

Digital IC Design
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Lecture - 04
MOS Transistor Current Expression

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$$\Rightarrow Q_p' = \left(\sqrt{2\epsilon_{si} |\psi_s| q N_A} \right) W/L$$

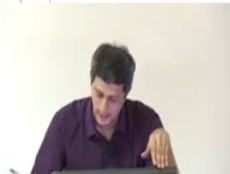
$$\Rightarrow Q_p = \sqrt{2\epsilon_{si} |\psi_s| q N_A}$$

$$V_{GB} = \left(\psi_s - \frac{Q_p'}{C_{ox}} \right) - \frac{Q_s'}{C_{ox}}$$

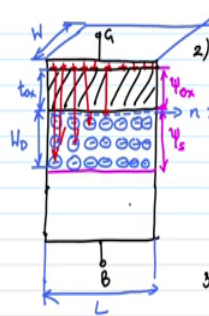
$$Q_s' = -C_{ox} (V_{GB} - V_{TH})$$

$$V_{TH} = \psi_s - \frac{1}{C_{ox}} \cdot \sqrt{2\epsilon_{si} |\psi_s| q N_A}$$

↓
 $\epsilon_{si} \epsilon_0$



(Refer Slide Time: 00:18)



2) $V_{GS} > 0 \rightarrow$ DEPLETION

$n \gg n_i^2/N_A$

$$n_s = n_B e^{\frac{q\psi_s}{kT}}$$

$$\psi_s = \frac{kT}{q} \ln \left(\frac{n_s}{n_B} \right)$$

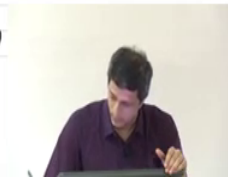
PINNED \rightarrow when $n_s = N_A$

3) $V_{GS} > V_{TH} \rightarrow$ INVERSION

$$n_s = \frac{kT}{q} \ln \left(\frac{N_A}{n_i^2/N_A} \right) = 2 \frac{kT}{q} \ln \left(\frac{N_A}{n_i} \right)$$

$$\boxed{\psi_s = 2 \frac{kT}{q} \ln \left(\frac{N_A}{n_i} \right)}$$

So, clearly even after doing all of this and inverting the channel right, I cannot still cause a current to flow through this device because there is an oxide on the top right. So, somehow the gate has created some free electrons, but I cannot do anything with it because there is an oxide on top and current cannot flow through the oxide. It is an insulator.



So, this concentration is N_A this is N_D and so on. So, now, I go ahead and do the same exercise, I apply a large enough potential. I am able to invert the channel by first creating mobile charge carriers here. And I am also able to simultaneously generate some free electrons with some concentration out there ok. Now, if I apply a lateral electric field right the vertical electric field is actually creating the inversion charge for. If I now apply a lateral electric field then I can cause these free electrons to move correct.

So, this guy let me call, let me say I will ground this terminal; without loss of generality I am picking one and I am going to ground it ok. This I will apply a potential V_D ok, this is called the drain and this is called the source you. So, now, if I apply a drain voltage right across this these two terminals clearly that potential is going to drop across the entire channel right.

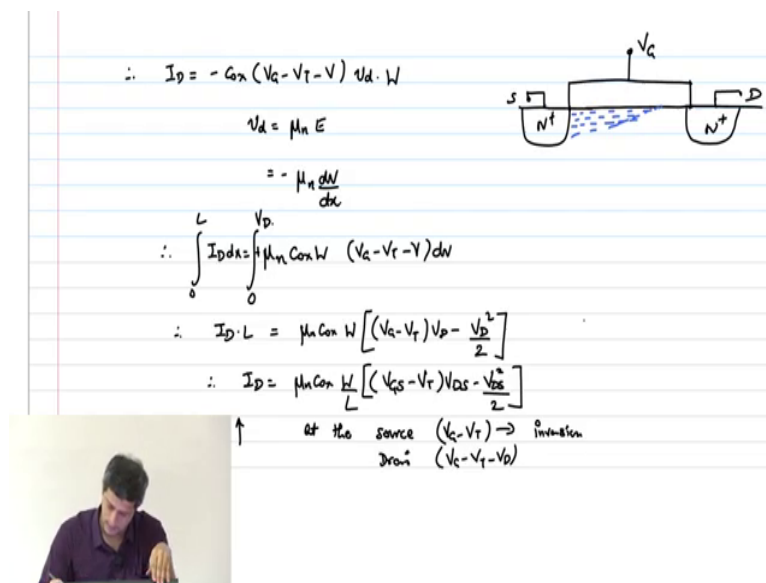
So, if you look at the way this is happening I have this going as x right and I have a potential in my channel at a distance x ; I will consider a small portion dx here ok. So, what is the potential here I am going to call that potential as $V(x)$. So, what is $V(0)$? 0 . $V(L)$; right where this is L as usual it is V_D ok. So, now, what is the amount of inversion charge that is available for conduction in that small element dx with width w ?

See that thing is already at a potential V , over and above that I need V_G minus V_T ; V_G minus V_T that can cause an inversion there. So, therefore, the net charge that is going to be available Q_I will be what? What did we say earlier? It is minus C_{ox} into V_G minus V_T minus V ok. Because what is happening is at the source the potential is whatever V_G minus V_T you apply you will get that much of inversion charge, but as you go that potential is going up in a channel also.

So, therefore, you will have V_G minus V_T minus V . Therefore, if you look at the elemental charge dQ right, no this is Q let me call it Q_I actually, because I want the charge per unit area. Therefore, the dQ should be what? It is just Q_I I mean Q_I into the area element. What is the area element? w into dx right.

So, therefore, I have dQ at prime is minus C_{ox} into V_G minus V_T minus V into W times dx right. So, I just differentiate with respect to time, this has to be what is, this has to be a current and this is called a drain current ok. Equation of continuity says that the drain current has to be the same through the channel it cannot be different. So, it does not it is not going to depend on the position right otherwise you will have charge accumulating in one place ok. So, therefore, I can and this dx by dt ; what is this? This is basically drift velocity right.

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$$\therefore I_D = -C_{ox}(V_G - V_T - V) v_d \cdot W$$

$$v_d = \mu_n E$$

$$= -\mu_n \frac{dV}{dx}$$

$$\therefore \int_0^L I_D dx = \int_0^{V_D} \mu_n C_{ox} W (V_G - V_T - V) dV$$

$$\therefore I_D \cdot L = \mu_n C_{ox} W \left[(V_G - V_T)V_D - \frac{V_D^2}{2} \right]$$

$$\therefore I_D = \frac{\mu_n C_{ox} W}{L} \left[(V_G - V_T)V_D - \frac{V_D^2}{2} \right]$$

↑ at the source $(V_G - V_T) \rightarrow$ inversion
drain $(V_G - V_T - V_D)$

Therefore, the drain current can be written as minus C_{ox} into V_G minus V_T minus V into drift velocity. Now what is drift velocity? It is mobility times electric field right. So, if I apply an electric field right the thing cannot just arbitrarily accelerate through the lattice, because now there are lot of atoms sitting in the lattice.

So, the electron will have some mean free path it will collide then some recombination happen and all that this is you know a continuous generation recombination process that happens. So, therefore, it is not; it is not that the electron can just accelerate as if it is an free path and therefore, it gets related to the electric field through the term called mobility right.

So, if you look at the this velocity this will be mobility of electrons into the electric field and; obviously, this is minus μ_n times $d v / d x$ and wait there is a w here. Therefore, I am now going to say I_D equals y plus I think the right notation is μ_n let us just keep that μ_n $\mu_n C_{ox}$ into W into $V_G - V_T - V$ into $d v$ and $d x$ I will bring this side and I can integrate this 0 to l .

This will go from what? V will go from 0 to V_D and therefore, I_D into L because now I_D is not the function of x , ok. It is uniform throughout the channel and therefore, I_D comes out of the integration I_D into L is $\mu_n C_{ox} W$ into $V_G - V_T$ into $V_D - V_D^2 / 2$. Therefore, the drain current is $\mu_n C_{ox} W$ by L into; I will now reference it to the source. I said that the source was grounded.

So, it is just $V_G - V_D$, I will make it more general $V_G - V_T$ into $V_D - V_D^2 / 2$; clear. Any questions here? So, the current is actually a drift current, it is not a diffusion current like it is in a p n junction. In a p n junction the minority carriers actually diffuse, here majority minority might be might be confusing because in the channel the electrons is actually a minority right.

So, forget about that, but it is a drift current that is causing. So, that is why the relationship is linear or quadratic with respect to applied potentials and not exponential; p n junction is exponential because of the diffusion component alright this is a drift component that is causing the current clear.

So, now let us see what happens when my V_D keeps increasing I will just keep increasing V_D , at the drain at the source end what is the net potential? At the source end the gate minus the potential at the source end what is it. So, at this source $V_G - V_T$ right will

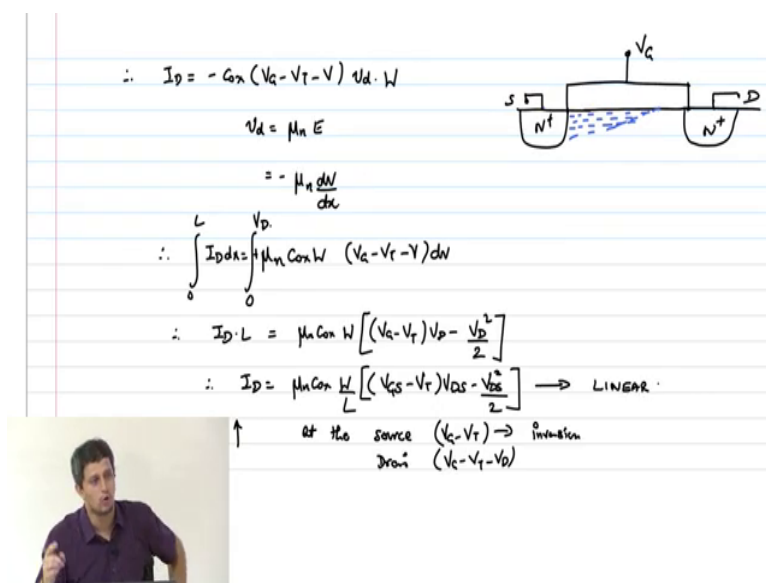
cause inversion right. So, effectively it is just V_G minus V_T that is able to cause inversion there, my question was still formed ok.

At the drain what will happen? V_G minus V_T minus V_D . Now, if V_D goes above V_G minus V_T what happens here? This term becomes negative right; which does not make sense because basically we are now going below the threshold voltage which means there is not enough inversion charge there right. So, therefore, what happens is the current quadratically goes up until a certain point and after that the current just saturates, because at the drain end you cannot have this term going negative ok.

So, the channel sort of what happens is they call it a pinch off region ok. So, if I have my drain source and my oxide and gate sitting here, as I increase N plus N plus, as I increase my V_D the electron concentration will be like this, free electron concentration its non-uniform. At the source end its going to be higher because it is just V_G minus V_T there.

But at the drain end it can sort of pinch off beyond the point and if I continue this process it will just pinch off like this ok, this is a very crude picture, but effectively the point is that the current just saturates ok.

(Refer Slide Time: 15:04)



$$\therefore I_D = -C_{ox} (V_G - V_T - V) V_d \cdot W$$

$$V_d = \mu_n E$$

$$= -\mu_n \frac{dV}{dx}$$

$$\therefore \int_0^L I_D dx = \int_0^{V_D} \mu_n C_{ox} W (V_G - V_T - V) dV$$

$$\therefore I_D \cdot L = \mu_n C_{ox} W \left[(V_G - V_T) V_D - \frac{V_D^2}{2} \right]$$

$$\therefore I_D = \mu_n C_{ox} \frac{W}{L} \left[(V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right] \rightarrow \text{LINEAR}$$

↑ at the source $(V_G - V_T) \rightarrow$ inversion
 Drain $(V_G - V_T - V_D)$

And therefore the I_D will simply become $\mu_n C_{ox} W$ by L into V_{GS} minus V_T ; V_{DS} the maximum value is V_{GS} minus V_T ok. So, minus V_{GS} minus V_T the whole squared by 2, basically what I am saying is that V_{GS} minus V_T minus V should be greater than 0. Only then you can have an increase in charge at the drain end, it can happen only till V is less than V_{GS} minus V_T right or V_D sorry; V_D is V_{DS} actually V_{DS} is less than V_{GS} minus V_T right.

And therefore, this current if you simplify this will become W by L into V_{GS} minus V_T the whole square by 2. And this region quite; obviously, is called saturation and the previous current equation was basically called linear region ok.

We look at the plots in the next class, but if you neglect this V_{DS} squared by 2 for small changes right around something in the analog domain this is basically it operates in a linear

region it behaves almost like a resistor ok, by neglecting a V_{DS} squared by two term ok. Of course, there is no dependence on V_{DS} in saturation region ok. So, I will stop here in the next class we look at the $I_D V_D$ plots $I_D V_G$ plots and so on.

Thank you.