}}$. If you want to be more precise, then you can always consider this $\sqrt{2}$ factor but most of the time during analysis we do not consider this. And if you do not consider this $\sqrt{2}$ factor then you might see some difference in your actual result compared to what theoretically you have calculated. So, whenever you calculate the $\omega_{ugb}$ and phase margin, you might be off by 4° or 5°; if you do not consider this $\sqrt{2}$ factor.

Now this A(s) is open loop and the pole is in left hand side of s-plane. So, intuitively you can say it's a stable system. But we are supposed to look at the left hand side pole of overall closed loop system not A(s) alone.

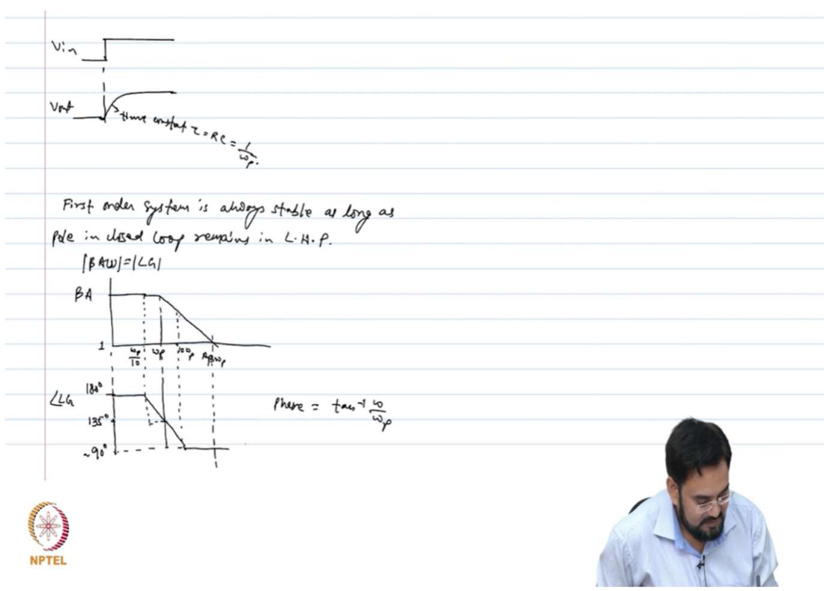
When you say the pole location left hand side or right-hand side of s-plane that is defined for your closed loop system. And when you say phase margin and gain margin, that is defined for the open loop gain. So, there are two differences here. When you are analyzing by looking at the pole location; whether it is a left half plane or right half plane, then you are actually looking at the closed loop system.

And when you analyze a system in terms of phase margin and gain margin then you are doing it for your loop gain (Aβ). Analyzing a closed loop system is more complicated, that is why we always analyze this Aβ or loop gain because it simplifies everything.

So, in the first order system, the step response will be exponential. And the time constant of this step response is given by

$$\tau = \frac{1}{\omega_{p'}} \text{ where } \omega_{p'} \text{ is closed loop pole and } \omega_{p'} = A\omega_p$$

The location of the closed loop pole will change when you connect in the feedback. And for the first order system, it will become gain times of your open loop pole.




The bode plot for loop gain (Aβ) is shown in above figure. Since it is a negative feedback 180° phase shift is already there at 0 frequency or DC. So, you have to start bode phase plot with 180° and you cannot start with 0°. Some people start with -180° in order to show that pole has a negative phase. If you start from 180° then you are going towards 0°. If you start from -180° then you are going towards -360°. Ultimately, 360° is same as 0°.

For first order system, phase drop will never be more than 90° because after $10\omega_p$ phase is saturated to -90°. The condition for instability is; the magnitude of loop gain should be greater than 1 and the phase is 180°. This condition is called Barkhausen criteria. From this unstable point, you have to see how much margin you have. For first order system, this phase margin is 90°.

$$\text{Phase Margin} = 180° - \tan^{-1}\frac{\omega_{ugb}}{\omega_p}$$

$\omega_{ugb}$ is the frequency at which loop gain is unity. That is why we call it unity gain frequency or unity gain bandwidth. Whatever the phase contribution you have at $\omega_{ugb}$ from the pole $\omega_p$; that will determine your phase margin.