Transmission lines and electromagnetic waves

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Lecture – 03

Introduction to Finite Difference Method

So, we will begin with this lecture. So, in this lecture we are going to start at a tangent to what we had seen before, right.

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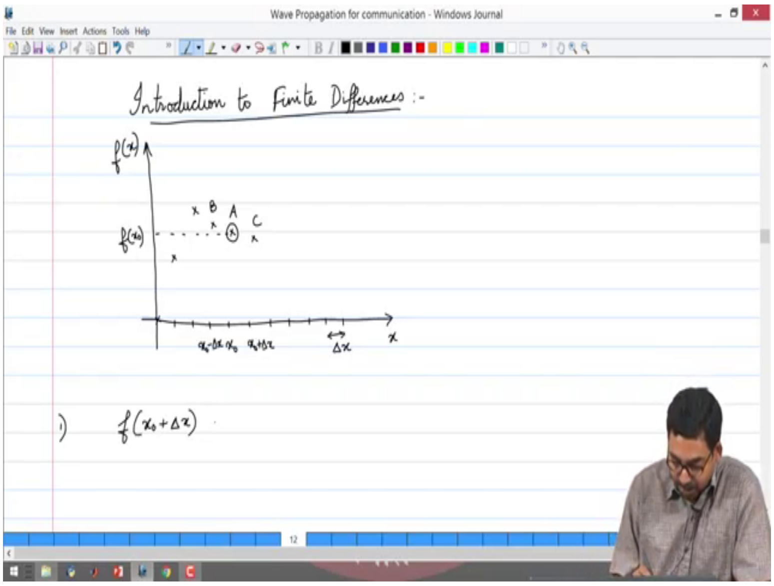

So, I am going to introduce to you how to use computers to solve partial differential equations and I am going to approach this with simple finite difference technique, ok. In the previous class, we had stopped with the wave equation and the solution to the wave equation in an analytical form, all right. And towards the end of the class I mentioned that we will try to solve these equations using a computer, ok. And we are going to do this throughout the course hand in hand. We will be solving the set of equations using the computer and we will also be trying to solve it analytically, all right to just go hand in hand, all right.

So, I will start with absolute basics. Let us say that I have an axis, right an xy axis and I am having uniformly sampled points in the x direction or the x axis, ok. The spacing between the two consecutive points in x axis is Δx, ok and in the y axis I have a function of x, ok. And at each of these x I am plotting the value of f(x), ok and let us say that I have this f(x) defined in the following manner let us say that it starts at 0 comma 0 here, ok. So, these are the points that define f(x) for different x. We do not know what the function is. These are the points that are given to us, ok.

You could imagine this to be in a spreadsheet. There are two columns one is uniformly distributed x, I mean not uniformly distributed, but uniformly sampled x. So, x is going from 1, 2, 3, 4 so on and so forth, and f(x) is given to be some values, all right. And the question that is asked to us is at a given point, ok, all right let us mark this point, all right. And we call this point as A. Can you find the slope of f(x)? Right. Can you find f'(x) in other terms? Right.

So, I will mark the x coordinate corresponding to this point A as x_0, ok and the y coordinate I will mark this to be as $f(x_0)$, ok. And the question that is asked is find the slope of this curve f(x) at point A, all right. So, it is $x_0, f(x_0)$. Now, since the x axis is distributed uniformly I can write down the x coordinate of the sample next to A, all right. So, to the right hand side of A I will be having the x coordinate to be $x_0 + \Delta x$ and at the point prior to A, on the left hand side the x coordinate will be $x_0 - \Delta x$. I am going to mark these points as B and C, all right.

So, when we ask question what is the slope at point A, there are many answers possible, but over the period of time I have found that the students invariably identified one particular method of finding the solution at point A. They always say that it is the slope of the line joining B and C, ok. So, they take the neighbouring point prior, neighbouring point after joining those two points, then they try to find the slope, but when asked for an interpretation on why they did that there was no analytical approach present, ok.

So, today we will just try to see what is the best way to find slope at point A and how this is useful in solving partial differential equations that we were seeing in the previous class, ok. So, let us begin now by the 3 scenarios which are possible, all right.

You could find the slope of line AB or you could find the slope of line AC or you could find the slope of line BC. These are the 3 options which are possible. Since, there are 3 options possible we have to be very clear in identifying the best possible method and the best possible method is the one that will give you the least error, all right in estimating the slope at point A. So, one of the ways to estimate the error and obtain the method that gives you the least error is by expanding the function as an infinite series and then trying to see where you truncate the series and the point of truncation gives you the order of the error, all right. And I will illustrate this in a systematic manner by using a Taylor series approximation, right.

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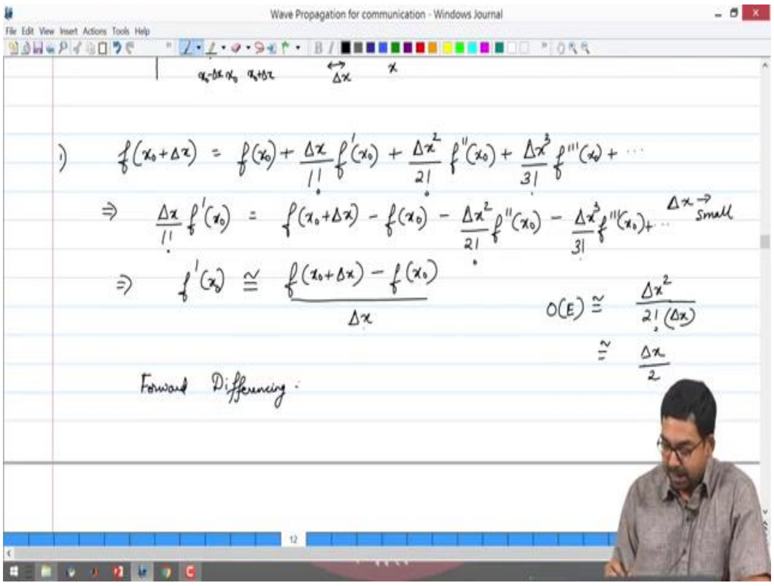

So, I will start with $f(x_0 + \Delta x)$ which means the value of the function at point C, ok. So, that x coordinate is $x_0 + \Delta x$ and the y coordinate will be $f(x_0 + \Delta x)$. This can be written down as a Taylor series expansion. So, you will have

$$f(x + x_0) = f(x_0) + \frac{\Delta x}{1!} f'(x_0) + \frac{\Delta x^2}{2!} f''(x_0) + \frac{\Delta x^3}{3!} f'''(x_0) + ..$$

One of the things that we need to pay attention to is here we are trying to estimate the value of the function at x_0 plus Δx coordinate provided you know the value of the function at x_0 and you know its first derivative at x_0, second derivative, third derivative, nth derivative. So, the series here says that if you know the value of the function and if you know all the derivatives present at that particular point you should be able to estimate what will be the value of the function at the next point that is what the Taylor series is telling us.

Now, this function can be a converging series or a diverging series. For a converging series Δx has to be less than 1, so we will first put down these points, all right. So, we are saying that Δx has to be small then it is a converging series and I would be able to estimate the value of $f(x_0 + \Delta x)$ very accurately, all right. It also means that if if Δx is very small as the number of terms grows on the right hand side, the value of the coefficient will keep on shrinking. So, the way you truncate the series, all right you can make a learnt way of doing this truncation. You can say where you have to truncate and what order of error you can expect. So, it is a really analytical way of truncating the series

But the problem that was asked to me was what is the value of $f'(x_0)$ That is the unknown that we are trying to find out, so, we will bring the unknown on the left hand side and all the other quantities to the right hand side, right. So,

$$\frac{\Delta x f'(x_0)}{1!} = f(x + x_0) - f(x_0) - \frac{\Delta x^2}{2!} f''(x_0) - \frac{\Delta x^3}{3!} f'''(x_0) + ..$$

Now, this equation says here that in order for me to find out the left hand side which is which is the $f'(x_0)$ which is what we are interested in finding. We need to know the value of the function at the neighboring point f $f(x_0 + \Delta x)$, we need to know the value of the function at x_0 and we need to know more details. We need to know the second derivative, third derivative, fourth derivative extra.

And it is a real problem because we do not even know the first derivative yet, but we are expected to have all the other derivatives to estimate the first derivative which means we already know where we have to truncate this series, right. We do not know what the second derivative or third derivative is as yet, so the best way to proceed would be to truncate the series at the place where we do not know the second order derivative and say that that is going to be many orders of error, all right.

So, we can say that

$$f'(x_0) = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

And the order of the error is going to be the value of the coefficient of the first term that you are truncating that is

$$OE \approx \frac{\Delta x^2}{2!\,\Delta x}$$

$$\approx \frac{\Delta x}{2}$$

So, if you were to estimate the slope at point x_0,

$$f'(x_0) = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

And we already know that this corresponds to the slope of the line AC, in the diagram that we have drawn. So, we are first estimating the slope at point A to be the slope of the line AC, ok. Now, we know from conventional mathematics that as Δx tends to 0. This is the definition of the derivative, ok. So, as Δx tends to 0 this is the definition of derivative, so you will obtain the derivative of the function at point x_0, ok.

Now, this method of trying to find out the slope at point A given the value of the function at the next point or ahead of it is known as forward differencing. So, you are replacing the derivative with the difference, all right and to find the difference you are using the value of the function ahead of it, all right. So, this is known as forward differencing, ok. And it has an error of $\Delta x/2$. Let us have a look at the other option.

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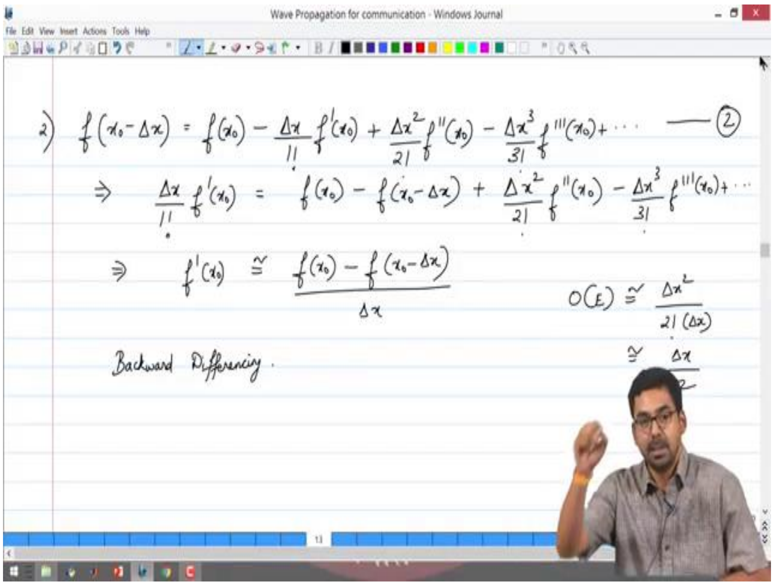

Let us write down the Taylor series for $f(x_0 - \Delta x)$. This is going to be

$$f(x - x_0) = f(x_0) - \frac{\Delta x}{1!} f'(x_0) + \frac{\Delta x^2}{2!} f''(x_0) - \frac{\Delta x^3}{3!} f'''(x_0) + ..$$

Again the assumption is Δx is very small, it is a converging series. And we are supposed to be estimating f'(x_0) which is the unknown quantity. So, the unknown quantity is moved to the left hand side. So,

$$\frac{\Delta x f'(x_0)}{1!} = f(x + x_0) - f(x_0) - \frac{\Delta x^2}{2!} f''(x_0) - \frac{\Delta x^3}{3!} f'''(x_0) + ..$$

Again, it says that in order for me to estimate the first order derivative I need to know the value of the function at the current point, I need to know the current value of the function at the point prior to it and then I need to know all the derivatives other than the first derivative to estimate the first derivative. We clearly have nowhere to truncate our series now. We do not know the second derivative third derivative extra. So, we have to truncate the series at this place with Δx square by 2 factorial.

So, we will write down instead of an equal to symbol we will put approximately equal to because we are truncating this series we can say that

$$f'(x_0) = \frac{f(x_0) - f(x_0 - \Delta x)}{\Delta x}$$

And the order of the error is going to be the coefficient of the f’’ divided by Δx from the left hand side, all right. So, we will be having approximately

$$OE \approx \frac{\Delta x^2}{2!\,\Delta x}$$

This technique where you estimate the slope at point x_0 by using the value of the function at the current point and the point prior to it is known as backward differencing, ok. If we closely look at the results for the order of error for forward and backward difference, we find that there is no real difference over here the order of the error is

$$OE \approx \frac{\Delta x}{2}$$

for both the cases. So, both of them are equally good choices for you to find the slope at point A, ok.

Now, I will mark the first Taylor series expansion as equation number 1 and I will mark the second Taylor series expansion over here as equation number 2, all right. So, both 1 and 2 are trying to find the slopes of lines a with point A and the neighbouring point either forward or backward, ok. So, in this

$$f'(x_0) = \frac{f(x_0) - f(x_0 - \Delta x)}{\Delta x}$$

we found the slope of the line A B, all right, in our diagram and we are saying that is the value of the slope at point A.

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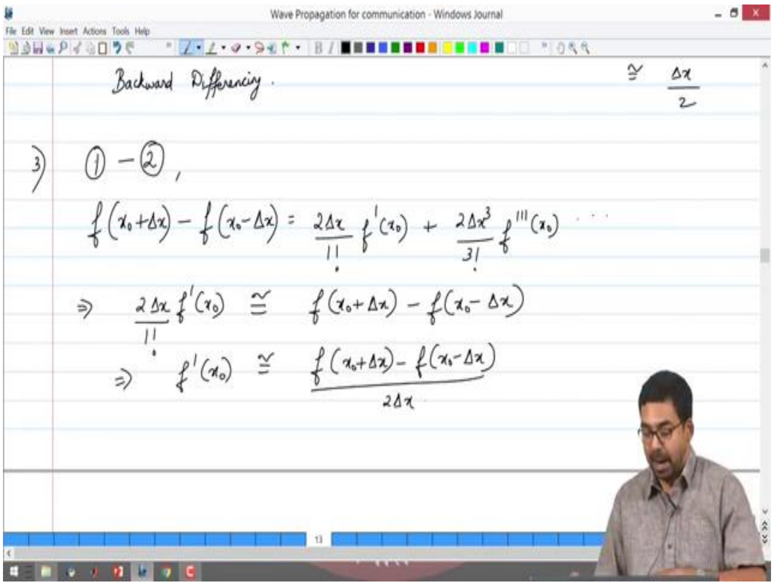

Now, we can go ahead and say that we will take question 1, and we will subtract equation two from it, ok and try to see what happens, all right. So, on the left hand side I will have

$$f(x_0 + \Delta x) - f(x_0 - \Delta x) = 2\frac{\Delta x}{1!} f'(x_0) + \frac{2\Delta x^3}{3!} f'''(x_0)$$

.

Now, once again the unknown term that we are interested in looking for is the first order derivative or the slope of the line, so that can be brought to the left hand side. So,

$$2\frac{\Delta x}{1!} f'(x_0) = f(x_0 + \Delta x) - f(x_0 - \Delta x)$$

one can say that this is

$$f'(x_0) \approx \frac{f(x_0 + \Delta x) - f(x_0 - \Delta x)}{2\Delta x}$$

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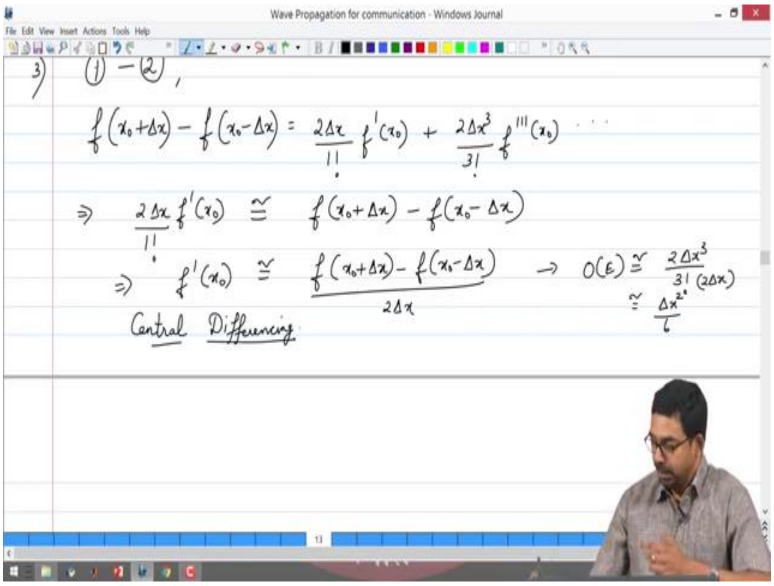

And we write down the order of the error is going to be the coefficient of the first term that you are truncating that is going to be

$$OE \approx \frac{2\Delta x^2}{3!\,(2\Delta x)}$$

$$\approx \frac{\Delta x^2}{6}$$

So, in this particular case the way you are finding the slope at point x_0 if you are taking the point prior to it you are taking the point after it and you are joining that and then you are trying to find the slope of that line, all right. So, in this case, obviously, the error turns out to be lesser than what you would get in a forward or a backward difference most of the times the students get it perfectly correct, but they do not have a systematic way of explaining it that is it, all right. Majority of the time I found that the students give the exact solution that is needed to find the slope, but it is just that I wanted to put this approach forward because I can build on it for different things related to this particular course.

Now, the method that we have used now using the point ahead of it and using the point prior to it is known as central differencing, and it is also clear that given an option where you have to choose between forward backward and central differencing it is wiser to choose central differencing because the error is going to be lesser for you to estimate the first order slope, ok. That is the take away message for the first order derivative.

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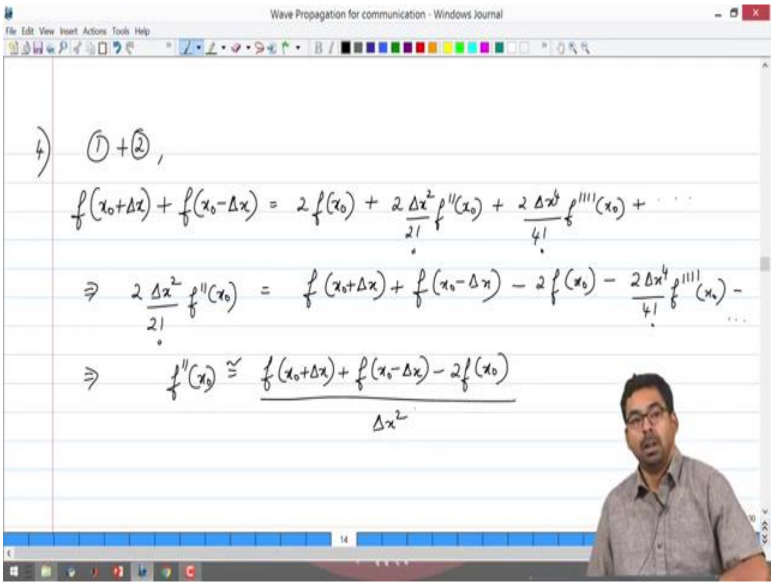

Now, let us go one step further. Let us take equation 1, and add it with equation 2, ok. So, on the left hand side you will have

$$f(x_0 + \Delta x) + f(x_0 - \Delta x) = 3f(x_0) + 2\frac{\Delta x^2}{2!}f''(x_0) + \frac{2\Delta x^4}{4!}f''''(x_0) + ..$$

The first order derivative terms will cancel due to alternating signs and then you will end up having the second order derivative coming into the picture.

So, I have just taken the Taylor series expansions 1 and 2, and I have added them together.

Now, this is a very special equation according to me. The right side does not have the first derivative, the right side does not have the third derivative. But the least order derivative that is present on the right side is second order. So, anyway I do not know the second order derivative of the given function. So, I would like to find that out. So, that is the unknown quantity. I would like to move that quantity to the left hand side, all the other quantities to the right hand side, all right.

So, I can say that

$$2\frac{\Delta x^2}{2!}f''(x_0) = f(x_0 + \Delta x) - f(x_0 - \Delta x) - 2f(x_0) - \frac{2\Delta x^4}{4!}f''''(x_0) \pm$$

Now, obviously, I am trying to estimate the second order derivative, all right and I do not know what the 4th order derivative is. So, it is clear where I have to truncate this particular series, I have to truncate it at the 4th order derivative. So, I am going to rewrite in such a way that I have $f''(x_0)$ on the left hand side and a truncated series on the right hand side, right. So, I will just write this down

$$f''(x_0) \approx \frac{f(x_0 + \Delta x) - f(x_0 - \Delta x) - 2f(x_0)}{\Delta x^2}$$

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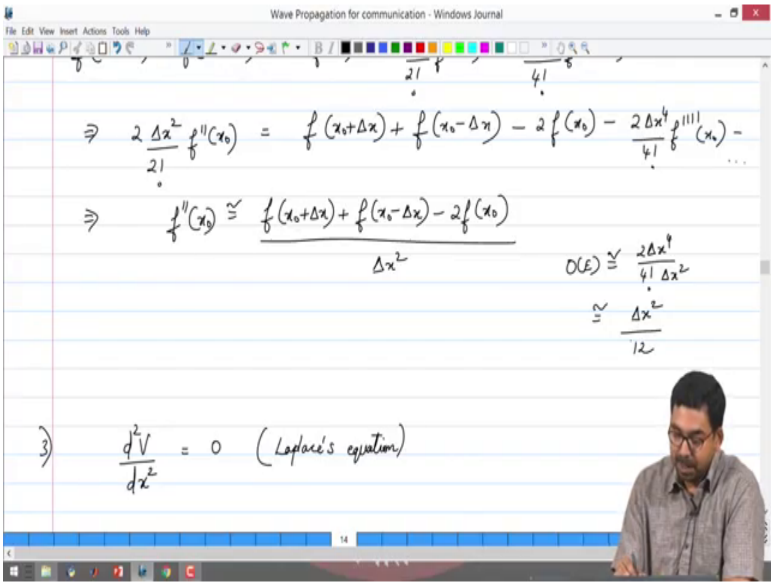

And the order of the error that I am going to be having is approximately

$$OE \approx \frac{2\Delta x^4}{4!\,(\Delta x^2)}$$

$$\approx \frac{\Delta x^2}{12}$$

It is a very peculiar result, ok and I will highlight to you why according to this equation you can estimate the second order derivative of a function at a point without finding the first order derivative.

The right hand side is telling you that you need to know the value of the f $f(x_0 + \Delta x)$, you need to know the value of the function at x_0 - Δx, you need to know the value of the function at x_0. If you know these 3 and if you know the spacing between the consecutive points in the x axis you can estimate the second order derivative directly.

It is not the way in which we do conventional mathematics for finding derivatives. Usually for finding the second order derivative we will find the first order derivative and then do a differentiation again to find the second order derivative, but this equation here is telling you that second order derivative can be found out directly from the value of the function, but it does not stop there. It also tells you that the error involved in finding a second order derivative is actually much lower than trying to find the first order derivative which is also something that you would not normally expect, ok. So, it is a very important result that tells you that second order derivatives can be obtained easily if you know the values of the functions at different points, ok.

So, having known this how do we connect this to the equations that we have got in the prior class? We must remember that at the end of the last class we had wave equations for voltage and wave equations for current. The wave equation was a partial differential equation. It had spatial derivatives on one side, time derivatives on the other side, they were partial differential equations of the second order, all right.

Now, we know how to write down second order differential equations in terms of differences and that is where we are going, but directly starting with the wave equation will be a little tedious. So, we will start with a very simple case and slowly build up towards the wave equation, ok.

Now, in the previous class when we had the wave equation, right we must have had some special derivatives on the left hand side and time derivatives on the right hand side let us start with a very simple case where the times derivative is assumed to be 0 that is the voltage is constant with respect to time, ok. If you do that, ok you do not have a dependence on time anymore, all right. And you will end up with

$$\frac{dV^2}{dx^2} = 0$$

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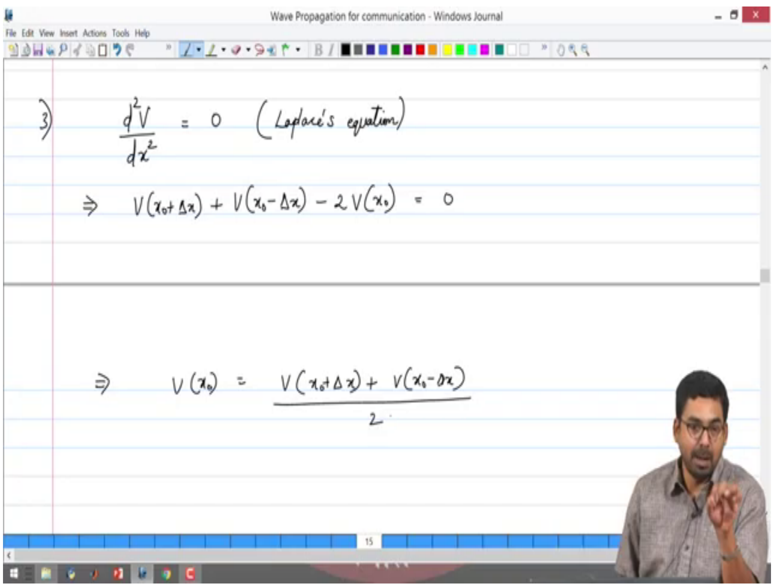

So, I will go back to the wave equation that we had before, right.

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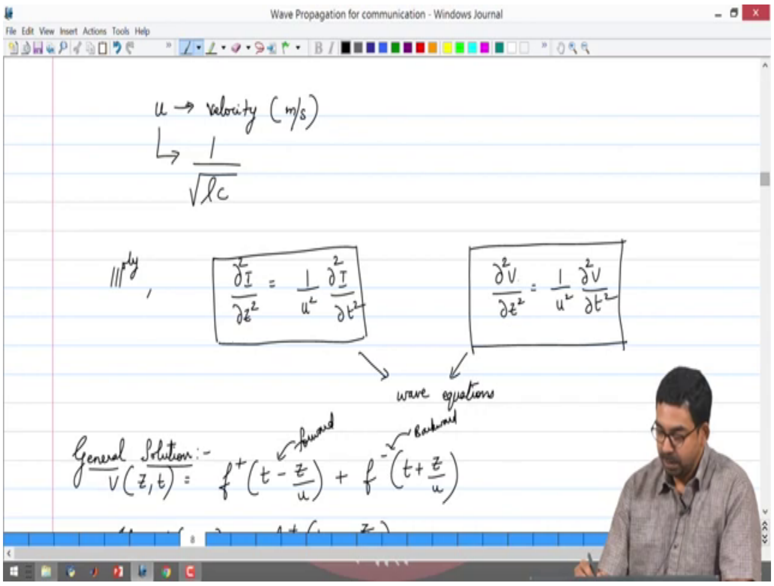

So, I had

$$\frac{\partial^2 V}{\partial z^2} = \frac{1}{u^2}\frac{\partial^2 V}{\partial t^2}$$

I am assuming that the voltage is not changing with respect to time. So, you can replace this quantity with 0, since a voltage is not varying with respect to time it is not an independent variable anymore. So, I can replace the partial derivative with an ordinary derivative. So, I am writing d square V by d z square is equal to 0, since I have used x for a independent variable in this class I am just writing that down as

$$\frac{dV^2}{dx^2} = 0$$

Now, this equation is known as the Laplace equation, all right. And it can be derived from Gauss's law, but in this class we do not want to go into general solutions extra. We want to see how purely using a computer without any knowledge about general solutions we would be able to solve a given differential equation, right

Now, looking at this $\frac{dV^2}{dx^2} = 0$, I know how to write the second order derivative of voltage with respect to x because I have a Taylor series expansion that tells me precisely how to do it. So, I refer to the form that I have over here, all right.

$$f''(x_0) \approx \frac{f(x_0 + \Delta x) - f(x_0 - \Delta x) - 2f(x_0)}{\Delta x^2}$$

So, I should be able to write down the voltage a I mean this equation in a difference form, all right. So, I am going to say that this equation actually means that

$$V(x_0 + \Delta x) - V(x_0 - \Delta x) - 2V(x_0) = 0$$

When an equation like this is placed in front of a student, the difficulty that they have is identifying an unknown and identifying the known quantity which is the same problem that students usually have. Even if a differential equation is given to them the first step is actually identifying what is unknown then only you can solve for it all right.

Now, I have an equation over here that tells me

$$V(x_0 + \Delta x) - V(x_0 - \Delta x) - 2V(x_0) = 0$$

The natural question is what is unknown, all right. Now, let us formulate this problem in such a way that the unknown is very clear, all right. So, for this purposes we will say that x_0 is the point where I am currently, all right and I would like to determine the voltage at that particular point x_0 + Δx is the point ahead of me where the voltage is known and x_0 - Δx is the point prior and the voltage is already known and the unknown is the value of the voltage at point x_0, ok.

So, if this is the case

$$V(x_0) \approx \frac{V(x_0 + \Delta x) + V(x_0 - \Delta x)}{2}$$

So, in other words it says it says something that normally a person would do. Suppose, you have to find the value of an unknown voltage at a point and their given value of the voltages at the neighboring point the first tendency that we will be having is to find out the average of the point before and point after and saying that should be the voltage and that is exactly what Laplace equation is telling you. If you have an unknown voltage at a point all you need to do is find out the average of the neighbouring point voltage that should be the voltage at the point that you are trying to find the unknown, all right.

Now, let us increase the complexity a little bit. Now, we are solving this in one special dimension. Let us go for two special dimensions, ok. So, $\nabla^2 V = 0$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

but now voltage is the function of both x and y, ok. So, I have point number 4.

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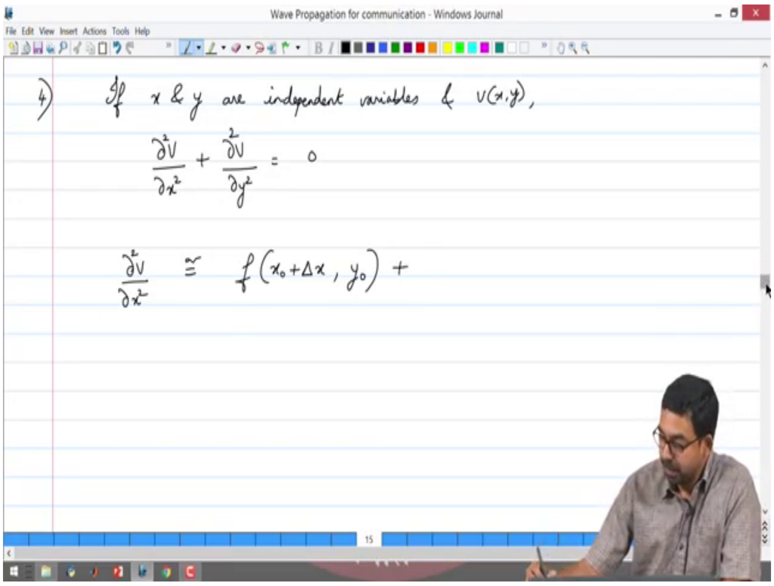

So, if x and y are independent variables and voltage is a function of x and y, ok. Then Laplace equation will tell you that

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

Thus for in our Taylor series we have not seen how to find out partial derivatives, we have assumed only one independent variable x_0 I mean a x and we have tried to find out what the value of the derivative will be. We have not tried to find out what the derivative will be partial. So, we have to rewrite. Fortunately it is not that difficult at all, ok.

So, we will go back to our original equation, all right.

$f''(x_0)$ is given by the expression on the right hand side. So, I am going to just borrow this expression and tell you how to do the partial differential a I mean partial derivatives, right. So, $\frac{\partial^2 V}{\partial x^2}$, right can be approximately written as

$$\frac{\partial^2 V}{\partial x^2} \approx f(x_0 + \Delta x, y_0)$$

So, here what happens is you will keep the y coordinate fixed when you are trying to find out the derivative with respect to the other coordinate that is it. So, I will be having plus, a plus or minus whatever, it is, it is plus, right.

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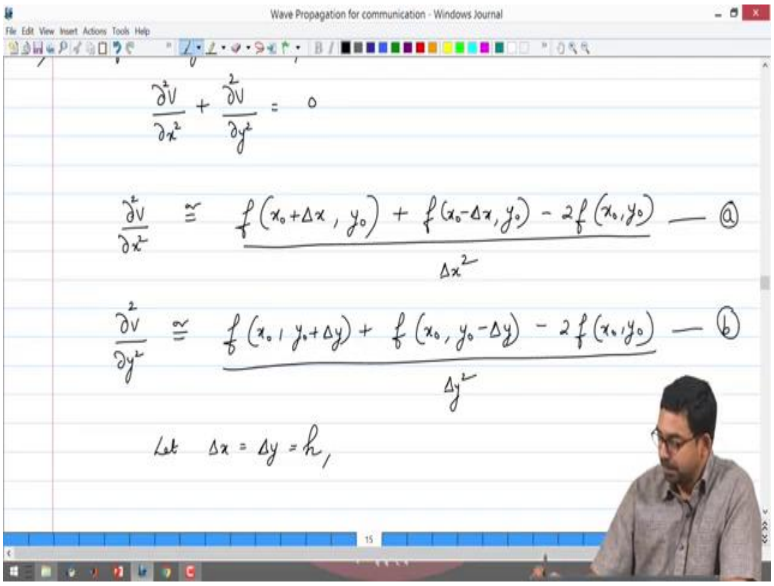

$$\frac{\partial^2 V}{\partial x^2} = \frac{f(x_0 + \Delta x, y_0) - f(x_0 - \Delta x, y_0) - 2f(x_0, y_0)}{\Delta x^2}$$

This is the way to find the partial derivative and it is very simple. All you need to do is keep the coordinate all the other coordinates fixed and try to find out only the derivative with respect to one independent variable at a time.

Similarly, you can write down

$$\frac{\partial^2 V}{\partial y^2} = \frac{f(x_0, y_0 + \Delta y) - f(x_0, y_0 - \Delta y) - 2f(x_0, y_0)}{\Delta y^2}$$

Now, we can make a few assumptions to make our calculations a little bit easier. Laplace equation says that

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

Now, I look at $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}$, and I have denominators. I would like to make these denominators equal so that I can push it to the right hand side and get rid of the denominator completely, all right.

So, it means that if I am going to have a uniformly sampled points in x I am going to use the same value of Δx to give my Δy also. So, it means that I am dividing a region into small squares with side of Δx, ok. So, I can say that let Δx = Δy, let us say it is equal to some h, ok. Then I can take these two equations, so I will mark this as a, mark this as b.

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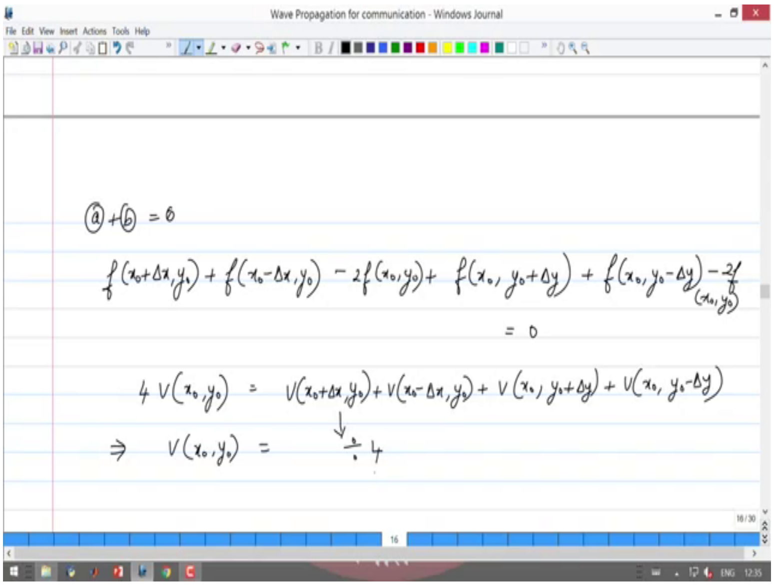

a plus b equal to 0 is Laplace equation, so I can write this down as, ok

$$f(x_0+\Delta x, y_0) + f(x_0-\Delta x, y_0) - 2f(x_0, y_0) + f(x_0, y_0+\Delta y) + f(x_0, y_0-\Delta y) - 2f(x_0, y_0) = 0$$

Again, now we have an equation. The first thing you have to determine from the equation is what is the unknown. Bring the unknown to one side and all the other quantities to the other side, that is when you get clarity. In these problems most of the times the point that you are currently looking at, so its x_0 comma y_0 is the unknown quantity given that you are given some finite values of voltages at the other points, all right. So, we were solving for voltage. So, I will just replace f with V in the next step so that I am consistent with what I had done before, all right.

So,

$$4V(x_0, y_0) = V(x_0+\Delta x, y_0) + V(x_0-\Delta x, y_0) + V(x_0, y_0+\Delta y) + V(x_0, y_0-\Delta y)$$

$$V(x_0, y_0) = \frac{V(x_0+\Delta x, y_0) + V(x_0-\Delta x, y_0) + V(x_0, y_0+\Delta y) + V(x_0, y_0-\Delta y)}{4}$$

So, the Laplace equation actually tells you that at a given point x_0 y_0, you have to take the average of some neighbouring points. Graphically, we can draw this to understand a little bit better.

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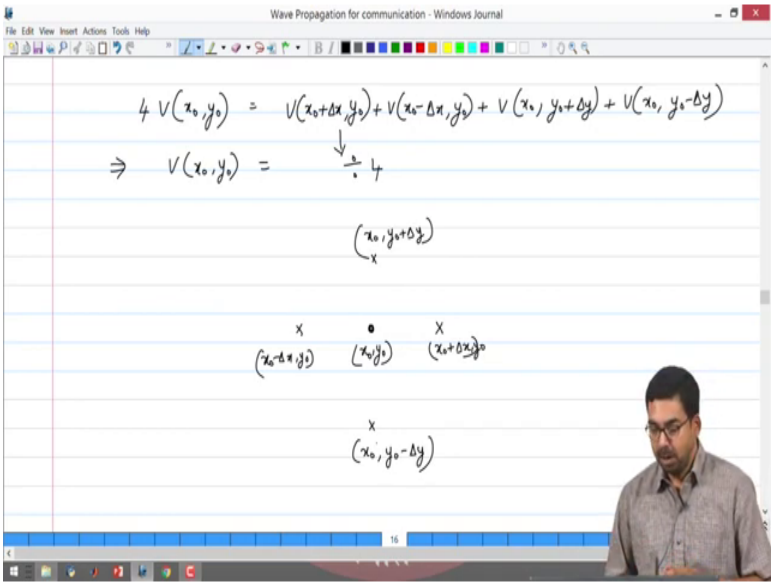

If I have x_0,y_0 to be present at some location and the value of the voltage is some V, ok x_0 + Δx,y_0 point is on the right side of it, ok. x_0 - Δx , y_0 is the point immediately to the left of it x_0 , y_0 + Δy is above it and x_0 , y_0 - Δy is below it.

So, I have to find the values of voltage at all these places and add them together divided by 4, which again says that the Laplace equation is telling you that you have to find the average value of the nearest neighbouring points that is it. So, the mathematical form of the Laplace equation becomes very clear once you start writing it down in the different form, ok.

So, given that this is the scenario with Laplace equation, all right we will start with some basic algorithm development to solve Laplace equation and then we will try to determine the first a, mean we will try to determine the voltage inside of a capacitor first we have to understand how the algorithm is going to work, we have to figure out what is the meaning of a boundary condition in these kinds of systems and then we will go towards wave equation we will try to understand the effects of time that is how we are going to be proceeding, all right

Now, what could be algorithm b? All right. So, let us start with the problem statement and then develop an algorithm to find out what is going on.

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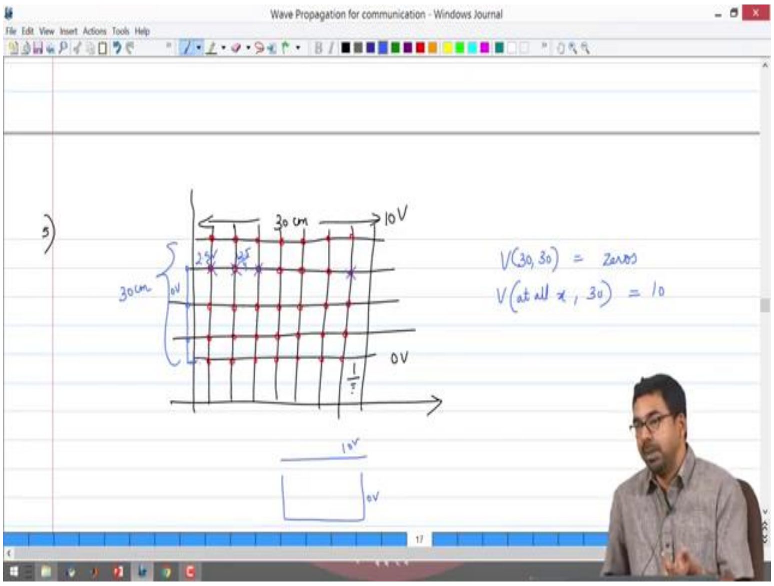

Let us say that I want to estimate the value of voltages inside of a capacitor, ok. I want to estimate the values of the voltage inside of a capacitor, the dimensions of the capacitor are given to you and some voltages applied to the plates are given to you, ok.

So, let us say that I have a plate on the top which is at say 10 volts, ok a plate at the bottom that is grounded, I can mark this as 0 volts. I want to be able to estimate the value of the voltage in between that and that is given by the Laplace equation, all right. I want to use a computer to solve it . Give me a diagram for a voltage distribution between these two plates, ok.

Consequently, I can also use this value of the voltage distribution to give me the value of the fields and how the electric fields are going to go from one plate to another plate, ok. So, this is what is given to us, all right. So, the first step would be to identify the sizes. So, the question should also give you let us say that this is 30 centimeters wide you know like this, ok.

Let us say this is 30 centimeters the material filling that is air, all right not that it really matters here, but its air, all right. I have a 30 centimeter long line. So, this is a cross section of a parallel plate capacitor that you are seeing, 30 centimeter long line, 30 centimeter long line. So, the first step that I have to do is discretize this, all right

So, discretizing this means that I have to draw some axis and I have to divide this region into small squares, correct. I need to figure out what my Δx is going to be Δy is going to be and we know that we want have Δx equal to Δy and I want to have a convergent Taylor series which means that Δx should be less than equal to 1, ok.

I can start with the simplest case where Δx is equal to Δy is equal to 1 and I can work my way through. So, this means that I will be drawing some axis like this and I will be dividing this region in x and in y, ok. And what I want to do is at the intersection of these gridlines at each of these points, all right, I want to be able to find out the value of the voltage, ok. I want a bottom plate that is 0, ok, let us say the top plate is all 30. I mean 10 volts, ok top plate is 10 volts bottom plate is 0 volts I want to be able to find out in between them, ok. Now, how do I go about this problem?

Now, when you are writing a computer algorithm you would have figured out the quantity that you are trying to calculate is the voltage, it is going to be a function of x comma y and in terms of representation of this data on a computer you will simply use a 2D matrix. Voltage will be a 2D matrix, it will consist of saying we have to give this distance let us say this is also 30 centimeters.

So, you will have 30 points on the x axis 30 points on the y axis, so you will have a voltage in the form of a matrix which is 30 comma 30, all right. At each of the x comma y you will be determining what is the value of the voltage, all right, but we do not know the voltages inside all we know is the top plate is 10 volts and the bottom is 0 volts.

So, the tendency is when you create a matrix the first thing you do is initialize the matrix to some values and the most common way of initializing for an engineer is putting everything to 0, all right. So, we create a matrix V of 30 comma 30 is equal to 0s, all the values in that matrix are 0s. Once we have given that everything is 0 we can go ahead and start with the top line, the top line is going to be having 10 volts. So, we have to figure out some y coordinate for it. So, I am going to say that y at all x comma 30 is going to be equal to 10, right.

This is how I would say that the top line is going to be having a voltage of 10 volts, ok. And the remaining points I do not know. The bottom most line is going to remain at 0 volts. So, I would like to determine what is going on. So, I start with this point over here the value of the voltage is unknown. I want to use the Laplace equation. So, I say that the unknown voltage at the crossed place is going to be the average of the nearest points. So, I take the point above that is going to be 10 volts, the point on the right hand side I have initialized it to 0 volts, so it is going to be 10 plus 0, the point at the bottom is also 0 volts unfortunately I do not have a point on the left hand side, ok.

I have only 3 points, so that means I have to figure out what I am going to do with this point on the left hand side, all right. So, here we need to pay close attention to what we are going to do. We will start with the elementary way of doing. If we do not know something we will make it 0. That is the easiest way to proceed, if we do not know something we will make it 0.

Obviously, I do not have a point to the left, so I am going to make it 0. But what does that mean? That means, that I am creating some points on the left hand side which are having 0 volts, all right, that means, the bottom plate of the capacitor was at 0 volts and the side wall also becomes 0 volts.

So, the capacitor actually looks very weird now we started with a parallel plate capacitor, but the diagram for the capacitor now actually looks like this: this is at 10 volts, this is at 0 volts, the same thing would happen to the right side, ok. But before that let us start with the point again the crossed point is going to be 10 plus 0 plus 0 plus 0 divided by 4.

So, we will mark it as 2 and half volts, ok. So, I will mark it as 2 and half volts. I go to the next point and I am going to cross it. This is the point I am looking at. I have 10 plus 0 plus 0 plus

2.5 divided by 4, so 12 and half divided by 4 is going to be the value I am storing here. I go to the next point. I do the same thing 10 plus 12 and half divided by 4 plus 0 plus 0 divided by 4. So, I find out. I go to the right side once I go to the right extreme. I do not have a point on the right hand side for me to use the Laplace equation. So, I will use 0, ok.

It has its consequences. You originally wanted to estimate the fields of a or the voltage distribution of a parallel plate capacitor, but we are changing the capacitor configuration itself. What does that mean, and how do we go about it, and what is the meaning of boundary conditions is what we are going to be expanding, ok.

So, I will stop here, all right. In the next class we will go over the algorithm for doing this and the correct algorithm for finding out the voltage distribution in the parallel plate capacitor. We will quickly write a program to do this to gain some familiarity on how to take partial differential equations and quickly write a program using Taylor series approximation, identify an unknown quantity, use the known quantities to find out the values of the solution, ok.

This will give us confidence for identifying the unknown for fixing the correct boundary conditions and then we will go back to the wave equation. We will introduce the concept of time, see how it works and then we will start gaining visual a you know representation of what is going on with the telegrapher's equation.

So, I will stop here. We will meet in the next class.