Transmission lines and electromagnetic waves
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Lecture – 31
Phase Velocity and Group Velocity

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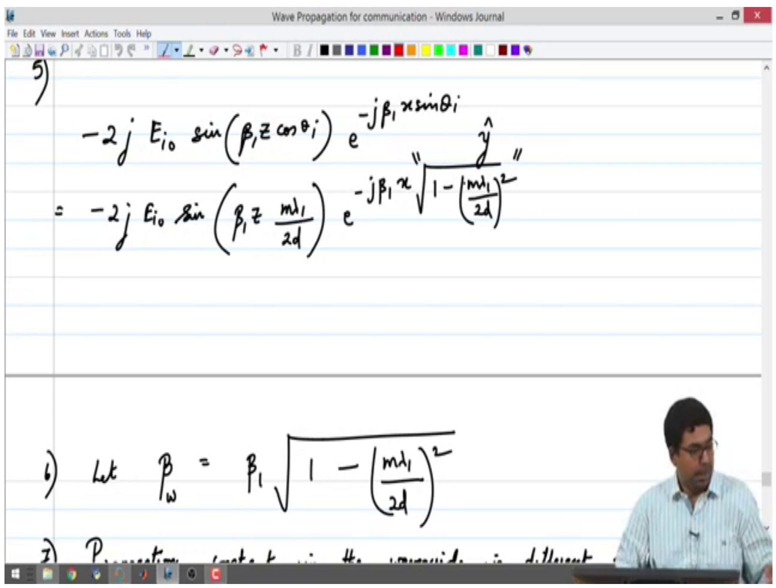

We will get started I think we have seen the basics for the parallel plate and the rectangular wave guide right. Now, we are going to just plug a few holes that we have overlooked all right and that we are going to do by a combination of two things we are going to use octave to visualize a few things right. And we are also going to make some arguments based on analytical a formulation right. And the two things that we are going to be talking about right one is a with regards to the Phase and the Group Velocities right the second thing is related to dispersion right.

So, I am going back to the derivation that we had for the parallel plate waveguide in order to pick up one formula that we had all right. And a I have here the expression for the phase constant within the waveguide β_w right and its

$$\beta_w = \beta_1 \sqrt{1 - \left(\frac{m\lambda_1}{2d}\right)^2}$$

So, I will go ahead and begin the lecture with that right.

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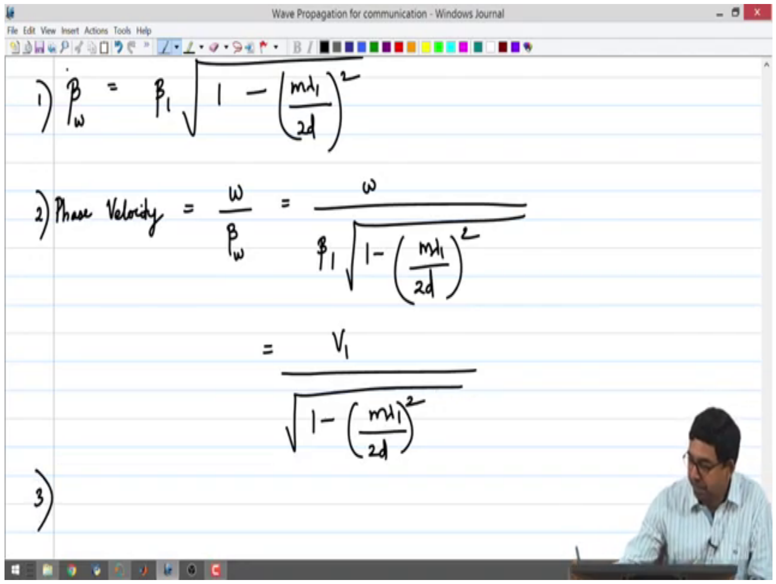

Right, so, I have β_w. So, β_w is the phase constant of the wave guide is equal to β_1. β_1 corresponds to the phase constant that is filling the parallel plate ok so, it is the homogeneous medium that is filling the parallel plate if it is vacuum you will have the phase constant of vacuum.

$\sqrt{1-\left(\frac{m\lambda_1}{2d}\right)^2}$ where λ_1 is the medium all right in the homogeneous a I mean a medium that is filling up the waveguide ok that is λ_1. And then you are having d which is the separation between the two plates and m corresponds to an integer that will tell you about the number of half cycles of the electric field you will have in the transversal directions right.

So, we start with this all right and then we will slowly build up to some holes that we need to plug in right. The first thing that we want to talk about is the velocity all right. Now, I am going to make a clear distinction ok and I am going to make a distinction between two quantities phase velocity and group velocity. And I will also show you how they differ right visually right.

So, the term that we have been seeing as velocity in the case of transmission lines was $\frac{\omega}{\beta}$ and then we went to plane waves which was again $\frac{\omega}{\beta}$ and now also we are dealing with $\frac{\omega}{\beta}$ all right. This $\frac{\omega}{\beta}$ is known as phase velocity tells you how fast the phase travels it means that you will pick up a point in your sinusoid all right corresponding to a particular angle say you pick up a peak. And you say that if your wave is a cosine and you say that that angle could correspond to a 0 degrees. And you track that particular peak how fast its traveling then that would be your phase velocity all right.

So, you are saying that the phase angle is traveling through space under your consideration with a certain velocity ok. So, the phase velocity is just telling you how fast the phase is traveling within your space a given amount of time right. So, here we are just talking about ω divided by β ok and in this case I want to talk about the phase velocity in the wave guide all right. So, I am just going to make this β_w ok and I have

$$Phase\ velocity = \frac{\omega}{\beta_w} = \frac{\omega}{\beta_1\sqrt{1-\left(\frac{m\lambda_1}{2d}\right)^2}}$$

Now, $\frac{\omega}{\beta_1}$, ω is the angular frequency of the wave that you are passing through this waveguide. β_1 is the phase constant of the homogeneous medium $\frac{\omega}{\beta_1}$ is going to be the velocity in that free space medium all right. If you are considering that you are having two parallel plates and you have free space or vacuum in between $\frac{\omega}{\beta_1}$ will correspond to the velocity c ok.

So, again you can say that this can be written as

$$Phase\ velocity = \frac{\omega}{\beta_w} = \frac{v_1}{\sqrt{1-\left(\frac{m\lambda_1}{2d}\right)^2}}$$

where v1 is the velocity in the medium that is filling up the region between the two plates in the parallel plate configuration ok.

Now, there are also a few other things that we have to look at all right before we get to the visualization part right ok.

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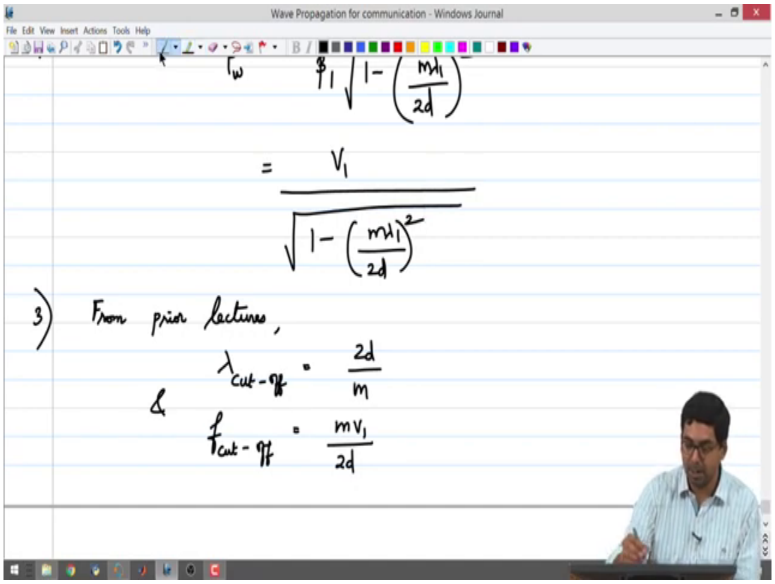

Now, from the previous lectures right, ok I have also written down an expression for cutoff wave length. And then I also wrote down an expression for cutoff frequency I said that in order for your wave to have a propagation constant that is real right through your waveguide you needed to be at a frequency higher than the cutoff frequency or a wavelength that is lower than the cutoff wave length.

And we had written the expressions in the prior classes instead of copying I have to scroll a lot of pages. So, I will just write down the expressions that we had right. So, I had

$$\lambda_{cut-off} = \frac{2d}{m}$$

and the expression that I had for the frequency it's

$$f_{cut-off} = \frac{mv_1}{2d}$$

So, for your wave to travel right you needed to have $\lambda < \lambda_{cut-off}$ or $f > f_{cut-off}$ right so, frequency. This is what we have seen. So, hence now I can go back to my expression for the phase velocity ok and rewrite the phase velocity right in terms of λ cutoff and frequency cutoff.

So, if I have a look at the expression the expression has $\frac{2d}{m}$ all right. So, it has $\frac{m}{2d}$ coming into the picture. So, I can just say it is $(\lambda_1/\lambda_{cut-off})$^2.

So, I can make a simple substitution and write this phase velocity in terms of the cutoff wave length.

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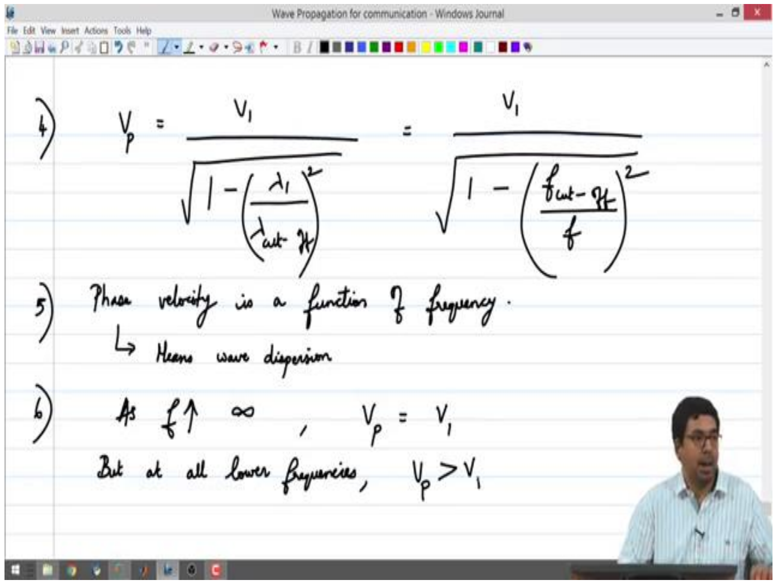

So, I can say that the phase velocity is going to be equal to

$$v_p = \frac{v_1}{\sqrt{1 - \left(\frac{\lambda}{\lambda_{cut-off}}\right)^2}}$$

ok I have a ratio of wave length.

So, I have $\frac{\lambda}{\lambda_{cut-off}}$ I could also write this ratio in terms of frequencies assuming a velocity ok. So, I can just write this in terms of $\frac{f_{cut-off}}{f}$ all right. f is the operating frequency right frequency of your signal that you are using f cutoff is determined by the dimensions of the waveguide all right and the mode that you are choosing.

So,

$$f_{cut-off} = \frac{mv_1}{2d}$$

depends upon the medium that fills the parallel plate depends upon the mode they are choosing m. And depends upon the construction of the parallel plate that is the how far away the two plates are going to be all right. And f is the frequency that you have chosen to operate it with as long as it is higher than f cut off you will be having wave guiding action right.

Now, we have to see a few things after we have written down the phase velocity in this format all right. The first thing that we see is the phase velocity ok is a function of frequency, phase velocity is a function of frequency because I have left hand side vp on the right hand side I have in the denominator f all right which is the frequency that I am going to operate, that means, the phase velocity is a function of frequency yeah all right.

So, if you have different values of frequency coming into the right hand side you will end up with the different phase velocities consequently on the left hand side all right. Now, this means that the phase velocity is dependent upon the operating frequency and such a condition all right where your velocity the phase velocity is dependent upon the operating frequency we call it as a dispersive medium or a wave dispersion ok. So, ok.

So, you could actually have a parallel plate configuration with vacuum present between, we know that vacuum is a medium where we consider the speed of electromagnetic wave for all frequencies to be constant at$3 * 10^8$ meters per second all right even if you did do that what will end up happening is.

If you are going to choose a frequency where your construction of the parallel plate acts as a wave guide that is you are choosing a frequency above the cutoff right. Even though you have used a nondispersive medium you will end up having dispersion simply because of the waveguide construction itself all right.

So, a homogeneous medium that is otherwise non dispersive ok. In the case of vacuum the phase velocity is just ω divided by β it is homogeneous medium all right it does not have any frequency dependence all right. So, it is going to be a fixed number ok.

So, what happens here is an in the case of the wave guide you will end up getting dispersion for medium that is otherwise non dispersive this is the first thing that we have to look at. The second thing and the more interesting thing is that as the operating frequency increases let us say it goes all the way to very high numbers and I call this as the infinity as the frequency is very very high ok vp becomes equal to the v1 right, vp is equal to v1. So, f is very very high infinity.

So, that term becomes divided by infinity. So, that 0. So, v1 divided by square root 1. So, vp is equal to v1 right, but at all lower frequencies I notice that vp is actually greater than v1 ok it is v1 divided by square root 1 minus some quantity ok which means that your denominator is going to be less than 1 and your vp is going to be greater than v1 all right.

That is the reason I want to make a clear distinction with the phase velocity in the case of the waveguide. The phase velocity looks like it is going to be greater than or equal to v1 ok, but a many times understanding this and making some logical interpretations of this will become very difficult ok that is why we need to now figure out what this phase velocity actually means in the engineering realm and how we can put a correct visualization to understand this right. So, first of all we can write down a few things and then fire octave and see what happens right.

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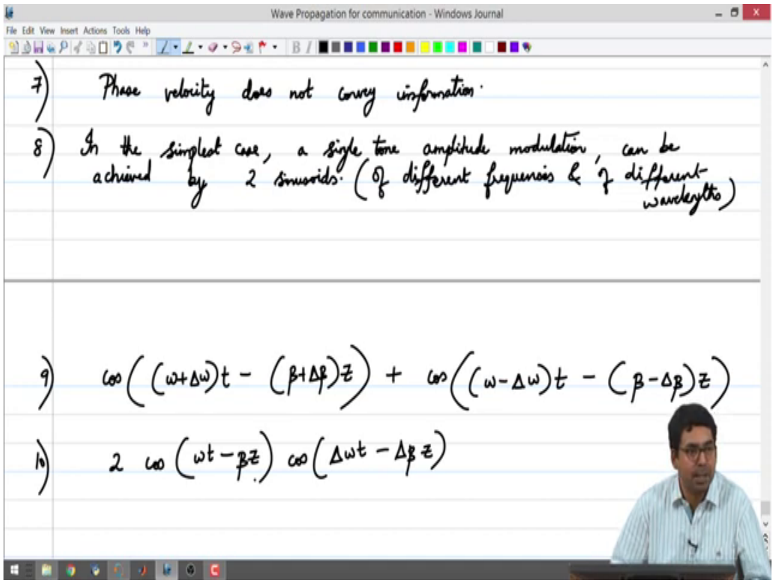

So, the general notion here is that the phase velocity does not convey this is what normally we say phase velocity does not convey information, but what does that mean right we have to understand.

So, in electrical engineering especially in the undergrad you would have had some courses which deal with the fundamentals of communication where you would have talked about say amplitude modulation frequency modulation extra. And there could have been advanced courses which teach you about different digital modulation formats extra right.

But we are going to take the simplest case in order to communicate information all right. The electrical engineers would use what is known as a carrier signal and then a modulating signal all right you will have a carrier signal and then you will have a modulating signal and in the case of the simplest scenario.

So, we will take a simple scenario right ok let us say that I want to convey an information which is of a single tone or a single frequency all right. And I want to use amplitude modulation because that is the first modulation format that you actually learn in your undergraduate right

So, I want to be able to use amplitude modulation. So, I am will I will be having a wave all right I am going to super impose another wave because I want to do single tone modulation I just want to convey another sinusoid using this carrier ok this is a case that we are considering.

So, I want to do a single tone amplitude modulation ok. And a so, you need at least two waves to achieve this you need something which is known as a carrier and the other one which is going to modulate this carrier with the information that you want to transmit and the information that we want is a single tone. So, it is going to sinusoid of some frequency that is what we want to communicate all right. So, you can achieve it you can achieve it using two sinusoids one is a carrier and the other one it is a modulating signal that is all right.

So, what would the mathematical formulation be now, let us try to see what we have already learnt and try to put it in a perspective over here right. We have seen that a forward traveling wave would be represented by say $e^{-j\beta z}$ all right. And then you will be having e to the j ω t coming into the picture with respect to time for a sinusoid.

So, you can say to the $j\omega t - \beta z$ all right and the real part of that would be a $Cos(\omega t - \beta z)$ and the imaginary part would be a $Sin(\omega t - \beta z)$ ok. This is what we have already learnt in transmission line plane waves extra right.

So, let us say that I want to transfer information from a source to a receiver and I am going to use a forward wave from the source going to the receiver and I just want to track the real part of it ok. So, what I am going to do is I am going to write down some expressions. So, it is going to be a cosine ok now I am saying that as in the simplest case a single tone amplitude modulation can be achieved by two sinusoids ok.

Now, instead of looking at it as $\omega t - \beta z$ ok. Let us say that I am going to pick up two sinusoids of two different frequencies and two different wavelengths all right. So, I am going to make this a little bit clearer you cannot say just two sinusoids they could be just differing by phase all right.

So, I want to be clear two different frequencies and of different wave lengths, that means, different beta. So, I am choosing two waves of different ω and different β ok. Now, I am going to say that a in order to highlight this difference I am just going to use you know some delta ω and delta β difference between the two waves.

So, what I am going to do is I am going to take two waves all right and I am going to write down the expression right,

$$Cos\big((\omega + \Delta\omega)t - (\beta + \Delta\beta)z\big) + Cos\big((\omega - \Delta\omega)t - (\beta - \Delta\beta)z\big)$$

So, I have $\omega t - \beta z$ form one of the waves or the sinusoids that I am using can be written as ω plus delta ω t minus β plus delta β z ok.

And I am going to use another sinusoid and I am just representing it as ω ops ok I am having two sinusoids of slightly different frequencies and slightly different wavelengths and I have just added them up together ok. So, one of them should give me information about a I mean the resulting expression should give me an information about the carrier and the modulating signal.

So, all I am doing is I am taking two signals I am just adding them up together ok the two signals are slightly of different frequencies and slightly of different wave lengths all right. Now, I can use some trigonometric identity. So, this is of the form cos a + cos b. So, I could simply use

$$2\cos\left(\frac{a+b}{2}\right) - 2\cos\left(\frac{a-b}{2}\right)$$

2 cos a plus b by 2 cos a minus b divided by 2 ok.

So, I could use this formula to expand what is happening over here. So, I get end up getting

$$2Cos(\omega t - \beta z)Cos(\Delta\omega t - \Delta\beta z)$$

Now, if I were to take these components individually I could say that the first sinusoid is having a phase velocity of ω divided by beta. The second sinusoid or the cosine here all right is having a phase velocity of delta ω divided by delta β ok.

Now, the real question is $\frac{\omega}{\beta}$ and $\frac{\Delta\omega}{\Delta\beta}$ are they very different are they same do they convey the same information if they do not convey the same information which one conveys the information extra all these things start to come into the picture all right. So, what we are going to do is we are going to take this expression and we are going to try to plug it into octave and see what the visualization looks like and what conclusions we can draw from there all right.

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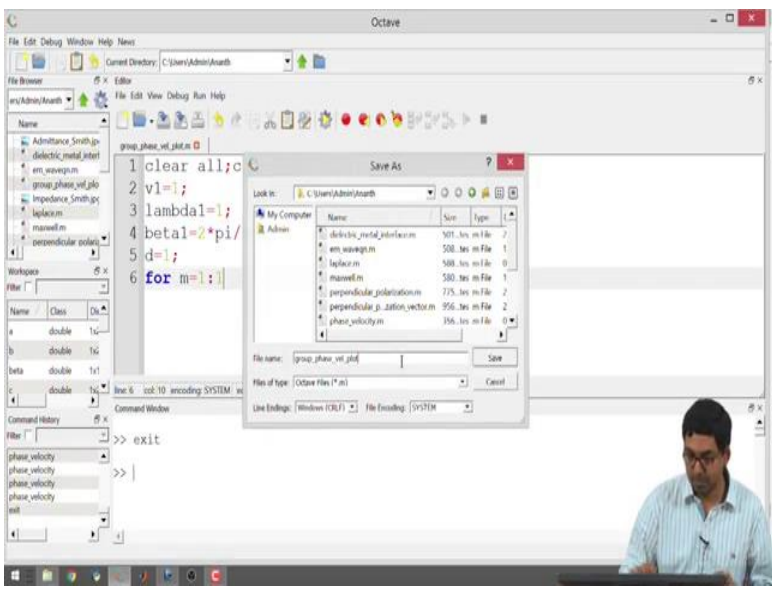

So, I am going to fire octave ok.

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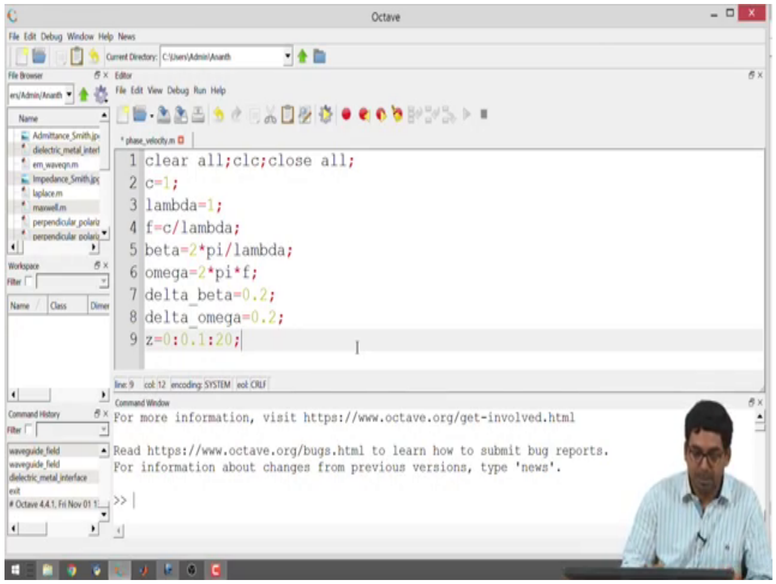

Ok, and a I am going to start with definitions of the terms that I want in that expression all right. First of all a I am not going to use $3 * 10^8$ meters per second to be the velocity I like to use the velocity to be equal to 1 in air. And then everything is relative to that, but currently I am worried about vacuum because anyway it is a nondispersive medium and it could fill a parallel plates a perfectly valid medium. So, in some normalized units I am just taking c equal to 1 this means that I am considering only ϵ_r and μ_r ok.

So, $\frac{1}{\sqrt{\epsilon_r \mu_r}}$ where ϵ_r is equal to 1 μ_r is equal to 1 right. It is just giving me a relative velocity of c equal to 1 right. And I would need λ considering it as 1 ok I want to calculate the frequency of this wave I can just do c by λ the frequency of operation is say c/λ ok and in order to use the formula I need also to find out β and ω all right.

So, β = 2π/λ. So, given a λ of 1 I can say that my β is going to be equal to 2π right. ω is going to be 2πf ok one more thing that I will need in the way I have written the modulation problem is $\Delta\omega$ and $\Delta\beta$ all right.

So, since it is $\Delta\omega/\Delta\beta$ I am going to consider some small values of $\Delta\omega$ and $\Delta\beta$ right. So, I am just going to say I will consider $\Delta\beta$ is equal to say 0.2 right. And then I will say $\Delta\omega$ let us make it 0.2 also ok you can play with these values and we can see what happens right. And I also have cos $\omega t - \beta z$. So, I need to construct a space in z direction all right.

So, I am going to assume some 20 points are a you know 200 points in z direction and see what happens. So, I will just go with z is going from ok z is an array just going from some 0 to 20 units ok and I want to see the evolution with respect to time.

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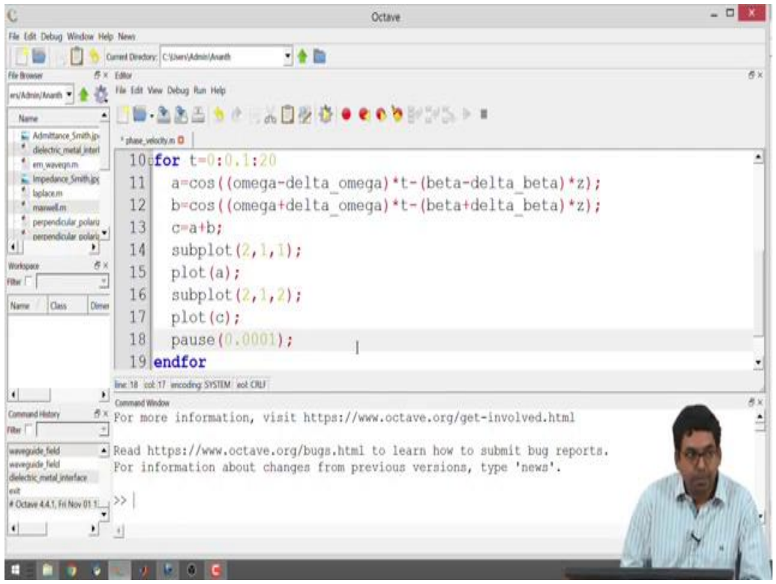

So, I am going to take some time is equal to 0 also 20 units of time right ok. So, I am going to write down the expression for the first wave that I am using I am going to call that wave as a. And I have

a=cos((omega-delta_omega)*t-(beta-delta_beta)*z

b=cos((omega+delta_omega)*t-(beta+delta_beta)*z

So, we had two waves and we were just adding them up. So, I am just going to do c is equal a plus b. So, I am going to create two plots to highlight the differences between a and a plus b ok. So, I am going to have two plots one on the top and one on the bottom. So, first I will just plot the wave a that is equal to some $Cos((\omega - \Delta\omega)t - (\beta - \Delta\beta)z)$ it is just a wave ok.

And I will have another plot at the bottom and I will plot the sum of these two for all instants of time right. Since, I want to see the evolution with respect to time I want to give a small pause command right ok and I am going to run this program all right.

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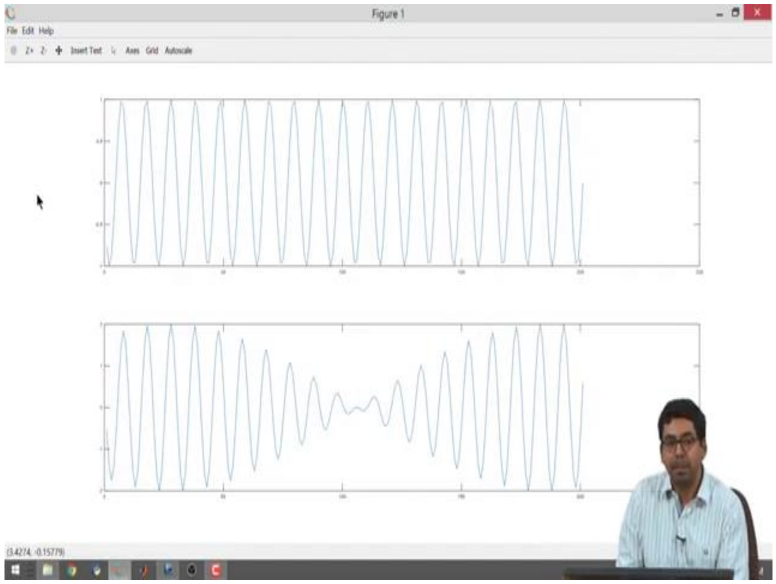

There are two things that I noticed the top one is a simple sinusoid we know that it is moving forward in the positive z direction with respect to time all right.

So, the top one is just some $\omega t - \beta z$ form. The bottom one is a combination of the two waves that you have considered of slightly different frequencies and slightly different wavelengths you have added them up together. And I noticed that there is a difference and a commonality between the two. The commonality that I see is phase velocity all right if I take a point on the sinusoid on the top and see how it is moving if I see a point over here and see how it is moving they are going to move in identical ways ok, but because I have added another wave there is also some information that is coming in as the envelope of this signal all right.

And we usually say that the envelope is the one that carries the information all right you could say that perhaps this entire region could correspond to some information corresponding to bit one something like that all right. So, you can say that that information has got so many waves all right, but the envelope is the one that is actually conveying the information in the most elementary case of amplitude modulation ok. So, once again I will run this all right.

So, I think in the bottom figure one should understand that the phase velocity is the velocity with which you are tracking a particular angle and it is how fast it is going in space with respect to time. The group velocity is actually going to tell you something about how fast this envelope is going to be traveling ok. So, this is the distinction that people can make between phase velocity and group velocity ok group velocity tells you something about the envelope that actually carries the information right and phase velocity is simply the oscillations and you are just tracking a point of constant phase right.

So, it is a very simple example and you can also play around with these numbers for example, you can make this 0.3, 0.3 extra ok right and you will notice you know if the β became higher. So, you will notice more modulation with respect to space right I can also make them you can you need not make them equal could also make it like for example, 0.2, 0.3 extra you could also make them you will notice some drifting patterns extra right all right.

So, you can go back play with it, but a this is the most elementary form of modulation that we start learning in the communication when we want to transmit some information, that is there is a carrier and there is an envelope to the carrier and that envelope is the one that is carrying the information right.

So, now, we go back and we have a look at our waveguide scenario in a similar manner all right all right.

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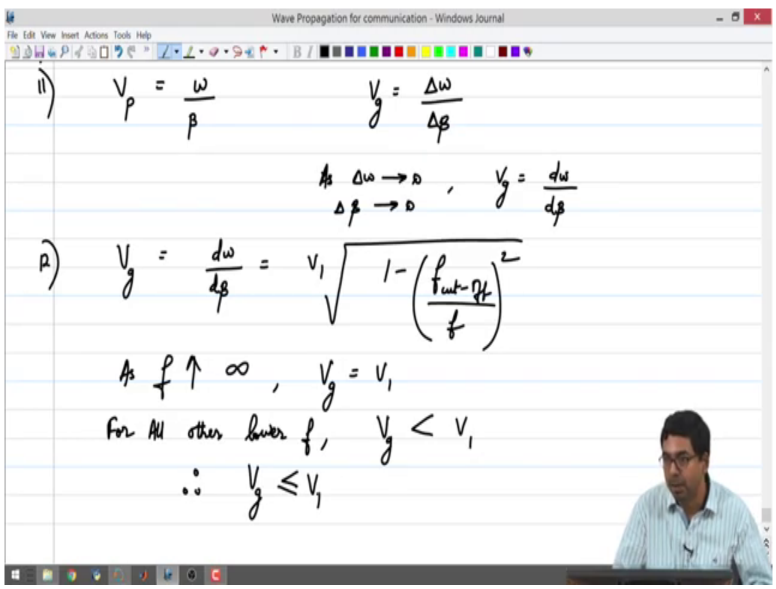

So, I can now write down the phase velocity of this entire combination right. Phase velocity is $\frac{\omega}{\beta}$ that is how fast your point of single phase is traveling through this space with respect to time. And then there is another term which is actually telling you how fast the envelope is moving all right.

So, I can just write down that is coming from the second sinusoid you can also call that as the phase velocity of that particular term all right, but in this case we are calling it group velocity because it is telling you how fast the velocity of the modulated signal is going from one side to the other side.

So, it is

$$v_g = \frac{d\omega}{d\beta}$$

and you could also choose very small difference in frequencies and wavelengths for example, your $\Delta\omega$ could be very small $\Delta\beta$ could be very small you could mix those two signals and you will end up sending information ok.

So, we can say that a you could just write this as

$$v_g = \frac{d\omega}{d\beta}$$

So, there is a difference in the way we have written. So,

$$v_p = \frac{\omega}{\beta}, v_g = \frac{d\omega}{d\beta}$$

You can always take the expression that we have written for β for the waveguide and do a dω/dβ all right or you could do one by dβ/dω it is up to you all right.

So, we have the expression for β in terms of ω you can differentiate the two sides and you will notice that a vg for the waveguide will look like

$$v_g = v_1\sqrt{1 - \left(\frac{f_{cut-off}}{f}\right)^2}$$

Let me just show what we had for the phase velocity for the phase velocity we had

$$v_g = \frac{v_1}{\sqrt{1 - \left(\frac{f_{cut-off}}{f}\right)^2}}$$

but if you did use dω/dβ you will end up getting vg and that is why v1 multiplied by the same term.

So, there is a difference between phase velocity and group velocity. And let us look at the obvious differences the obvious differences that we can look at is what we did for phase velocity as f increases let us say it goes all the way to infinity very very high values right, right as f goes to very high values values much higher than the cutoff frequency ok.

So, you will have this term becoming 0 your vg is equal to v1 all right. So, at infinite frequency you cannot distinguish between phase velocity and group velocity ok. The second thing that we notice is for all other f's lower than infinity all right all other finite values of f right we notice that vg is less than v1 ok.

So, square root 1 minus something all right if your f is finite you will end up getting v1 square root of something which is less than 1 right. So, you will end up getting vg less than v1 this is in since stark contrast with the phase velocity

So, here we are saying that the group velocity in general is going to be less than equal to the velocity in the homogeneous medium suppose you constructed with vacuum group velocity will be less than the velocity in vacuum for the same signal ok.

So, it is not uncommon to have phase velocity higher than velocity in the bulk medium all right that does not signify that you are transmitting information faster or anything it just mean that phase is changing fast, but the information is always traveling at speeds less lesser than equal to the speed of light in that medium right. And if you consider that medium to be vacuum information cannot be transmitted in vacuum at velocities higher than the velocity of the electromagnetic wave itself ok.

So, it is perfectly fine to have phase velocities higher than the velocity in the bulk medium that does not mean you have overcome speed barriers all right, that does not mean you are entering into some domain where you are traveling faster than the speed of light and all that no. Group velocity is the term that actually dictates the information content that you are trying to send and the information content has an upper bound for the velocity ok.

So, we are having vg less than or equal to v1. Now, with this idea let us expand or write another simple program to just to see what is happening with vg and v p. One of the things that I notice common between vg and vp is that at frequencies which is infinite both of them asymptotically come to the velocity in the bulk medium this case v1 all right.

The second thing that I noticed is vp is operating frequency dependent for a waveguide vg is also operating frequency dependent for a waveguide all right just that the way they are dependent is seems to be slightly different, but let us make a plot ok and see what happens with the a you know modes of these a what happens to the phase and the group velocity right. So, I am going to open octave again maybe I will just write a new program ok.

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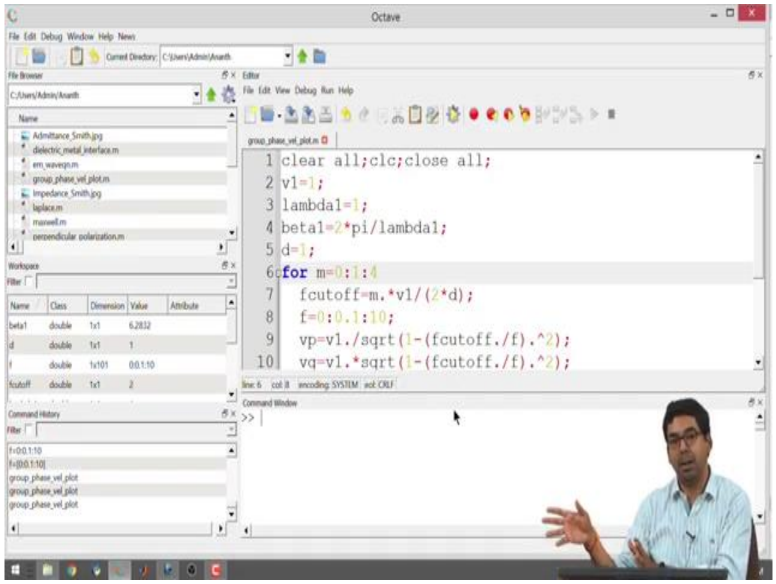

Once again I am going to take some variables that I will need to draw this all right. I have the formula which a which says that I need to be able to define v1 for both phase velocity and group velocity I also need the values of f cut off f that is all ok.

So, I will go ahead and say that I am going to assume v1 t to be equal to 1 vacuum all right. So, in some relative terms I am using some medium. So, speed is 1 over there right. I also would like to give more information right λ_1 equal to 1 you can also calculate β_1 in that medium to be $\frac{2\pi}{\lambda_1}$ right ok. And a I think the formula needs ok for finding out the

$$f_{cut-off} = \frac{mv_1}{2d}$$

So, v1, I have already defined I need to define d and I need to define m all right. So, I am going to take ops all right. So, I am going to take unknown quantities d to be equal to 1 all right d is equal to 1. And a let us say I want to draw this for multiple modes all right ok for modes going from 1 to 4 all right I want to calculate f cut off first all right.

fcutoff=m*v1/(2*d)

that is the formula we have and a for each value of m, I will be having m multiplied with v1 all right divided by 2 d ok.

You can also do without the dot over there they are all just simple numbers you can just make m*v1/(2*d) right. Phase velocity the formula that we have is ok let us also define another term right operating frequency has to be defined. So, the operating frequency I wanted to go from 0 all the way to 10 all right.

In this case ten is a high frequency that is it ok. So, I have v1 divided by square root of 1 minus f cut off divided by f. I am using the dot slash dot to the power because f is an array all right f is having its a vector its having 0 to 0.1 to 10. So, it has a I want to take each element and multiply to be double sure everything I am putting dot slash dot to the power extra all right. So, it will take element by element and calculate this ok.

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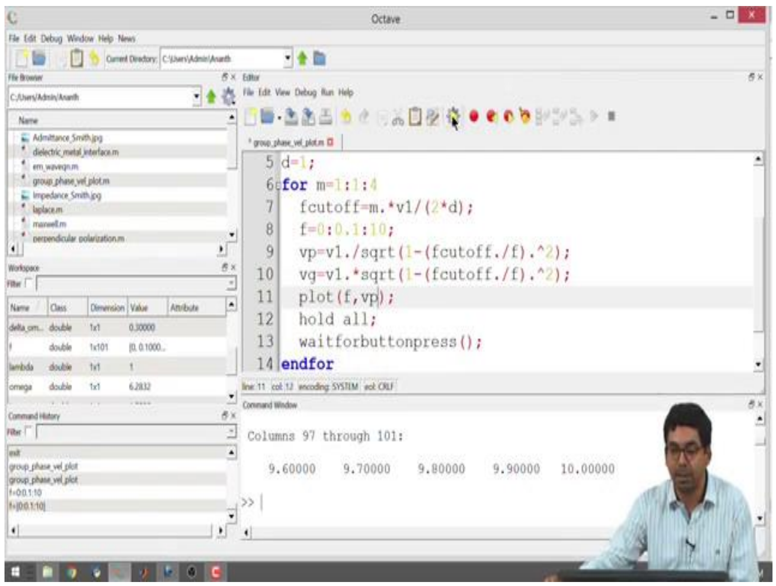

Student: (Refer Time: 41:23) equation there is an extra bracket.

Is there an extra blanket one two?

Student: (Refer Time: 41:31).

Ok Yeah, yeah I think its fine vg was I think it was not divided, but I think it was just multiplied yeah. So, I will just change the sign over there ok. And then I just need to make a couple of plots ok. So, I will just make. So, frequency comma the phase velocity all right ok because it is in a loop I want to plot for m equal to 1 through 4 all on the same plot I am just saying hold all the plot parameters as it is right.

So, X axis would be frequency Y axis will be the phase velocity in this case right. And a I think here as the number of modes changes its going to draw different plots I do not want to do this legend string because it is too much of coding.

So, I will just say after each plot I will just click the button all right then I know that it is going from m equal to 1 to 4. And then I will figure out which is corresponding to 1 and 4 because that is only one line of code and I will just write that as wait for button press right it will just wait for me to click my mouse button on the current active figure and that saves the trouble right ok. And to make it a little bit more interesting what I will do is I will also put f comma vg right the same plot.

So, I will have phase and group velocity maybe first I will do phase velocity to distinguish between the two right. And then I will come to the phase in the group velocity.

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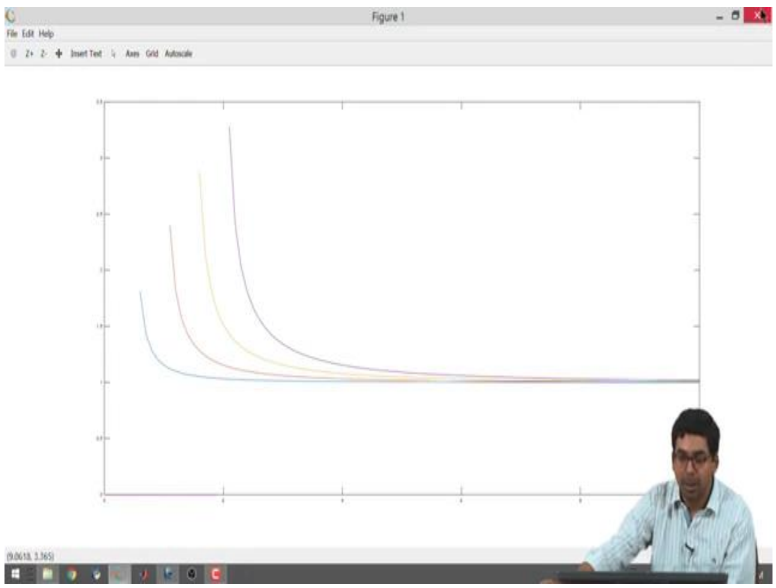

So, I will just run this right. So, I am having a plot for the phase velocity they should be corresponding to m equal to 1 all right.

So, it starts from here and then asymptotically it goes to 1. So, at very high frequencies the phase velocity has become equal to 1 which is my defined value of v1 at very low values of operating frequencies it seems to be going all the way to infinity it seems to be falling the trend I will make a click ok I have the next plot. So, this red plot has come in new all right it shifted to the right and it has some characteristics like this all right.

So, here also I notice that asymptotically it arrives at the velocity of the homogeneous medium and it start at infinity m equal to 3 m equal to 4. So, it looks like as I keep on increasing the modes all right the plot is going to start towards the right and it is always it is going to go towards asymptotically a you know arriving at that particular mediums where homogeneous velocity right.

I will just be going to make it right make it a little bit more interesting and I am just going to make it comma vg ok also now going to plot group velocities. Now, I know that the phase velocities were having this kind of a shape right they were going like this right whatever other shape comes should be due to the group velocity right ok.

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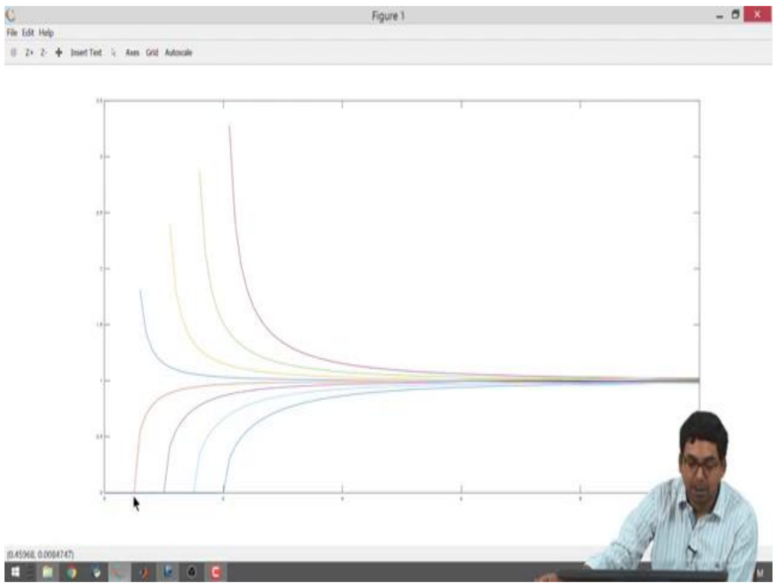

Now, I have something going on here I remember that this was phase velocity all right. And phase velocity was higher than 1 ok it was higher than 1 all right. In fact, it was going all the way to 2 it could go to infinity also right. And then it was asymptotically coming to 1 that was phase velocity the group velocity is lower than 1. In fact, it is 0 in the beginning and then it is going towards asymptotically to the value which we have design defined as v1 in our program right.

So, it is starting with low value of group velocity and going all the way and becoming v1. So, I will just make a click for the next higher mode all right and it keeps going. So, I know that this plot corresponds to the group velocities because its asymptotically approaching 1 from 0 ok. And the other plot is going to be for the phase velocity ok. So, this is how the phase and a group and the phase velocities would look like for your parallel plate waveguide all right.

So, you are having some curves that tell you that the velocity is changing with the operating frequency. So, you have kept the f cutoff fixed, that means, you have kept your, you know for a given mode the configuration of your parallel plate is fixed d is fixed medium filling in is fixed, but just with respect to frequency or having different group and different phase velocities. There is only one more detail that we have to add over here ok. Now, m we started with 1 going all the way to 4 all right we can always make m equal to 0 to 4 ok. And then redo this all right we are talking about m equal to 0.

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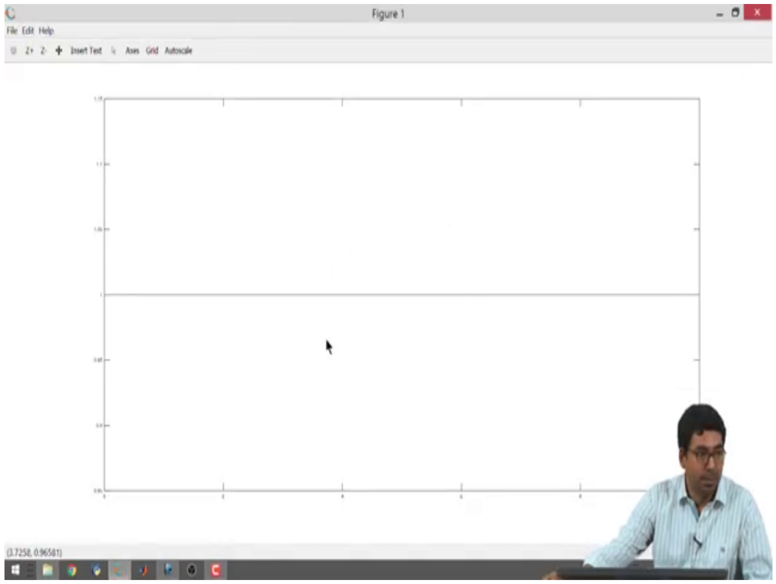

Now, I get a horizontal line straight which means the phase velocity and the group velocity have been plotted here they are one on top of the other. And they are forming a horizontal line at the value of the velocity equal to 1 for m equal to 0 in the case of a parallel plate waveguide. The phase velocity and the group velocity is equal to the velocity in that medium itself and the wave guide is not dispersive it is non dispersive all right.

This means that something is happening if you go back to the expressions for the fields that we had before you will notice that if you substitute m equal to 0 for the electric field expression that we had before in the parallel plate case you will notice that you will not have a component of electric field in the direction of travel it will become 0. Similarly, for the magnetic field you can use the curl equation find the magnetic field.

And if you try to plug in m equal to 0 there you will have no component of magnetic field along the longitudinal direction you will notice that electric field and magnetic field will be transversal they will also be perpendicular to each other and perpendicular to the direction of travel this means that m equal to 0 corresponds to a plain wave or TEM wave the case that we are already familiar with all right.

In this particular case what happens is you are launching a plane wave and you are keeping a wave guide and the wave guide has no effect on the plane wave because its traveling as it would in a homogeneous medium with a constant velocity all right. So, whether you had kept the waveguide or not the phase and the group velocities will be the same air eh all right. Secondly, if you keep the wave guide or if you removed the waveguide there is going to be no dispersion ok.

So, what does this mean this means that even though you can write down the fundamental mode of a parallel plate wave guide to be a m equal to 0 it corresponds to a case where there is no component of electric and magnetic field along the direction of travel it represents a pure plane wave. And the wave guide does nothing to it which means the intrinsic impedance of the waveguide is equal to the intruding intrinsic impedance of air does not reflect, does not scatter, does not do anything what does that mean, that means, that the wave guide even though you have kept two infinitely long sheets in vacuum and you are launching a plane wave the plane wave does not see it just sees the wave guide to be purely transparent ok.

So, a wave guide with m equal to 0 is just going to become transparent to a plane wave going in that medium for a parallel plate waveguide right. And if you have higher frequency I mean higher modes then only you are going to be having some dispersion coming into the picture ok.

So, these are some two small details that we had to see ok. Now, I think there are a few more things with respect to the rectangular wave guides that we have to cover the next class right. And following that we will be talking very briefly about wave guides with metals at the boundaries of the facets also what if you took a wave guide and you put a metallic facet at the input metallic facet at the output you will have a what is known as a cavity.

Suppose you did find a wave to excite it with the source from inside the wave has nowhere to go it should keep being built up in the cavity we will just see very briefly about right it very briefly not in great detail. And then we will try to have some experiment where we can estimate all right standing wave patterns frequencies extra we will do some experiments and then try to get an understanding for these standing wave patterns ok. So, I will stop here we will meet in the next class.