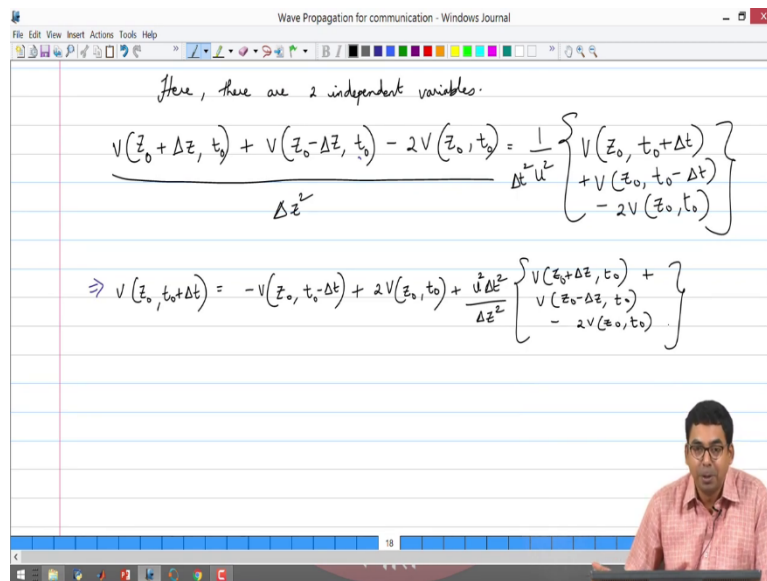


Transmission Lines and Electromagnetic Waves
Prof. Ananth Krishnan
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Indian Institute of Technology, Madras

Lecture - 05
Octave Simulation of Telegrapher's Equation

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We will begin with where we left. So, the last step that we wrote down was go back to the wave equation in the transmission line, take the voltage wave equation and write that down in the difference form. And when we write this down in a different form, we will have to look at what is the unknown quantity, bring the unknown quantity to the left side and all the remaining quantities to the right side.

And what I was mentioning yesterday was in these kinds of equations it is a good guess to a begin with the unknown quantity which is in future that is $t_0 + \Delta t$ is the quantity that you want to determine, all the other quantities are known. So, t_0 denotes current instant of time, all right and $t_0 - \Delta t$ includes the past. So, $t_0 + \Delta t$ is what you want to determine, ok. So, given current and past values you would want to determine what the voltage values are, ok. This is the way to formulate this problem.

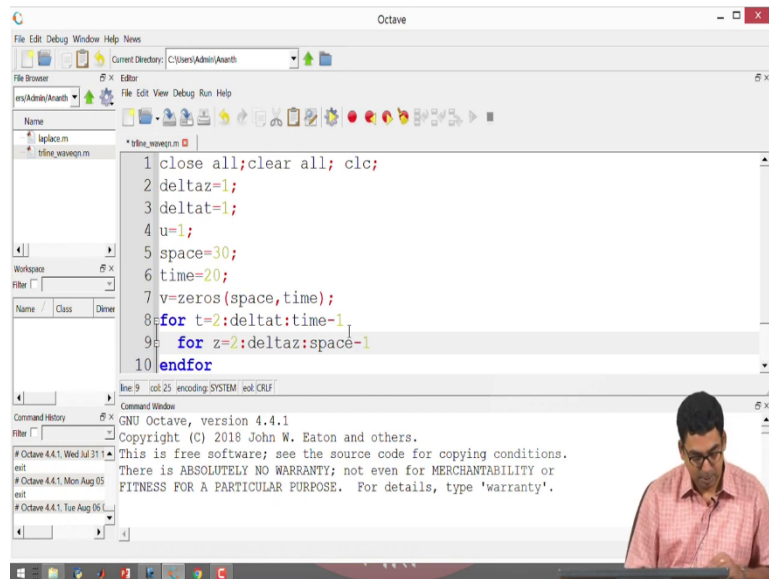
So, one can go ahead and actually write down what the unknown quantity would look like. So, I will just rearrange this. So, we have

$$\frac{V(z_0 + \Delta z, t) + V(z_0 - \Delta z, t) - 2V(z_0, t)}{\Delta z^2} = \frac{1}{\Delta t^2 u^2} \{V(z_0, t + \Delta t) + V(z_0, t - \Delta t) - 2V(z_0, t)\}$$

$$V(z_0, t + \Delta t) = -V(z_0, t - \Delta t) + 2V(z_0, t_0) \\ = \frac{\Delta t^2 u^2 (V(z_0 + \Delta z, t) + V(z_0 - \Delta z, t) - 2V(z_0, t_0))}{\Delta z^2}$$

This is what the rearrangement looks like, all right. And we can go ahead, open our computers, fire octaves.

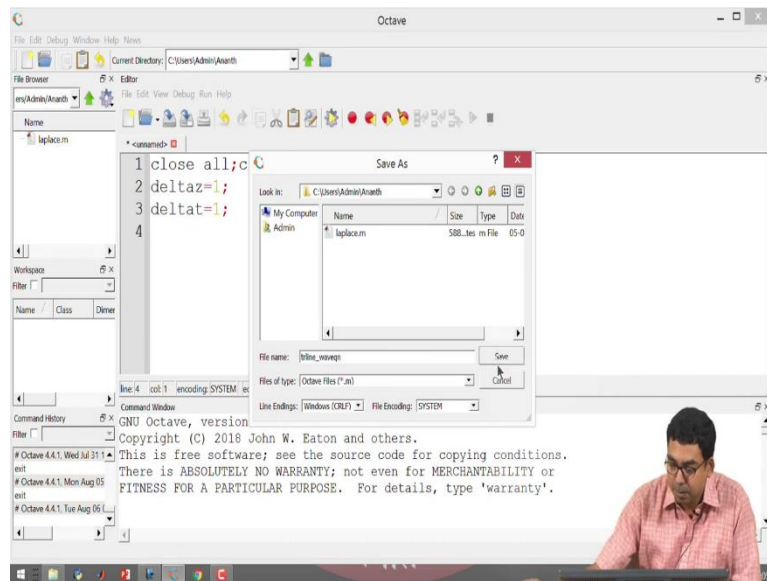
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I will begin the code with the standard statements, to close all open windows, clear the workspace variables and clear the display buffer in the command window, right.

So, now I am going to make a programmatic version of the equations that we just saw, right. So, I am going to start with the definition of variables. I am going to say Δt Δz is what we assumed to be 1, Δt is 1, ok and consequently the velocity u is going to be 1, all right. Because Δz is 1, Δt is 1, so in this case we are going to take the velocity to be 1, all right.

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I am going to now take the length of the transmission line to be some 30 units, all right. So, it is like 30 pieces. So, you can say that the transmission line is divided into 30 cells, each of them consisting of an l and c in the form of a ladder that we made, right.

So, I am going to say that the space the transmission line is spanning 30 units, right and I want now this is a simulation for time evolution of voltages, all right. So, we will have to start at some time and end the simulation at some instant of time, right. So, we have to tell what the ending time is going to be. I am going to say that you know some time is equal to 30, all right and even begin with smaller values of time, so right.

So, the voltage is going to be a two-dimensional matrix, right. It is going to look like this. Initially, I do not know the values of voltage at all points in space and at every instant of time. So, I am going to initialize it to 0s for all instances of time and for all locations in space, ok. That is how we initialize. So, it is going to be a two-dimensional matrix. For every instant of time I am going to have a voltage value along the transmission line length, ok. So, the rows here represent the space, the columns here are represented by the time, ok.

Now, I will go to the actual part where we need to write down the solution. So, if we look at the expression that we have here we will need to start at some instant of time, all right and we have to continue to evaluate with respect to the current instant of time what is the value of the voltage at the next instant of time, all right.

So, we have to start with the values on the right hand side here, all right. Initially, your t_0 is going to be 0, then you are going to have some Δt which is 1 in our program. So, every instant of time you are going to be calculating at a position and some instant of time based on the values of the voltage in that same position and in neighbouring positions and past instances of time. We just have to take this and code it in our program, ok.

So, I am going to use the for loop to achieve this. So, write for t equal to. By now you would already have guessed why I am using 2 to time minus 1, when we solve for the Laplace

equation we know that when we are taking central difference you will go with array a array index out of bounds, so you will have to start with the value next to the boundary and end with the value prior to the boundary, right, ok. And I am going to have for all points in the transmission lines in space you will need to calculate the value of the voltage, right. So, I am going to go ahead for z equal to, ok.

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```

7 v=zeros(space,time);
8 for t=2:deltat:time-1
9   for z=2:deltaz:space-1
10    v(z,t+1)=-v(z,t-1)+2*v(z,t)+(u^2*deltat^2/deltaz^2)*(v(z+1,t)+v(z-1,t));
11  endfor
12  plot(v(:,t+1));
13  pause(0.1); I
14  v(2,:)=1;
15 endfor

```

Command Window
GNU Octave, version 4.4.1
Copyright (C) 2018 John W. Eaton and others.
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There is ABSOLUTELY NO WARRANTY; not even for MERCHANTABILITY or
FITNESS FOR A PARTICULAR PURPOSE. For details, type 'warranty'.

And inside this loop, I am having a time loop that will takes care of t_0 , $t_0 + \Delta t$ and keeps going till the end of the time that you have specified, all right. And at all instances of space, I mean at all positions in space for every instant of time is when we are calculating the voltage over here, right.

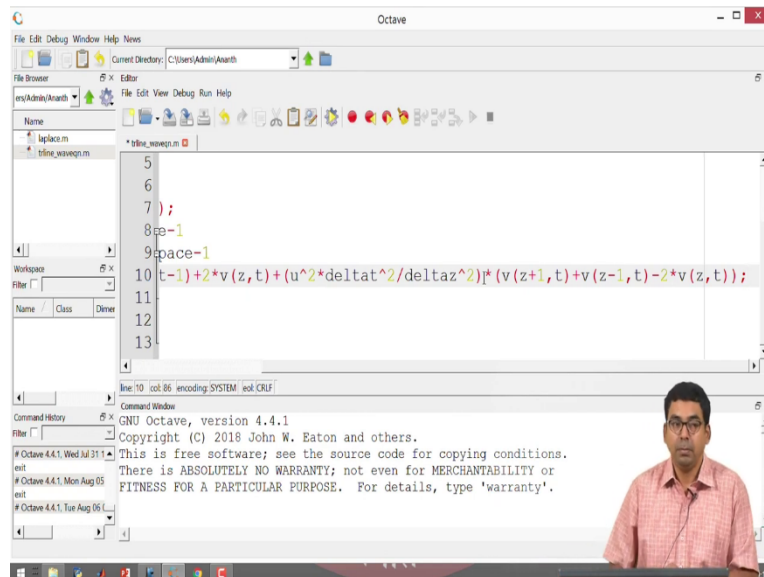
All I need to do is take the equation, now plug it into the form that can be easily found here. So, I am going to have v of z comma $t + 1$ is what I am looking for. So, instead of writing $t + \Delta t$ or $t_0 + \Delta t$, all right. So, I am just writing the current instant of time is going to be t in this loop, the next instant of time is going to be $t + 1$, right. That is all I am interested in finding. So, v of $t + 1$.

According to the equation what I have is minus v of z comma $t - 1$ that was $t_0 - \Delta t$ in the expression that we wrote, all right, plus 2 times v of z comma t , ok plus I had some term a u square Δt square by Δz square, all right. In this case u square is 1 Δt square is 1 Δz square is also 1, but for the sake of completeness I will go ahead and put that also in a in the formula. Anyway the coefficient is 1, one may choose to ignore it, but in future I may make use of it for some other purpose.

So, I will just go ahead and plug in those values, right u square 2, Δt square divided by Δz square. And I have some terms inside the curly braces. What have we written? All right. So, I will go ahead and to this as v of $z + 1$ comma t plus v of $z - 1$ comma t . So, these terms,

comma t means it includes the current time instant value at that particular position, right minus 2 times v of z comma t, ok.

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All I have taken is the expression that we have derived and I have just plugged it in a programmatic form into the program, all right. What are we writing here? Ok. Once we have done this, all right. The essence of the program is already over. It's now about how to set up this program to work, that is it, right.

Like we have seen in the case of the Laplace equation, we at every instant of time would like to watch what is happening in the transmission line, right. Suppose, I launch a voltage at some point I would like to know how the voltage is traveling through the transmission line with respect to time. So, at every instant of time I need to have a display that tells me what the plot of the voltage with space looks like. So, I am just going to go ahead and have a plot command, right, right.

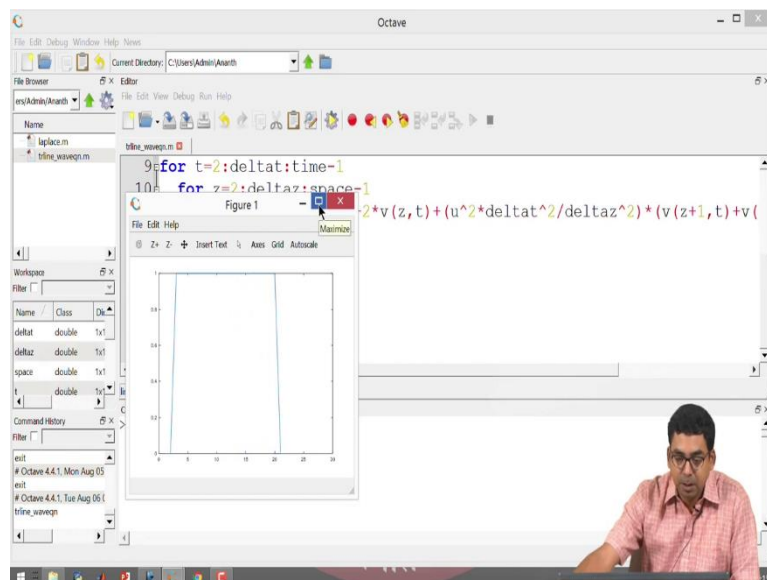
So, I am going to be having to plot the voltage at all points in space and the new calculated voltage you know, ok. And we also know that, yesterday we when we were looking at the previous lecture, if we do not give a pause command then chances are that you will see one and you will see one plot at the end of your program rather than seeing it every instant of time. So, I will just use a pause of 0.1, ok.

Now, everything is ready except that there are some small details that we will have to fix, ok. In the case of the Laplace equation solver, we did see that we had set up some conditions for example, the top plate is at ten volts the remaining plate where it is 0 volts extra. Here you are trying to solve the wave equation, all right, but we have not provided any source of the wave. We have to define the source, we have to define where it is going to become 1 volt or

2 volts extra and then watch it with respect to time, right. So, I am going to provide that information over here, ok.

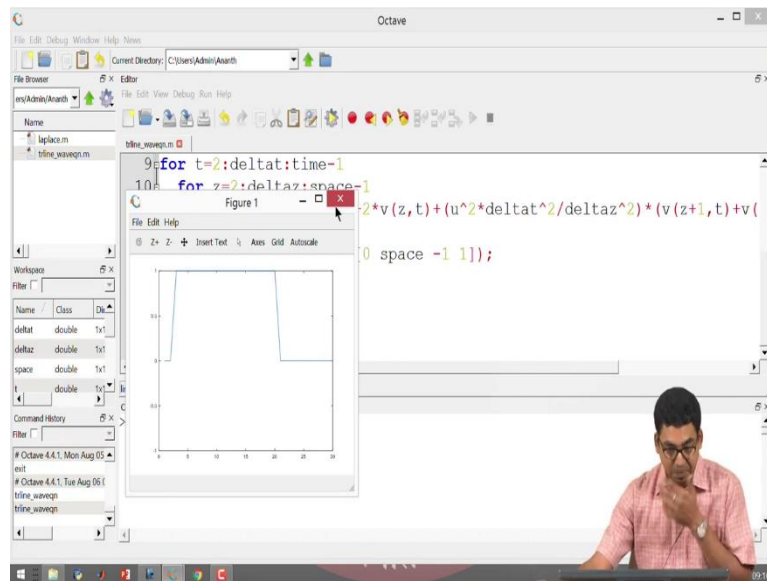
So, I am going to say that you know just going to place this in two places. So, in line number 8 I am saying that v of 2 comma colon is equal to 1. So, the spatial location of 2, ok. For all instances of time it is going to be at 1 volt. So, in other words, I am plugging in a battery there of 1 volt, it is going to be delivering 1 volt at the position 2 for all instants of time this will become apparent if I run the program ok. And what happens within the loop is there is a chance that because I am writing in this form I may actually update that to something else, because I am calculating the value of voltage. At the next instant of time at any instant of time I want the battery to give me the fixed voltage. So, I am just making sure that I am plugging in v of 2 comma columns equal to 1 again.

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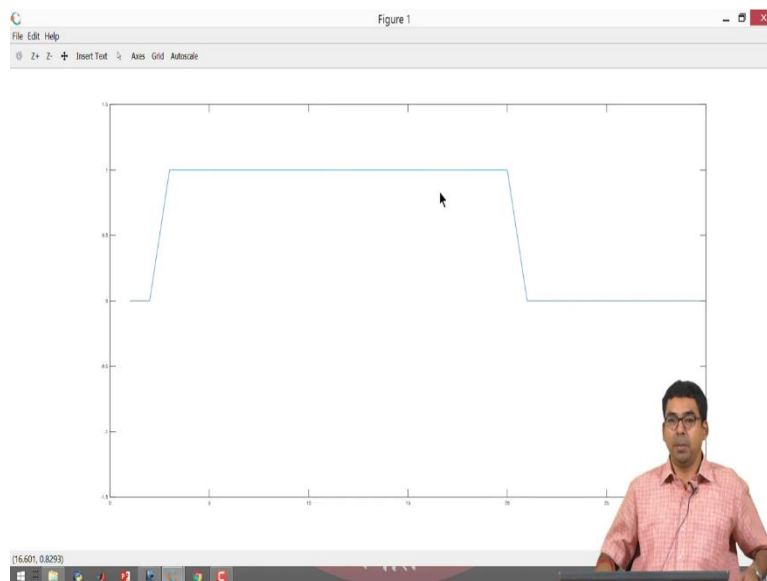


So, now let me just go ahead and run it, ok. Now, let me do this (Refer Time: 13:03) in a little bit more elegant manner. So, I will give an axis command, ok, ok, ok.

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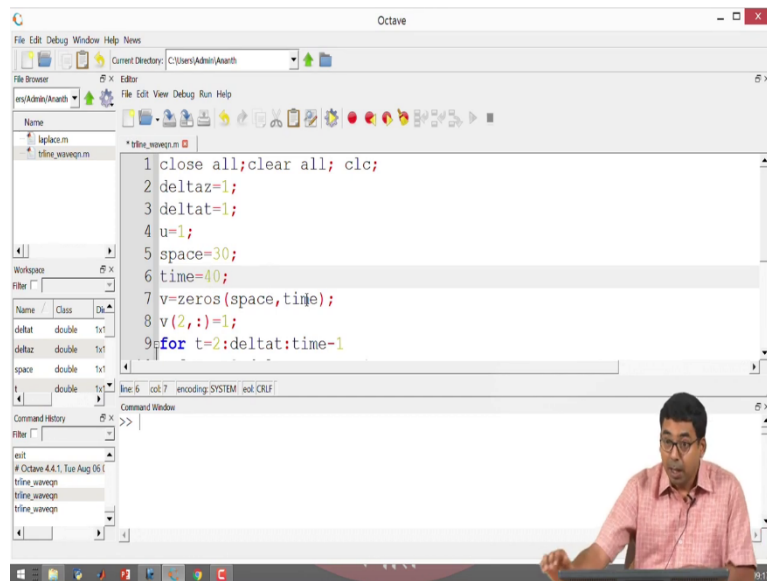
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So, here what is happening is at some instant of time in the beginning, all right for all instances of time we are going to be having position 2 to be giving out a voltage value of 1 volt. As time increases, you are just solving the wave equation and the wave equation is telling you that the wave is traveling inside the transmission line, right. So, at 20 instants of time it is going to have a velocity of you know 1 cell per unit time, that is it is going to travel Δz distance in Δt time because its velocity is u is equal to 1. So, at some instant of time it reaches the spatial location of 20.

This is the way to look at the wave equation for the voltage, all right. If you did give a source it is going to solve and it is going to tell you that there is going to be some wave front that is traveling along the transmission line.

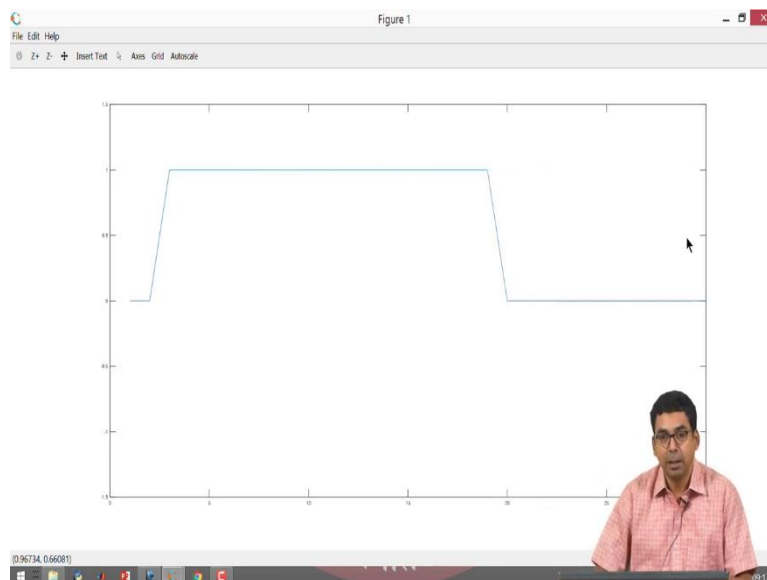
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```
1 close all;clear all; clc;
2 deltat=1;
3 time=40;
4 u=1;
5 space=30;
6 time=40;
7 v=zeros(space,time);
8 v(2,:)=1;
9 for t=2:deltat:time-1
```

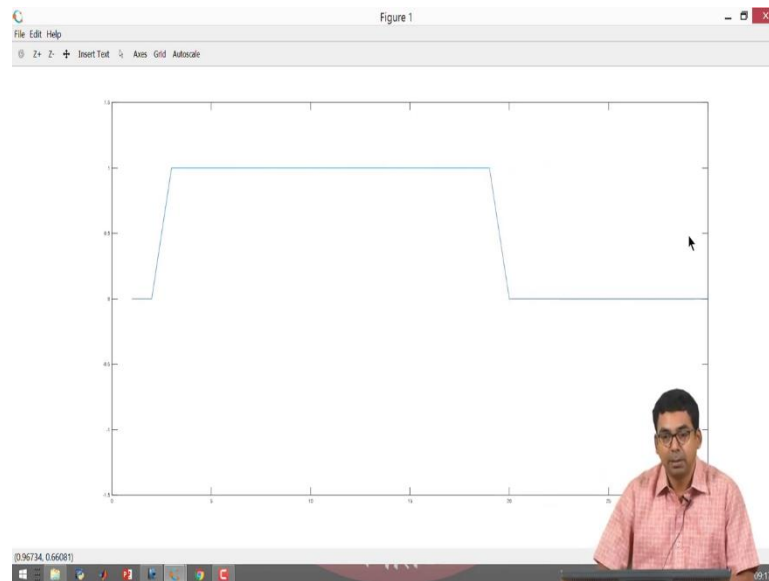
But some funny things happen in this and that is why I wanted to start in detail.

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I am going to increase the amount of time that I need for the simulation to say 40, ok. And I am going to run this again, I am going to notice that the voltage is going to the end and then something else is happening.

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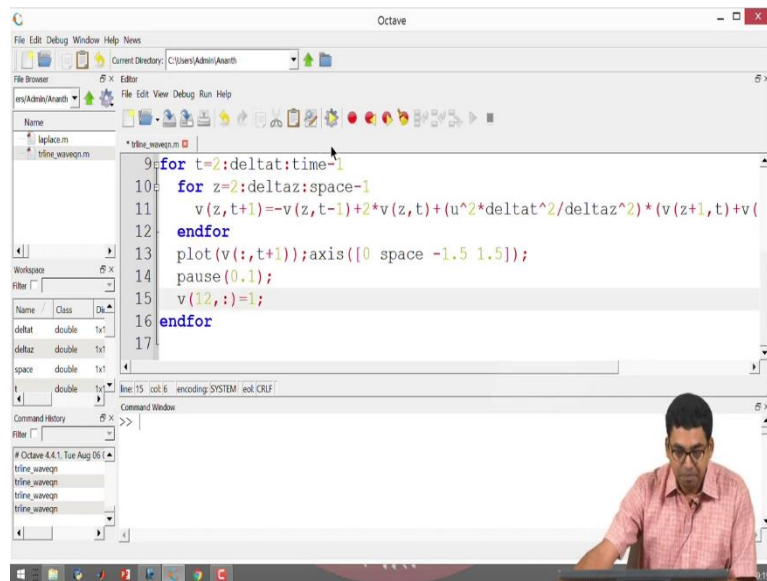
Suddenly, the voltage in the transmission line appears like it is doing something very vague, all right. It was supposed to have 1 volt throughout the transmission line, but, ok something is happening, all right.

It is clear that the voltage cannot suddenly become 0, all right. This part has to be understood more precisely. You had a forward voltage of 1 volt, on the right hand side end what could have happened is there was a reflection and the reflection now has to be understood in more detail. It just did not reflect 1 volt, it actually flipped the 1 volt in sign and then sent it back as a backward wave and you had a superposition of 1 volt traveling forward and minus 1 volt going in the backward direction and the superposition was actually 0, ok.

So, now we understand that the wave equation depending upon the boundary condition can tell you, at least programmatically, whether you are going to be having a reflection or not. Secondly, we also understand that the reflection is not only going to tell you something about the amplitude of the wave that is being reflected it is also going to consist of a sign change, ok. So, this is a very important thing.

But other than that it is a very simple scenario, all right. Suppose, I make this for I could do things that I cannot do with a real transmission line very easily.

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```
9: for t=2:deltat:time-1
10:   for z=2:deltaz:space-1
11:     v(z,t+1)=-v(z,t-1)+2*v(z,t)+(u^2*deltat^2/deltaz^2)*(v(z+1,t)+v(z-1,t));
12:   endfor
13:   plot(v(:,t+1));axis([0 space -1.5 1.5]);
14:   pause(0.1);
15:   v(12,:)=1;
16: endfor
```

The screenshot shows the Octave environment with a script named 'trine_waveeq.m'. The script implements a numerical solution for a wave equation. It uses nested loops for time (t) and space (z). The update equation for voltage v is: $v(z,t+1) = -v(z,t-1) + 2v(z,t) + \frac{u^2 \Delta t^2}{\Delta z^2} (v(z+1,t) + v(z-1,t))$. The plot shows the voltage profile over time, with a pulse at z=12. The workspace shows variables: deltat (double, 1x1), deltaz (double, 1x1), space (double, 1x1), and t (double, 1x1).

For example, I could say that the voltage at position 12 at all instances of time is equal to 1. So, this means that in the middle of the transmission line I am connecting a source. Normally one would not do it, but it allows me to do this, so I can play with this program and try to make deliberate changes and see what happens, all right.

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So, I would like to do this first, right. So, I noticed something, right. This is the way the voltage is going to be inside of the transmission line. It is not what one would expect normally. It is not just that you would send 1 volt and if you will have 1 volt everywhere.

First of all, when we began the program we noticed that from the position, there was a voltage wave front traveling to the right side there was another voltage wave front traveling to the left side. They go and hit the boundaries and depending upon the boundary conditions they get reflected. They can also have sign change in reflection. Due to this, at this instant of time where we have stopped the simulation the voltage profile across the transmission line looks like this, there is a voltage of 1 volt in some portion the remaining portion of the transmission line has 0 volts, ok.

Now, this needs a little bit more care and it tells us that we have to look at transmission lines in terms of reflections. We have to understand in a steady state what is going to be the voltage in the transmission line, all right. We also need to understand if you detect a 0 volt in transmission line, does it mean it is really carrying 0 volt or does it mean that there is a forward voltage and a backward voltage superposed that is giving you 0 volts. All these things have to be considered in great detail, all right. But this is a good starting point, all right because I have touched the word reflection.

Now, we will go back and look at what all we have done and where we are getting, all right. So, initially we started with the simple transmission line equations, all right. We made a model which was lossless and it gave us some partial differential equations for voltages and current, ok.

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Wave Propagation for communication - Windows Journal

$$\frac{\partial^2 V}{\partial z^2} = -l \frac{\partial}{\partial z} \left(\frac{\partial I}{\partial t} \right)$$

$$= -l \frac{\partial}{\partial t} \left(\frac{\partial I}{\partial z} \right)$$

$$= -l \frac{\partial}{\partial t} (-c) \frac{\partial V}{\partial t}$$

$$\frac{\partial^2 V}{\partial z^2} = l_c \frac{\partial^2 V}{\partial t^2}$$

Dimensional analysis \Rightarrow LHS units $\frac{V}{m^2}$
RHS units $l_c \frac{\partial^2 V}{\partial t^2} \rightarrow \frac{V}{s^2}$
 $\Rightarrow l_c = \frac{1}{v^2}$

Once we had the partial differential equations we tried to make the wave equation alright and we tried to use some general solutions to the wave equation and found out that the

voltage and the currents can actually be travelling in the form of waves. At that stage we did not know what to do with it more, all right, so we taught ourselves how to use a computer to solve partial differential equations. We started with the representation of first and second order partial derivatives and then as an example we saw we solved Laplace equation.

(Refer Slide Time 19:06)

The screenshot shows a Windows Journal window with the following handwritten content:

$$\Rightarrow f'(x_0) \cong \frac{f(x_0) - f(x_0 - \Delta x)}{\Delta x} \quad O(\epsilon) \cong \frac{\Delta x^2}{2! (\Delta x)}$$

Backward Differencing

$$\cong \frac{\Delta x}{2}$$

3) ① - ②,

$$f(x_0 + \Delta x) - f(x_0 - \Delta x) = \frac{2\Delta x}{1!} f'(x_0) + \frac{2\Delta x^3}{3!} f'''(x_0) \dots$$

$$\Rightarrow \frac{2\Delta x}{1!} f'(x_0) \cong f(x_0 + \Delta x) - f(x_0 - \Delta x)$$

$$\Rightarrow f'(x_0) \cong \frac{f(x_0 + \Delta x) - f(x_0 - \Delta x)}{2\Delta x} \rightarrow O(\epsilon) \cong \frac{\Delta x^2}{3!}$$

Central Differencing

Now, we did not make a very big change from the Laplace equation to this and we are already able to solve the wave equation. The only thing that is remaining is finishing what we started, all right. We still did not solve the telegrapher's equations. The telegrapher's equations were two in number, once we finish that the remaining parts are going to be a breeze because you will have a visual understanding of what the telegrapher's equations actually mean.

The difference between the wave equation and the telegrapher's equation is, the wave equation is decoupled that is the voltage is there on one side with respect to a time you take derivative on the other side you take with respect to space, but you do not observe what is going on with the the current at the same at the same time, all right. So, the only thing that is pending for us to complete this particular exercise is to go for solving the telegrapher's equations in the form that we have written. Let me go ahead and do that, all right.

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1) $\frac{\partial V}{\partial z} = -l \frac{\partial i}{\partial t}$

$$\frac{V(z+1, t) - V(z, t)}{\Delta z} = -l \left(\frac{i(z, t+1) - i(z, t)}{\Delta t} \right)$$
$$\Rightarrow i(z, t+1) = i(z, t) - \frac{\Delta t}{l \Delta z} (V(z+1, t) - V(z, t))$$

2) $\frac{\partial i}{\partial z} = -c \frac{\partial V}{\partial t}$

So, I will start with the first equation

$$\frac{\partial V}{\partial z} = -l \frac{\partial I}{\partial t}$$

right. This was one of the telegrapher's equations that we had.

Now, we have an idea about how to write these derivatives, right. These are first order derivatives and we could use forward difference, backward difference or central difference. Thus far I have been using central difference for all my programs, all right. But I am going to give you some confidence that I am going to be using forward difference, all right and I am going to be using backward difference to just illustrate that I could use either of them.

So, I am going to write down the left hand side derivative, all right with respect to space. So, I am just going to write this as

$$\frac{V(z+1, t) - V(z, t)}{\Delta z} = -l \left(\frac{i(z, t+1) - i(z, t)}{\Delta t} \right)$$
$$i(z, t+1) = i(z, t) - \frac{\Delta t (V(z+1, t) - V(z, t))}{l \Delta z}$$

This is going to be the difference form of the first telegrapher's equation, ok. So, $i(z, t+1)$ is the quantity that you want to calculate, $i(z, t)$ is the quantity that is that you are already aware of at the current position and then you have to take some voltages at neighbouring points, ok.

Now, let us also write down the difference form of the other equation in the telegrapher's equation. So, that will be

$$\frac{\partial I}{\partial z} = -c \frac{\partial V}{\partial t}$$

In this particular case, just to mix up things I am going to be using backward difference for the space in the left hand side. It may look very confusing, I just want to convey that you could use any of them, all right. There are some catches, I will not go over the catches in this course because there is an advanced course in computational electromagnetics that you can always do in a later stage, all right. But I think there is no harm in using a backward difference, using forward difference in one case, we can use backward difference.

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The screenshot shows a Windows Journal window with the following handwritten content:

$$\frac{V(z+1,t) - V(z,t)}{\Delta z} = -L \left(\frac{i(z,t+1) - i(z,t)}{\Delta t} \right)$$

$$\Rightarrow i(z,t+1) = i(z,t) - \frac{\Delta t}{L \Delta z} (V(z+1,t) - V(z,t))$$

2)

$$\frac{\partial i}{\partial z} = -c \frac{\partial V}{\partial t}$$

$$\frac{i(z,t) - i(z-1,t)}{\Delta z} = -c \frac{V(z,t+1) - V(z,t)}{\Delta t}$$

$$\Rightarrow V(z,t+1) = V(z,t) - \frac{\Delta t}{c \Delta z} \left\{ i(z,t) - i(z-1,t) \right\}$$

So, I am just going to write this down as

$$\frac{i(z,t+1) - i(z,t)}{\Delta z} = -\frac{c(V(z+1,t) - V(z,t))}{\Delta t}$$

$$V(z,t+1) = V(z,t) - \frac{\Delta t(i(z,t+1) - i(z,t))}{c \Delta z}$$

Now, let us pay a little bit of attention to these equations, and what is the strategy to solve these equations to make some sense, ok. First of all, the wave equation was decoupled, that means, your left hand side and right hand side has the same voltage or current, right. But here the left hand side for the first equation that we have, right is going to be dependent on the

voltage at the current instant of time and the voltage in future is going to depend on the current value at the current instant of time.

Now, these equations are not independent, they are coupled partial differential equations and we have not solved coupled partial differential equations before in this course, ok. So, we need to know a strategy on how we are going to do it. So, the way in which we are going to do it is for any given position let us say that we will calculate the current first. So, I have written this equation first, let us say that I have some boundary conditions or I have some source conditions given, so what I will be calculating at a given position I will be calculating the value of current at the next instant of time.

Once I calculate the current at the next instant of time, all right that current has to be used here in the bottom equation to calculate the voltage at the next instant of time. So, coupled partial differential equations here means that both have to be solved, all right one after the other and the solution that you get from one has to be plugged into the other. This gives rise to some serious thoughts, ok.

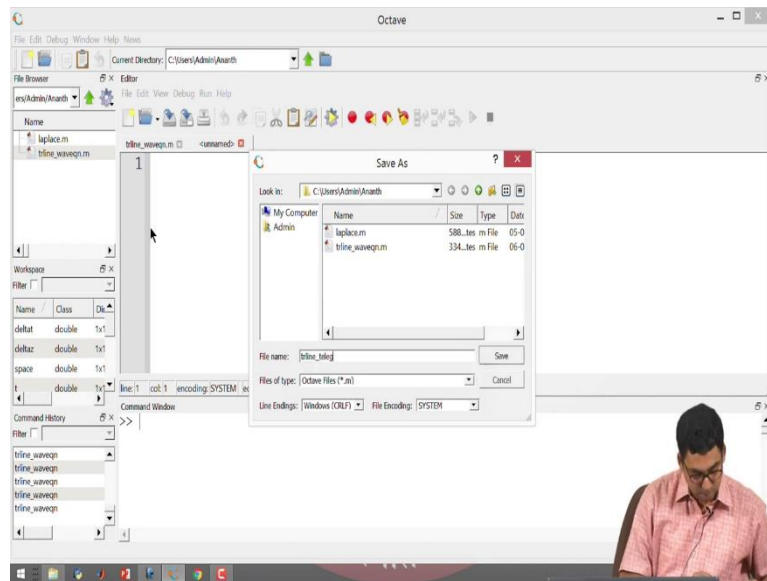
When we were trying to solve this equation $t + 1$ was the future over here, right $i(z, t+1)$ was the future, but once you have solved that is your present value correct. You you you I mean once you have finished solving you have estimated what is your current value. Then you have to plug the $t + 1$ whatever you had for current you will have to plug it as $i(z, t)$ and then you end up solving voltage for the next time instant for when you calculated current, ok.

What ends up happening is that you will keep calculating current at one instant of time, voltage at the next instant of time, current at the next instant of time, voltage and it keeps going like that. You are going to be solving in a way you know in a manner that is different from your Laplace equation or wave equation. You are solving two equations and they are coupled, that means, you have to take the solution from one and plug it into the other. This takes some time to understand, but these are not independent equations.

So, your $t + 1$ the quantity of current that you calculate, once you have calculated becomes the current value of I mean the present value of current that you are aware of. So, this has to be used as a present value. Once you have calculated $v(z, t + 1)$ you will have to go back, plug that as the voltage at current position and current type. This is what it means by doing the coupling. So, you have to do this back and forth, all right.

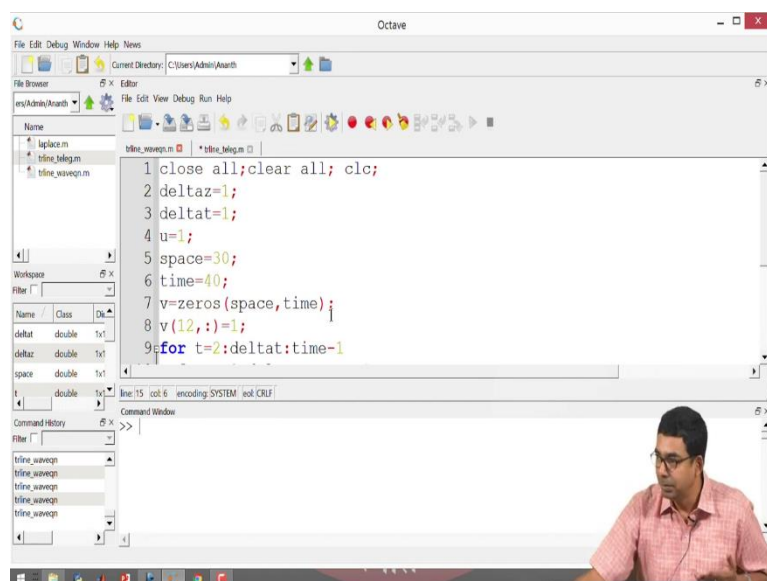
And this will let you know this will become familiar once we write down the code for this, ok. So, I will go ahead and write down the code for doing this, ok and it has some specific advantages over the code that we have just written for the wave equation, right, ok.

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So, start with the standard statements: close all, clear all and clc, right.

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Now, I am going to define my transmission lines and the way we had seen in the earlier theory classes. I want to say the number of sections that are going to be present in the transmission line, right. We are going to start with 100 small sections, ok. So, I am assuming that the transmission line is chopped into some small 100 pieces, right.

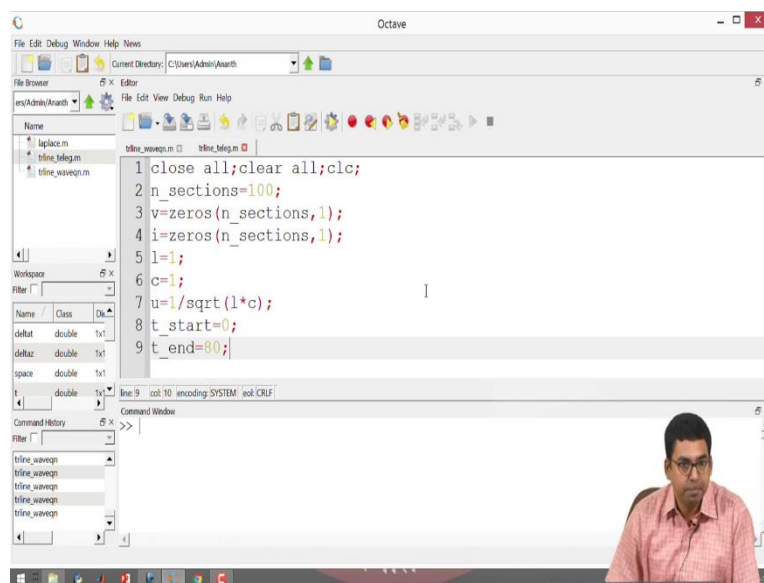
So, I am going to define my voltage. Now, I am making a slight change in the strategy, ok. Previously, the voltage in Laplace and the wave equation, right a I mean especially in the wave equation when we were using this voltage we tried to use two-dimensional matrices, all right

it consumes more memory then doing a one-dimensional, all right. And I am looking at this each time after I finished plotting, I do not have any use for the stored value, all right. So, at any instant of time I am going to be storing the value for the current time step, all right and I am going to be using that, I am going to be overwriting that again and again.

What that means, is if I have a look at the equations that we have arrived at $v(z, t + 1)$ is $v(z,t)$ minus some quantity, all right. This is your present, this is your future, all it says is the future value is the present value minus something, ok. So, I can just say that I will rewrite the future value in this present value itself, right. It is like writing a is equal to a minus something that is all. So, all I need is to store one instant of time and I am I am going to be able to calculate the values of the voltage in the current.

The same thing is going to be happening for the current, all right. It is a present value plus some you know term on the right hand side. So, I can say the future value with the same variable name is present value plus something, right that is how we are doing. So, we are trying to use a little less memory than before. So, we are making progress in the way we are doing the coding part, all right.

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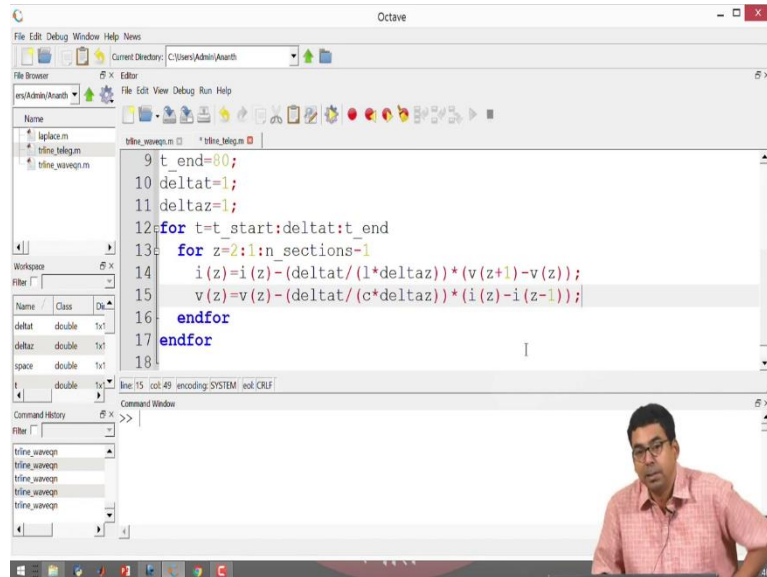


```
1 close all;clear all;clc;
2 n_sections=100;
3 v=zeros(n_sections,1);
4 i=zeros(n_sections,1);
5 l=1;
6 c=1;
7 u=1/sqrt(l*c);
8 t_start=0;
9 t_end=80;
```

Now, unlike the wave equation we also need to define our current. Current is going to have the same dimensions as the voltage vector that we have defined, ok. And just be clear and consistent with the class notes, all right I am using a ideal lossless transmission line and I am just assuming l to be equal to 1, whatever units you want to plug in you can plug in, right, c is equal to 1 and the value of the velocity, all right is 1 by square root $l c$. So, here it is obviously going to be equal to 1, right. So, the l have not put any units because my unit of space is going to be Δz , my unit of time is going to be Δt that is it, ok.

I also have to specify a few more things. I want to start at time t equal to 0, all right and I want to be able to finish. I would say you know 80 future time steps (Refer Time: 34:00), ok.

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```
9 t_end=80;
10 delta_t=1;
11 delta_z=1;
12 for t=t_start:delta_t:t_end
13     for z=2:1:n_sections-1
14         i(z)=i(z)-(delta_t/(l*delta_z))*(v(z+1)-v(z));
15         v(z)=v(z)-(delta_t/(c*delta_z))*(i(z)-i(z-1));
16     endfor
17 endfor
```

On top of this I have to give the Δt value because my update equation has Δt , Δz extra on the right hand side, right. So, I am just going to make life simple by choosing everything to be equal to 1, ok.

So, most of the definition is done. All we need to do is now plug in the equation that we have made using finite differences into these loops, and then we can see what is going to happen. Then we will have to compare what is happening here with the wave equation and then try to see what more we can get from here, right.

So, in order to do this, you will need to have a loop in time, all right. For t is equal to t start colon Δt colon t end, I said that in the case of octave if you do not give the middle argument it will keep incrementing by 1, here it is the same, but I am just being more precise because we are making progress in the way we are coding, right. And for all points in space, ok.

For every instant of time, for all points in the space that I am having I would like to calculate the values of current and voltage, all right. So, we wrote down the current equation first in our notes. So, we will do the same thing over here, right. Please understand that the way I am coding I have removed t as something needed from the array index because it is a one-dimensional array now, all right. Whatever is on the left hand side is the future value, whatever is on the right side is going to be the present and past values. I do not need a time index at all, ok. So, it will take some time to understand this, but these are slightly different from the equations before, these are known as update equations. You are taking the current value and adding something to it to update, ok.

So, the way we had written was i of z future value, this would have been written as i of z comma $t + 1$ in our notes, ok is equal to i of z , right minus the quantity was Δt divided by l

times Δz multiplied by v of z plus 1 minus v of z . This is the equation that we had written down. I just want to make sure it is the same equation that I have written, ok. So, Δt divided by $l \Delta z$ v of z plus 1 minus v of z looks the same, ok.

So, once I have calculated the current time step, I would like to calculate the voltage. So, I am having v of z equal v of z minus, all right Δt divided by, ok. So, we had used forward difference in one case and a backward difference in space for the other case, right and that is all. And the way we have written it we have actually coded it. So, this is your telegrapher's equation code. All we need to do now is set it set it up a little bit that is we need to give a source of a voltage or a current or something in this transmission line, then we have to give some plot commands to plot the current and the voltage and then we can start to go back and see what is happening. And in the next class we can go back to some theory on what happened and build on top of things. So, let us go ahead and finish this, ok.

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```

10 deltat=1;
11 deltax=1;
12 for t=t_start:deltat:t_end
13     for z=2:1:n_sections-1
14         i(z)=i(z)-(deltat/(l*deltaz))*(v(z+1)-v(z));
15         v(z)=v(z)-(deltat/(c*deltaz))*(i(z)-i(z-1));
16     endfor
17 subplot(2,1,1);
18 plot(v,'r');axis([1 n_sections -3 3]);
19 endfor
  
```

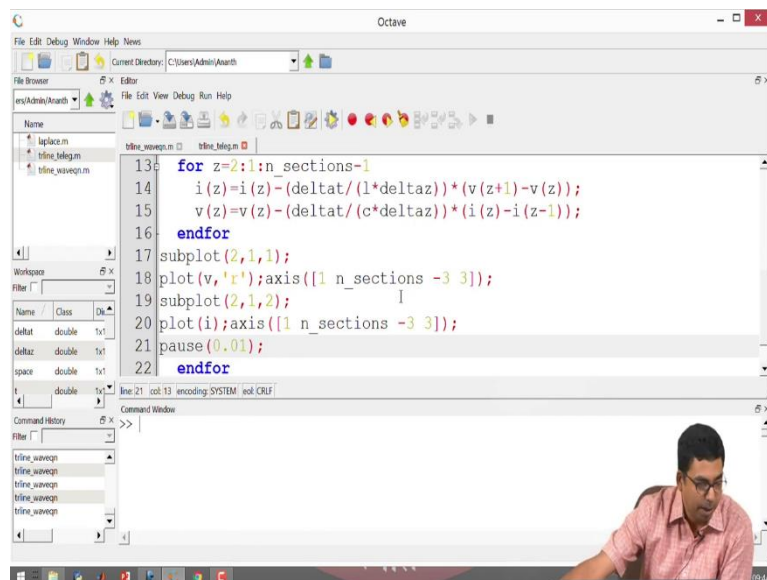
So, at every instant of time. So, the subplot of 2 comma 1 comma 1, just says that it is going to divide your screen or a plot into 2 rows, all right. 1 column, when you divide your plot into 2 rows and 1 column you will be having a plot on the top of your screen, plot in the bottom of the screen 2 rows, 1 column because there is only one on each a I mean on the top and the bottom. And then once you divide it you have to call it as plot number 1 is on the top, plot number 2 is the bottom. So, the subplot of 2 comma 1 comma 1 means that I am dividing it into 2 rows, 1 column and I am going to be talking about the first one on the top, ok.

So, I would like to have this because I would like to plot the voltage on the top and I would like to plot the current at the bottom. Previously, in the case of wave equations we could plot only one quantity at a time, all right, now I can plot both the quantities, right I can plot voltage

and current and I can draw different inferences from what is happening, right. So, I would like to plot voltage, right and I would like to plot voltage in red colour so that single codes just tell you to plot it in red colour, all right, ok.

And, the axis command here tells you it is of this form axis within a square bracket. What it is giving is x min x max for the x axis, y min y max for the y axis. I want to specify this because I saw already in the wave equation that when reflections happen and all that the value of the y axis keeps jumping here and there, I want to keep the y axis fixed, so that my concentration is on the voltage or the current quantity. So, that is what we had used in the transmission line equation also we had given an axis command, all right. If we do not give this it keeps going up and down the axis scales depending upon the current value of whatever voltage or current we have calculated. So, I want to keep it fixed, ok.

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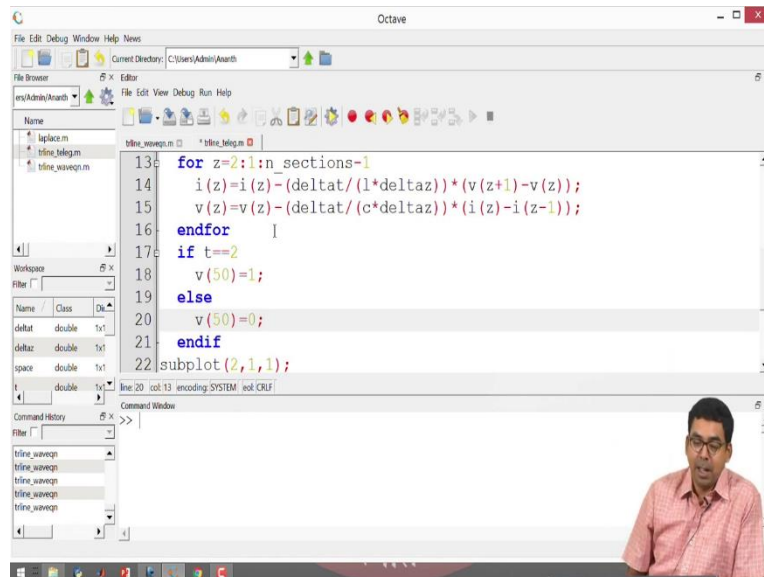
```
13 for z=2:1:n_sections-1
14     i(z)=i(z)-(deltat/(l*deltaz))*(v(z+1)-v(z));
15     v(z)=v(z)-(deltat/(c*deltaz))*(i(z)-i(z-1));
16 endfor
17 subplot(2,1,1);
18 plot(v,'r');axis([1 n_sections -3 3]);
19 subplot(2,1,2);
20 plot(i);axis([1 n_sections -3 3]);
21 pause(0.01);
22 endfor
```

So, I would like to plot voltage on the top plot, all right then the plot below. So, I am going to say 2 comma 1 comma 2. So, it is second a the, so the plot is divided into 2 rows, 1 column and I am addressing the second plot, all right. The second plot I want to be able to plot i, it is ok if it is in default colour because a I would like to distinguish between the two, right and I would like to give an axis which is going to be the same and as usual some pause command for me to actually have a look at each and every instant of time what is happening.

Now, everything is more or less said except that we have not defined our source, that is all right. So, we will go ahead and define our source, ok. In this case, the previous wave equation case, the kind of source I had defined, was on all the time. It is like a battery. You have closed the switch at equal to 0 and you are solving a wave equation trying to find out what the solution will look like, ok. So, in this case, we can do the same and we can do many other

things also. So, I will go ahead and do something which is slightly different, ok, so that you get a feeling for the amount of variety that one can do, ok.

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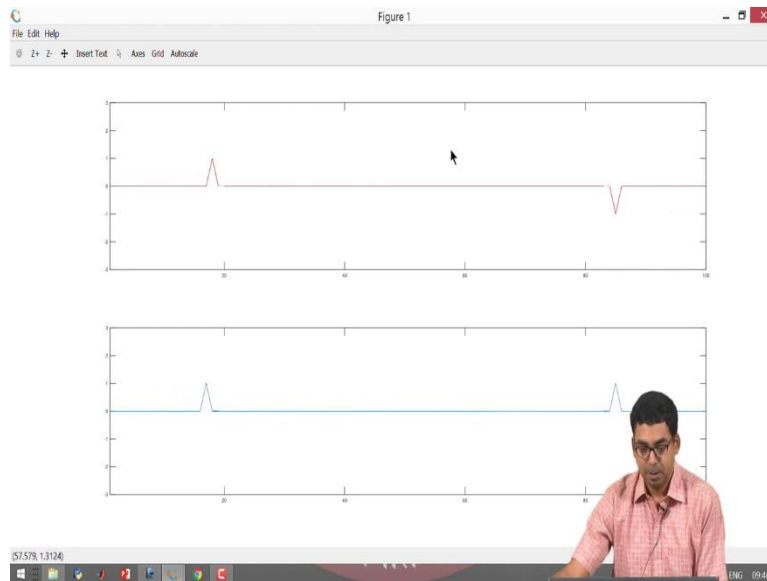


```
13 for z=2:1:n_sections-1
14     i(z)=i(z)-(deltat/(1*deltaz))*(v(z+1)-v(z));
15     v(z)=v(z)-(deltat/(c*deltaz))*(i(z)-i(z-1));
16 endfor
17 if t==2
18     v(50)=1;
19 else
20     v(50)=0;
21 endif
22 subplot(2,1,1);
```

This is also kind of a definition of the source. What I am doing here is at the instant t equal to 2, I am making the voltage at a specific point go to a value 1, at all other instances of time that point will be 0. In other words, it is like giving impulse, ok at some instant of time and then 0 volts or it's grounded at all the other instances of time.

I just wanted to plug this in because you should know that the way we have defined the sources, we have flexibility over that we could do impulse, we could do rectangular, we can do sinusoidal, we can do many things, ok. So, it just needs to register in your mind that you can play with all of these quantities. I will just go ahead run this one to make sure that the program is running, ok. It is indeed running, ok.

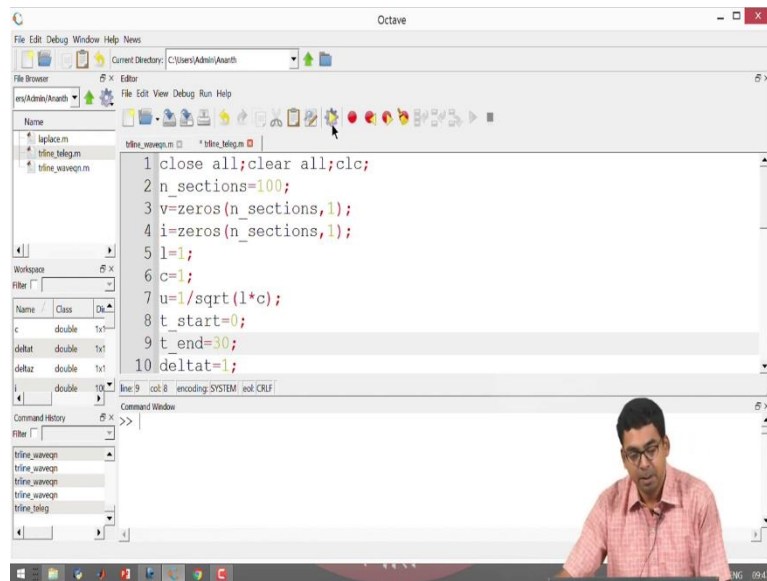
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Now, many weird things will be happening and that is going to be the subject of our discussion as far as the theory goes. Now, that we have completed one complete loop, we started with analytical part how to develop a model, we wrote down the partial differential equation, we wrote down the general solutions, we also verified what is going to happen with the computer, you should have a decent understanding of what is going on with the telegrapher's equation and the wave equation. But now let us use this program to drive the theory forward, ok.

So, let us see what happens when I do a few things and then decide how to carry on the theory, all right.

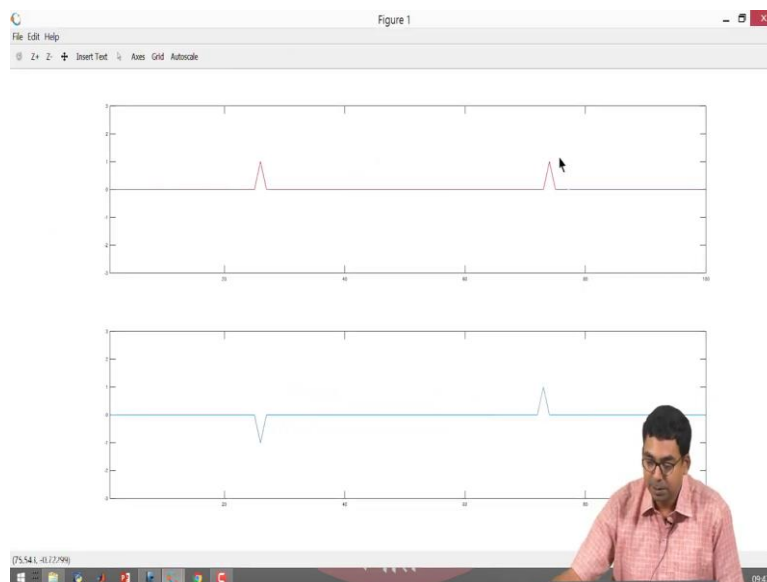
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```
1 close all;clear all;clc;
2 n_sections=100;
3 v=zeros(n_sections,1);
4 i=zeros(n_sections,1);
5 l=1;
6 c=1;
7 u=1/sqrt(l*c);
8 t_start=0;
9 t_end=30;
10 dt=1;
```

So, I am going to start with the ending time of the simulation to be 40 seconds, right or 40 units this or even 30 units, right.

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So, I am going to start. So, the red colour on the top represents voltage and the blue colour represents the current at the bottom. This information was not present to you in the wave equation case. Now, we are watching the current and voltage of the transmission line.

Next thing that I notice is I had set my source in the center, regularly in transmission line you may not be able to do this, but for our understanding we could do this in a program, right. I set an impulse at some instant of time and the impulse travelled in the left and in the right in

the transmission line, ok. So, the voltage is going from the center to the other side. But notice the current, it is traveling to the right side with a peak at 1 and its traveling to the left left side with a peak at minus 1. First of all, this information was not available to me before when I was solving the wave equation, right. Now, this is a new piece of information.

Depending on the direction of travel of the voltage wave, the current wave is going to adjust its sign, that is what it says, right. If the voltage is going to travel forward the voltage and the current are going to be of the same sign. If the voltage is going backward the voltage and current are going to be of the opposite sign. This is a phenomenal piece of information, all right.

If you go back to your general solution for the transmission lines, ok, we had f plus of t minus z by u plus f minus of t plus z by u , to be the solution for the voltage. So, f plus of t minus z by u plus f minus of t plus z by u is what the current would look like, right. So, the current is actually doing weird things and it is adjusting itself to that direction.

Now, let us notice more things, ok. I will run this again. This time is going to increase the amount of time I am going to run it. The voltage is going along the transmission line to the left and the right end, right, all right, almost hitting the boundary, ok, almost hitting the boundary. So, now is a very critical stage. I am going to make it go a little longer and try to understand what happens. So, the voltage is traveling to the left hand to the right, ok. Now, that is something phenomenal.

The impulse on the right hand side bounced from the edge and flipped it side. However, the voltage way on the left hand side did not change its sign, it tells you that something is happening to the boundary, so the boundaries can reflect, but they can also alter the sign of the reflection. So, I think in our mind we are starting to already figure out where we are going. We are trying to understand the term reflection coefficients, right. The reflection coefficient is going to be having a magnitude and also going to be having a sign is what we are indirectly seeing.

Same way if you look at the bottom graph or current the current actually did not flip on the right side, but it flipped on the left side. So, something is happening. So, when the voltage flips, the current does not flip. When the current flips the voltage does not flip. This means that we need to understand what is going on on these edges, ok. So, it also means that a boundary condition would affect voltage in one way, the same boundary condition would affect current and some other way, ok. So, and we need to understand what the voltage profile and current profile in a transmission line would be given some set of boundary conditions.

And here we are going to start with some very simple boundary conditions, all right. We can start with say a transmission line is broken, it is an open circuit or since you have two conductors a transmission line can have a short between the two conductors say some piece of metal is touching both these pieces and you have a short circuit. These are the two extreme cases, all right. So, either you have an open circuit or you have a short circuit. These are the two boundary conditions that one can have in the transmission line to begin with.

And clearly, I can make out that maybe that because the voltage did not flip on both the sides, on one side I am having one condition on the other side clearly I am having another condition. Since, the current did not do the same thing on both sides the conditions are different. I think we need some analysis. We will go back to our theory and try to find out more about reflection coefficients and things.

Now, what you are seeing here can be used for many purposes. Imagine that you have a transmission line that has some open or short circuit at the end. Maybe you have a way to find it. All you need to do is send a pulse, wait for the pulse to come back. If the sign of the pulse is flipped, you can figure out what is going on. If the sign of the pulse is not flipped you can figure out what is going on. So, in other words you can detect whether the line is actually having a short circuit or an open circuit on the other end. So, it is a very important thing.

You can have a 1 kilometre long transmission line, and I want to figure out if there is an open circuit or or a short circuit. All I do is send a pulse, I wait for the reflection, when the reflection is positive I can make out something, the reflection is flipped, I can figure out something, all right. So, this is practically used to detect faults and cables actually, ok.

So, we will stop here. We will go back to the theory in the next class, try to build on this and then we will come back with the computer part and then we will keep augmenting one and the other, right. So, I will stop here.