Digital IC Design
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Lecture - 61
Two’s Complement Sign Extension

So, now next go ahead and try to do one Two’s Complement Multiplication, ok.

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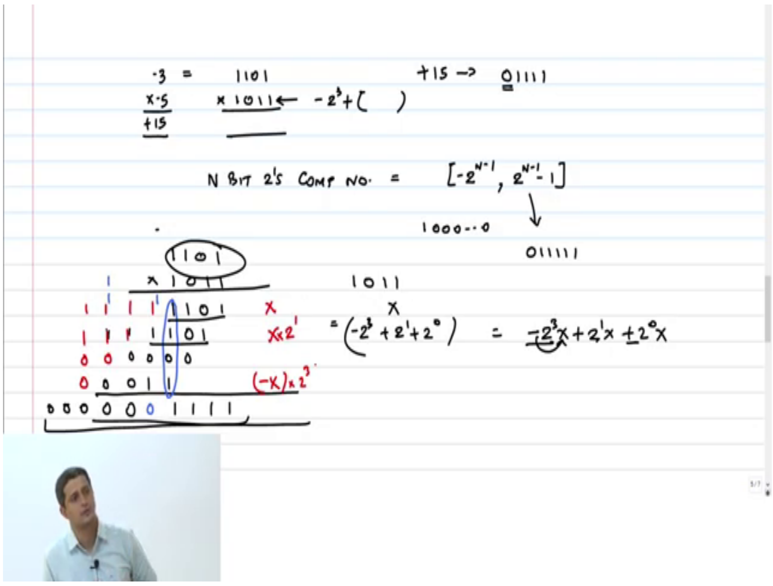

Let us say I want to do minus 3 into minus 5 ok. So, before that if I have an N bit 2’s complement number, what is the range of this representation?

Student: (Refer Time: 00:46).

Yeah?

Student: (Refer Time: 00:48).

So, this will lie between minus 2 power N minus 1 to 2 power N minus.

Student: (Refer Time: 01:06).

You are saying this, ok. What are the values corresponding to this the bit binary representations? Minus 2 power N minus 1 is what? 1 0 0 0 0 all the way. What is this representation? 0 1 1 1 1 1 ok, correct right. So, for minus 3 and minus 5 how many bits minimum number of bits do we need to represent this? 4 bits, right. So therefore, we can write this as what 1 1 0 1 and this is going to be 1 0 1 1 ok. So, let us go ahead and do this number multiplication and see.

So, we go ahead and do it as we did earlier. The first partial product to be 1 1 0 1. Second partial product to be shifted again, but the same number right 1 0 1 1. What about the third one? All 0's, so this will just be 0 0 0 0. Fourth one?

Student: (Refer Time: 02:39).

Yeah.

Student: (Refer Time: 02:41).

1 0; 1 1 0 1, 1 0 1 1 right. So, can you now add these numbers and tell me what you get. What are you supposed to get eventually by the way?

Student: (Refer Time: 03:12).

Minus.

Student: (Refer Time: 03:14).

Plus 15 right, this has to be plus 15. What is the representation for plus 15?

Student: (Refer Time: 03:23).

Very important plus 15 has to be 0 1 1 1, do not say 1 1 1 1 1; four 1's is equal what minus 1 in 2's complement representation. So, you have to you cannot leave this preceding 0 out when you are dealing with 2's complement numbers. So, you have to get 0 1 1 1 1 as your answer. So, do it and now tell me if you are able to get it or not. So, this is right.

Student: (Refer Time: 05:08).

Yeah.

Student: (Refer Time: 05:10).

So, if you try to discard the carry.

Student: (Refer Time: 05:14).

Yeah.

Student: (Refer Time: 05:20).

So first thing is, when you are doing 2's complement multiplication you have to be very careful about like I told you about the preceding 0, that will change the value completely. So therefore, to say that this is the first partial product is equal to 0 0 0, is it right? What should I do?

Student: (Refer Time: 05:52).

You have to do what is known as sign extension right. You have to do sign extension to get this right. So, first thing is we will look at that.

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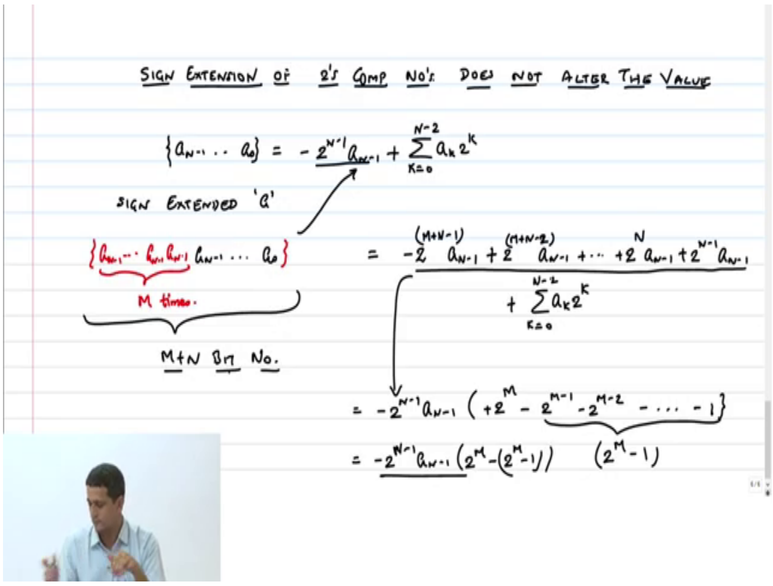

Sign extension of 2’s complement numbers. Let us say I have a representation a N minus 1 a naught right; a naught equals minus 2 power N minus 1 a N minus 1 plus summation k equal to 0 to N minus 2 a k 2 power k, right. Now what am I doing? I am what is sign extension, its taking this a N minus 1 bit and repeating it k times or repeating it M times ok. I am going to create a number which is sign extended a.

So, what is that value it is a N minus 1 all the way to a naught, and I am going to repeat this guy: a N minus 1 a N minus 1 all the way how many times I am going to sign extended by M bits. So, by sign extending it M times this now becomes a how many bit number? M plus N bit number right. So, this guy if you look at overall its an M plus N bit number ok.

So, now what I am going to do is, I am going to simply show that this M plus N bit number is exactly equal to this in volume. So, if this is an M plus N bit 2's complement number what is the value?

Student: (Refer Time: 08:39).

Minus 2 power.

Student: (Refer Time: 08:45).

M plus N minus 1 into a N minus 1 plus 2 power M plus N minus 2 a N minus 1, so same bit. All the way down to what 2 power M no N a N minus 1 right, this is by sign extended value; plus the old sum now, not the old sum. Remember now a N minus 1 will get multiplied by plus 2 power N minus 1 now; plus 2 power N minus 1 a N minus 1 plus the remaining part of the sum k equal to 0 to N minus 2; we put that on the next line plus summation k equal to 0 to N minus 2 a k 2 power k.

Now, can you take a N minus 1 out common right and show that previous guy reduces to there you know. Can you show that this term value is nothing but this way? So, just take a N minus 1 common and minus 2 power N minus 1 out from here, this term I am going to write it as minus 2 power N minus 1 a N minus 1 into the remaining terms that is plus 2 power M minus 2 power M minus 1 minus 2 power M minus 2 all the way down to what, minus 1. What is this summation equal to?

Student: (Refer Time: 11:03).

This summation is 2 power M minus 1 right. Therefore, we will get minus 2 power N minus 1 a N minus 1 into 2 power M minus 2 power M minus 1, which is nothing but minus 2 power N minus 1 a N minus 1, right. Therefore, sign extension of 2's complement numbers does not alter the value ok. So now, let us go back I am do the same example that we did by correcting for the sign extension, ok.

So, we had what? Minus 5 into minus 3 is it what is it or minus 3 into minus 5; minus 3 into minus 5 or let me just go back here and corrected ok.

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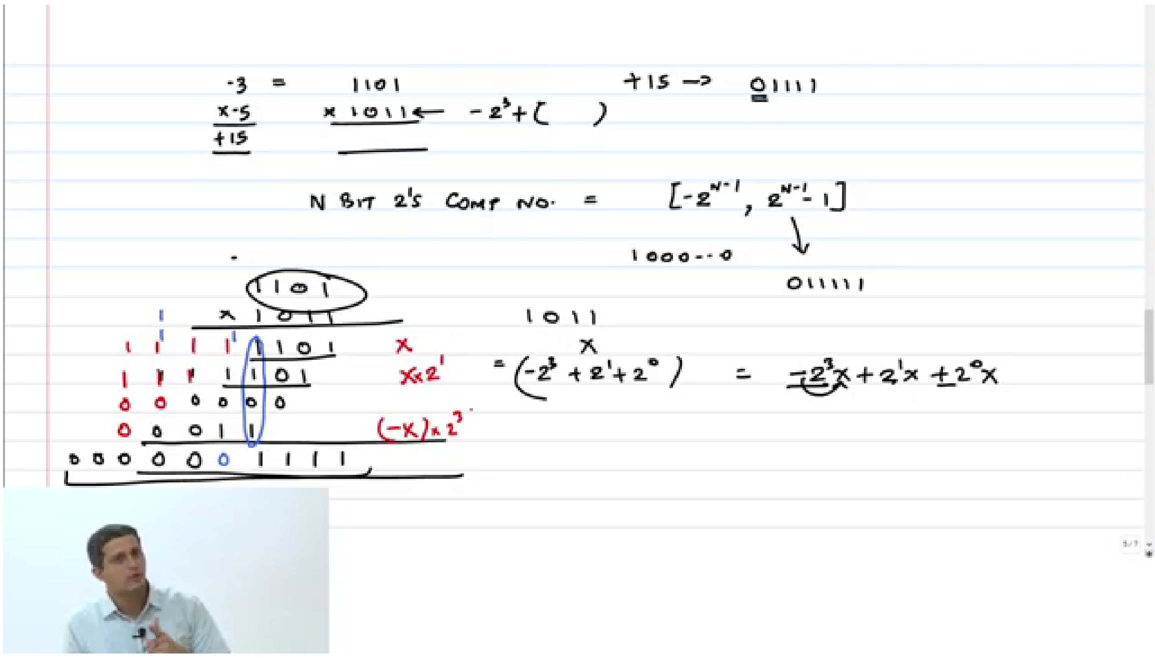

So, the sign extended bits are now like this, right. For this also its going to be like this this will be 0. What about this last term here? Is this correct? Now do the addition tell me what you get? 1 1 1 1 1 right, 0 1; is not it this what you get what went wrong?

Student: (Refer Time: 13:14).

Sign extending?

Student: (Refer Time: 13:19).

Sign extend or you have to consider the sign of the multiplicand. In the sense this last partial product this guy we said is 1 1 0 1 because you are multiplying with the 1 there, but there is actually a minus 1. See this number here is minus 2 power 3 plus something. So, when I am multiplying by the MSB alone its actually a negative number. And therefore, what I am to put here is not just that value, but the negative value of that guy right. So if I get minus 5, I have to put plus 5 there right no minus 3 into minus 1 I have to put plus 3. So, this has to be 1 1 0 0, correct.

Now what do you get? I add one here I get 1 here 1 1 plus three 1's. So 1, I get a let me put the carries in a different color ok. So, I get a carry of 1 here, because I am adding this three 1’s 1 1; now I add four 1’s what do I get? 0 right. And the carry will go where, here right. Now, what do I get here? 1 plus 1 0 I get a carry of 1 here now again I get four 1’s ok. So, I get 0 ok.

Now this is a very critical and a very settle point that you have to follow. I can sign extend these numbers to how many our bits I want and yet by answer should be consistent. So, if I tell you that the answer is 0 1 1 1 or 0 followed by four 1’s or 0 0 0 followed by four 1’s or any number of 0’s followed by four 1’s the answer is still minus plus 15. The arithmetic should be such that how many ever bits I sign extend I still get that answer.

Previously we said we have to drop that carry that is not correct. The carry is going to be used in order to allow the sign extension to as many bits as you want, you get it? So, now for example here, if I sign extended this further right 0 and then put another 0 here then this carry would still do the something, will give me four 1's two 1's get me all 0's. There this is a very settled point go back look at the arithmetic that we did here, look at the arithmetic mistake that we made earlier.

The carry cannot be dropped or the carry is not the MSB of your 2's complement multiplication. The carry is going to enable sign extension to as many bits as you want. Which means, if I give you to unsigned numbers and say how many bits is the final answer: it is M plus N minus 1, no problem. If I give you 2's complement number and ask how many bits should the final answer be, of course it needs a minimum number of bits.

But beyond that I will reverse the question I will say you tell me how many bits you want, I will perform carry propagation until you get the number of bits that you want. You understand this question in 2's complement itself becomes meaningless, beyond the minimum number of bits that is, ok. Please go back workout.

Student: (Refer Time: 17:39).

Yeah.

Student: (Refer Time: 17:43).

Correct.

Student: (Refer Time: 17:47).

So, carry will go to two bits away.

Student: (Refer Time: 17:52).

So, you that is what I am saying you work out this example for more sign extension, you will see that the same 4 2 pattern will keep repeating. And it will ensure that these preceding values will always be 0's, whatever you do. That final carry will be 1, nothing can be done. So, you tell me how many bits you want, I will truncate and give you because that is all; that is all you need. The carry I will just discard at that point.

Student: (Refer Time: 18:25).

Why we are doing only the.

Student: (Refer Time: 18:29).

Correct. That is because you look at the mathematical representation right; this is you know if you take this 1 1 0 1 I mean 1 0 1 1 this is basically minus 2 power 3 plus 2 power 1 plus 2 power 0. And we are now multiplying with this number right, let us say this is x, x into this. So, if I write this it will be minus 2 power 3 x plus 2 power 1 x plus 2 power 0 x that is what we are doing here.

Wherever there is a plus we retain x as it is, right. 2 power 1 x means we have shifting by 1 and retaining x as it is. Wherever there is a minus sign I am observing this minus into x here, it will become minus x into 2 power 3. And therefore, I am shifting by 3 bits an putting minus x there. So, if you want me do label this the partial product should be x x x into 2 power 1 minus x into 2 power 3.

That is how we are achieving ok, clear. That is why I am doing the math here because, only then you can be convinced of some of the settle points.