Transmission lines and electromagnetic waves
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Lecture – 29
Rectangular Waveguide

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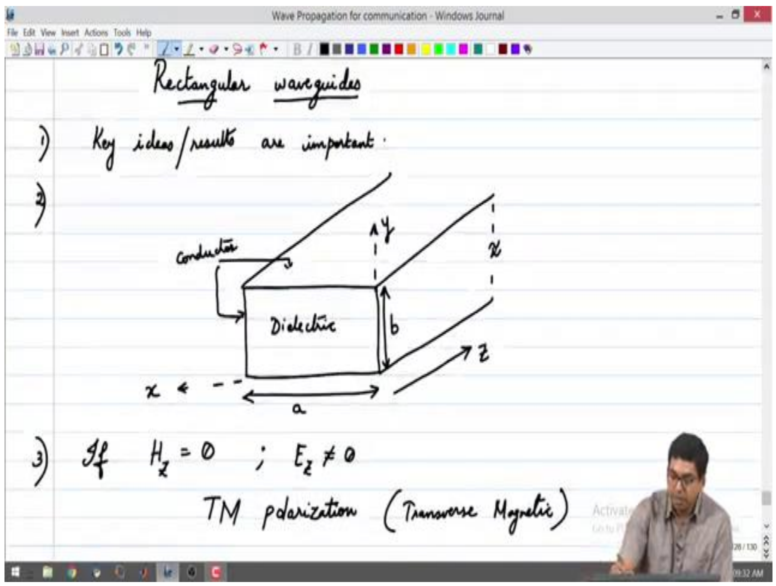

I think we will get started right. So, I think today we are going to see about a Rectangular Waveguides ok. So, on the prior classes we have seen about parallel plate waveguide and we have seen that there exists a concept called cut-off frequency and that makes the waveguide behave like a high pass filter. This will pass some frequencies above the cut-off frequency and below the cut-off frequency you will have an exponential decay right that is what we have seen.

So, now we are going to extend this concept to two spatial dimensions right rectangular waveguide would just means that just mean that you will have a metallic wall on all four sides forming a rectangle in the most generic case all right. And you will launch the electromagnetic wave in one side of this tube all right, which is formed with four metal a plates and then on the other side you are able to receive the information all right.

However, majority of the concepts are very similar, but the manner in which we are going to arrive at the mode results all right are going to be a little different. Previously we had just used an extension to a metal dielectric interface and we had got the results for a panel plate waveguide right.

Now, what we are going to do is we are going to look at another approach all right another approach is looking at the wave equation all right. And using a lot of background information to come up with general solutions which satisfy some requirement that we want that is information should go for travel from one side to the other side boundary conditions should be satisfied on all the metallic walls extra.

You force all these physical constraints and try to see if there are some generic patterns that the electric field can have within these waveguides. The reason for doing this is it is a more common method for finding out modes of a waveguide one. Secondly, if you were to use a program to find out the modes of a waveguide configuration, then this is probably the route that you would take all right. Typically, people would try to find out eigenvalues and eigenvectors for the wave equation for different boundary conditions right.

We will also see that using a simple tool ok. For now, we will do the analytical derivation the following classes we will also do a simple program ok which will calculate the eigenvalues and eigenvectors all right. Eigenvalues we will be telling you about the affective index all right and eigenvectors will be telling you about the field patterns all right.

We will be seeing how to do that using a computer also. So, this forms the base for that right. So, one of the things is I would like to stress here before we get into the mathematics part all right. I think it is ok if you just know the key ideas key ideas were results in the form of a graph or a picture I think if you remember that that is that is that is really good.

But the method in which we are going to do this is procedurally draining that is it is going to be you know it is going to assume that you know a lot about general solutions, you know a lot about differential equations how to solve them method of separation of variables some of those things you may have already learned, but you may not have touch for a very long time all right. So, it will become difficult to pick all the old points and put them together and that is fine we will go over it in a systematic way.

But if you remember the key ideas, the key ideas would be what is the mode all right what does a pattern for the electric field look like for different fundamental and higher order modes all right. What is the concept of a phase velocity, what does it mean to have a group velocity, what does it mean to have cut-off frequency these are some basic things and if you are able to remember that and if you are able to work around that then that is that is perfect right?

So, definitely the derivation is a really tricky there are many prerequisites there are too many assumptions also ok. So, we will start with this configuration first right ok. So, we are taking a configuration like this right and all the four sides are made up of an ideal conductor right and inside it is hollow. So, it is like a rectangular tube right and it is hollow as air or vacuum inside right this is the scenario we are considering first we will mark some axis which are very common in the waveguide community right.

Usually the direction of travel is marked as z right z axis is the direction of travel, that means, you launch from this side and you will expect the wave to come out on the other end of the z axis right. The transversal directions are usually marked to the x and y. So, we have y should be our x axis right ok. And let us say that in the most generic case this side is measuring a units and this is measuring b units it is the most generic case. One can always make equal to b during construction of these waveguides and you can analyze the consequences in the same way it is not a big deal.

So, here you are having dielectric. So, in this case we can simply start with vacuum right. So, just to indicate one more thing I will just mark this to be infinitely long ok. So, this tube is very long ok. All four sides are made up of conductor's ideal conductors ok. So, the goal is for us to find out the configuration of the electric field inside of it all right analytically.

And for this we will have to start with a few assumptions right. The first thing is we have to figure out what configuration do we have. So, in such cases this wave guides the direction of travel this case the z direction is known as the longitudinal direction ok. So, the direction of travel is known as the longitudinal direction right. You can remember that side is going to be really long you can call it as a longitudinal direction.

And the phase that we have drawn with four metallic conductors with the dielectric enclosing it that is talking about the cross section all right it is known as transversal direction all right. So, x y in this case form the transversal axis all right and z is forming the longitudinal axis.

Now, whenever we want to describe a plane wave all right, these configurations it is very a common to start assuming some components of electric field being present some components of magnetic field being present extra all right. So, you start with some polarization configuration and you then you build up your derivation right

So, in this case what I am going to say is going to start with a condition say magnetic field z component is 0 ok and I am going to say that Ez is not equal to 0.

We are going to start with this condition and I am going to proceed and see what happens right. Now such a case where along the direction of travel in a waveguide now remember that you are not going to be solving for plane wave solutions these are guided wave solutions all right.

So, you are looking for interference patterns between waves that are going and hitting all the walls and then forming a total electric field total magnetic field extra ok. So, in these cases Hz equal to 0 means the magnetic field has only transversal component does not have a longitudinal component at all. So, such a polarization is also known as TM ok.

So, we are beginning with this polarization configuration right and then we are going to see what happens all right. The alternate case that one can assume is that electric field z component is equal to 0 and z_0 equal to 0 and that particular case will be known as TE or transverse electric mode all right, transverse electric polarization. There are also many other modes which people would study in higher level courses ok you can have hybrid modes ok.

And you could have many other things also you can have for example, in fiber optics people study very complicated modes all right. So, in this course we are just seeing the foundations of it right and it should enable you to pick up some text or papers related to these and start studying that is the whole point right. So, this would form the basics, but higher than this you could always should be in a position to go back and read material and assimilate information right. So, we are not going to see all the different kinds of waveguides which are possible absolute basics we are going to be seeing.

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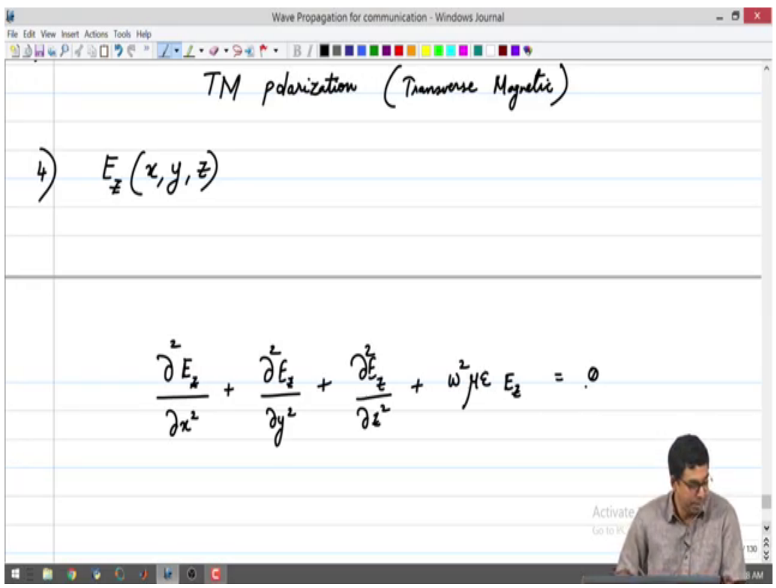

So, all right. So, let us a say that now that I have assumed my Ez not to be equal to 0 right. The goal is I would like to be able to find out the pattern of electric fields which are present here if it is going to form a standing wave if it is going to form a propagating wave if it is going to have for example, in parallel plate we saw there could be 1 half cycle or 2 half cycles 3 half cycles extra all right and in those cases one half cycle was out of phase by a 180 degrees compared to another half cycle and all that right.

So, what kind of a pattern would be there in such configuration is what we want to find out ok. So, for this we can say that clearly Ez is going to be a function of x, y and z coordinates all right. So, you have x, y and z coordinates Ez is going to be a function of all of them right.

Ez is a function of x and y simply because you have metallic walls ok it has to go to 0 on the interface all right. So, it is definitely a function of x and y it is also a function of z because you are assuming that you are launching a polarization which where Ez is not equal to 0 right.

And the direction of travel is along z direction. So, we know something about this right now right. We know that a we could expect some standing wave pattern in the cross section and we should be expecting some traveling wave along the Z direction right. We know what to expect all right and we are going to pick and choose those solutions and try to see if they are possible all right and if they are possible under what conditions would they be possible and all that right.

So, now since Ez is going to be a function of x, y and z we could always write down the wave equation here I am not using the curl equations because I want to focus first on the electric field patterns all right I want to look at the completely decoupled equation for Ez and then start doing the analysis before I bring in other components extra. So, one of the ways to do it is just go to wave equation try to substitute for Ez in terms of some solutions and try to analyze what could happen if you enforce some boundary conditions right.

So, we will start we will just say that I will use the wave equation ok

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} + \omega^2 \mu \epsilon E_z = 0$$

So, this is a tedious process now right now you have written the wave equation. Now you have to start writing down some general solution and start making some sense out of it and this is the tedious part. It requires you to have a lot of background already on pds unless you know a lot of things you will not be able to manipulate. So, I think I again emphasize that if you are able to remember some key results the form of a picture or in the form of a you know simple inequality or something like that it is good all right.

But the process of deriving you can have a base in future if you want something you can always go back and refer to it all right. So, I do not expect you to follow all the steps in the case of an exam or anything like that, but I think we will go over it in a systematic manner anyway to refresh a few things.

So, we want to find out solutions to the wave equation ok that is we want to find out Ez of x, y and z that is the objective. So, first thing that we figured out is what is the unknown? The unknown quantity here that we want to find out is Ez all right and Ez is a function of x y and z and we want to find out what Ez looks like with respect to x y and z that is all right.

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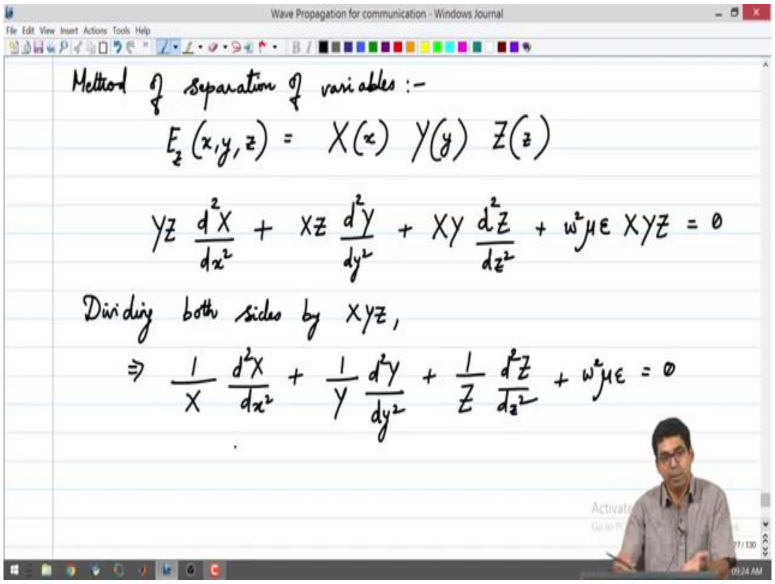

So, in this case we are going to be using, ok method of separation of variables all right. What this means? Is that since the Ez is dependent upon x y and z axis? Can we write down a general form for Ez all right, such that it is a function all right which is a product of 3 functions X which is dependent only on x, Y which is dependent only on y and Z which is dependent only on position coordinate z. What this means is you are having 3 functions.

$$E_z(x,y,z) = X(x)Y(y)Z(z)$$

X(x) which means that it is a function dependent only on the coordinate axis x. Y(y) is dependent only on the coordinate axis y. Z(z) is dependent only on the coordinate axis z and you take a product of these 3 all right. Is it possible to form a complete description for Ez ok the reason for doing that is then very easy. So, I can apply some boundary conditions for x, I can apply some boundary conditions for y all right and I can think about what happens the Z direction it allows me to think one coordinate axis at a time and then arrive at some simpler way of doing this right.

So, method of separation of variables here is just means that I am writing it down in terms of a product of 3 functions each of the function depend on only one independent variable in your wave equation right. So, if I were to assume this particular form the first thing I can do is take this form and substitute it in the wave equation above all right. So, this is the kind of solution I want what are the constraints what does X look like what does Y look like what does Z look like? Capital X, capital Y and capital Z I want to have a look at that.

So, I am going to substitute this a form into the wave equation above right. So, I will be having

$$\frac{\partial^2 E_z}{\partial x^2} = YZ\frac{d^2X}{dx^2}$$

So, since it is X(x) multiplied with Y(y) multiplied with Z(z) right and the derivative, partial derivative does not make much sense over here because I have x which is dependent only on x and y and z do not depend on you know x at all so, that means, that I just need to take the derivative of capital X with respect to small x and it is an ordinary derivative it is not partial right.

And I have Y Z just as constants coming out right. Same way

$$\frac{\partial^2 E_z}{\partial y^2} = XZ\frac{d^2Y}{dy^2}$$

$$\frac{\partial^2 E_z}{\partial z^2} = XY\frac{d^2Z}{dz^2}$$

This is what the substitution should look like and then you have the $\omega^2\mu\epsilon$ term coming into the picture all right ok. So, $\omega^2\mu\epsilon E_z$. So, that is I am just putting it as X Y and Z. So, just save the space I can write X(x) Y(y) Z(z), but it is already known to you right.

So, it is equal to 0 this is how the wave equation would look like and to make it a little bit simpler what I can do is I can divide both sides with capital X capital Y capital Z it is all right. So, I will just ok. So, I will have ok

$$YZ\frac{d^2X}{dx^2} + XZ\frac{d^2Y}{dy^2} + XY\frac{d^2Z}{dz^2} + \omega^2\mu\epsilon XYZ = 0$$

So, all we are doing is now mathematical manipulation and immediately we can see that we have some you know 4 terms that are added together the right hand side forms a 0 ok. Now, as we can also look at this equation in a different way I can say that you know I can evaluate

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + \frac{1}{Z}\frac{d^2Z}{dz^2} + \omega^2\mu\epsilon = 0$$

That is all this equation is telling me all right. So, I could always say that let me take one term at a time and form some equations all right and when I finally, put them together they have to just satisfy this equation wave equation over here.

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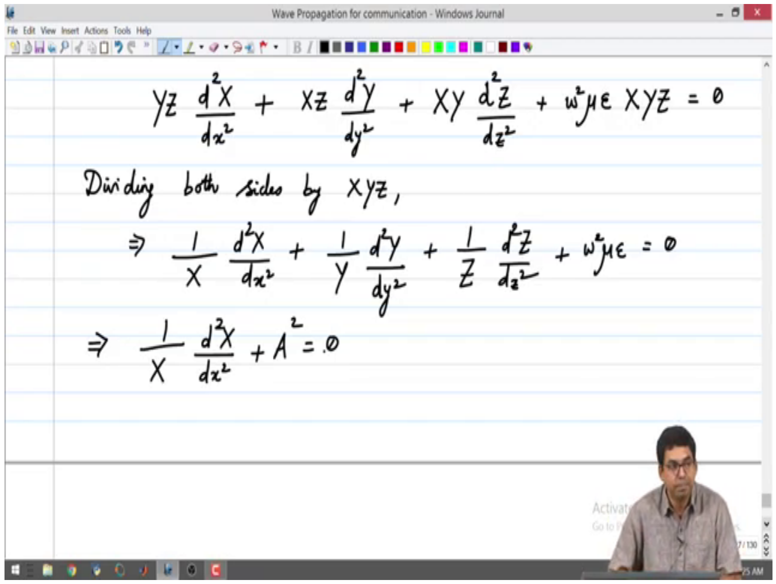

So, you can always write this down as a you can say that let us say that I have 1 by X let us say that I have plus some a square.

This could be one equation all right. It is not necessary that all the terms have to be equal to 0 just summed up together they have to be equal to 0. So, you can always have

$$\frac{1}{X}\frac{d^2X}{dx^2} + A^2 = 0$$

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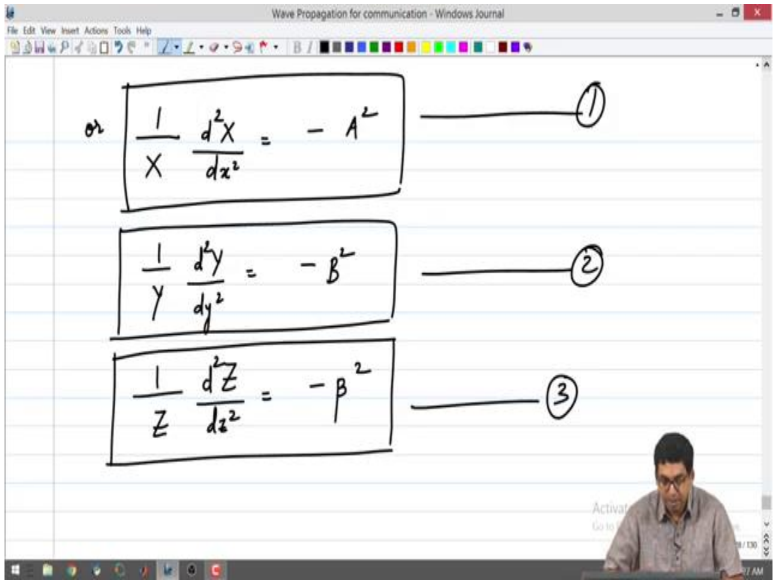

Or you can also write this as

$$\frac{1}{X}\frac{d^2X}{dx^2} = -A^2$$

Just it is the same thing all right written in multiple ways all right. I will mark this as say equation number one the advantage of doing this is that I get ordinary differential equation point number 1. Secondly, I get only one independent variable. So, I can write some simple solution forms which will satisfy this equation and then start to analyze what is happening right.

So, same way you can write down

$$\frac{1}{Y}\frac{d^2Y}{dy^2} = -B^2$$

And what I will also do is

$$\frac{1}{Z}\frac{d^2Z}{dz^2} = -C^2$$

That will make look like A B and C all are similar, but I want to make a small distinction over here all right I want to tell you again and again that I am looking for a particular kind of a solution which will tell me that Ez forms a standing wave pattern in x and y.

And some kind of a propagation characteristic in the Z direction I want to make sure that this is clear. So, what I am going to do is I am just going to replace this C^2 with β^2 to just tell you that

I am already looking for a propagating wave in the Z direction with a propagation constant of beta ok. So, I have just renamed it to beta square, but the distinction should be clear that I am on the lookout for something all right.

So, now I have 3 equations right. So, I could always write this in

$$\frac{1}{X}\frac{d^2X}{dx^2} + A^2 = 0$$

$$\frac{1}{Y}\frac{d^2Y}{dy^2} + B^2 = 0$$

$$\frac{1}{Z}\frac{d^2Z}{dz^2} + \beta^2 = 0$$

and I can start looking at general solutions which will satisfy them all right.

But I am making some assumptions, I know that from my parallel plate capacitor analysis that I can expect standing wave patterns when I have 2 conductors all right placed distance apart. Similarly, I have another two conductors placed this way. So, there should be some standing wave patterns coming into the picture and I am interested in knowing them all right and in the direction of travel I should be having a travelling wave and it should be like $e^{-j\beta z} + e^{j\beta z}$ just like your transmission lines these are some things that I know.

So, I am going to pick some solutions which will allow me to do that ok.

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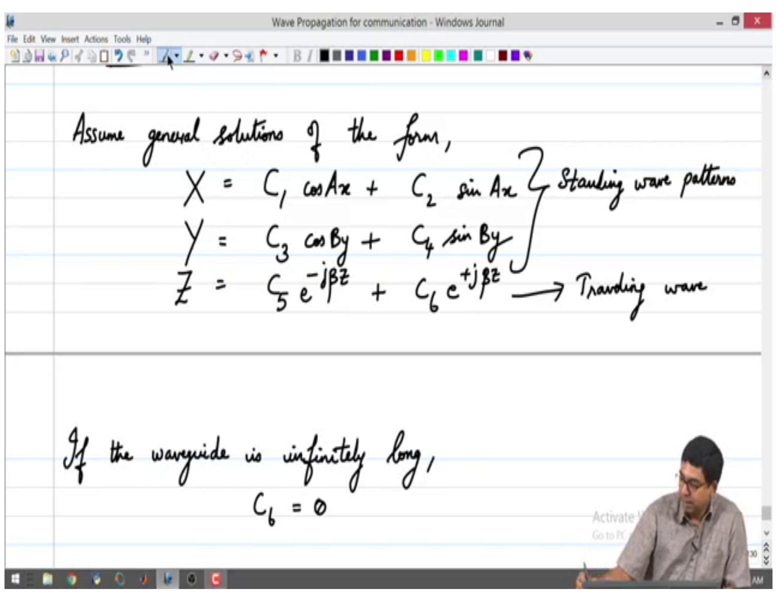

So, now we are going to take general solutions ok right. So, let us say that my objective is now to find a form for capital X Capital Y Capital Z and if I multiply capital X Capital Y Capital Z together I will get the form of electric field all right.

So, each of these equations 1, 2 and 3 the unknown quantity will be capital X Capital Y Capital Z ok. Now, I have to find out something for each, but I know that I want to find out specific kinds of them. So, I am going to take a general solution which looks like

$$X = C_1 CosAx + C_2 SinAx$$

because the function x is dependent only upon the coordinate X all right I have taken a general form all right.

The reason for taking this particular general form right is highly specific instead of being general all right in simply because I already know that I am looking for a standing wave pattern right. So, I have taken a form corresponding to a standing wave ok that is all. So, similarly in the Y direction also I know that I am looking for a standing wave pattern. So,

$$Y = C_3 CosBx + C_4 SinBx$$

And in the Z direction I have function

$$Z = C_5 e^{-j\beta z} + C_6 e^{j\beta z}$$

So, I will be having some constant C5 now I want to take the solution corresponding to a travel all right. So, I will just check $e^{-j\beta z}$. So, I have I know the physics of this already and I am picking solutions all right which will fit this a you know physics and then I am going to look back and see what inferences I can make whether the boundary conditions are satisfied all right.

You can have a check it all that later right, but first we are picking some form which we think will exist and then we are going to see and substitute in wave equation extra and you will see whether the solution actually satisfies and if there are no other issues for example, no issues with boundary conditions any other a physical interpretation then you will assume that your solution is valid and you will just proceed.

Now, this these expressions actually now tell you some form for the electric field that is if I take X(x)*Y(y)*Z(z), I will end up getting the expression for electric field, but it is a very complicated expression right. So, I am having

$$X = C_1 CosAx + C_2 SinAx$$

similar form for y and a traveling wave in z if I multiply all these three terms that would be the pattern for electric field all right. Now in order to reduce this a little further to make some interpretations it would be worthy to make some of these constant 0s.

Right it would be worthy to make some of these constants 0s all right. So, whenever we have large number of terms coming in like this in order to make some interpretations we can always try and see if there are ways to make many of these constant 0. So, I end up with the 1 or 2 terms then I can make very straightforward analysis as to what is happening. So, first I look at the traveling wave part ok.

I go back to the diagram and I look at the way I have drawn the diagram ok. I have drawn a semi-infinite all right or an infinite z axis you can say whatever ok. So, it is very long in the z axis, that means, if I were to launch an electromagnetic wave from one facet right and suppose some conditions are satisfied it is going to go to the other end all right and since it is infinitely long it is not going to hit a load or a you know interface of any kind and then travel back.

So, one of the things we can start by saying is let us assume that there is only a forward wave right and that the waveguide is infinitely long if that is the case then I can get rid of one of the constants. So, I am having $C_6 e^{j\beta z}$, I can say that if the waveguide, I can say that the constant becomes a C6 becomes equal to 0 then you do not have any term corresponding to backward travelling wave in their expression.

So, for us it just means that from the electric field expression you have got rid of one term all right and we are interested in more all right. So, we already know that we can apply some boundary conditions now and see what can happen? What could the boundary conditions be?

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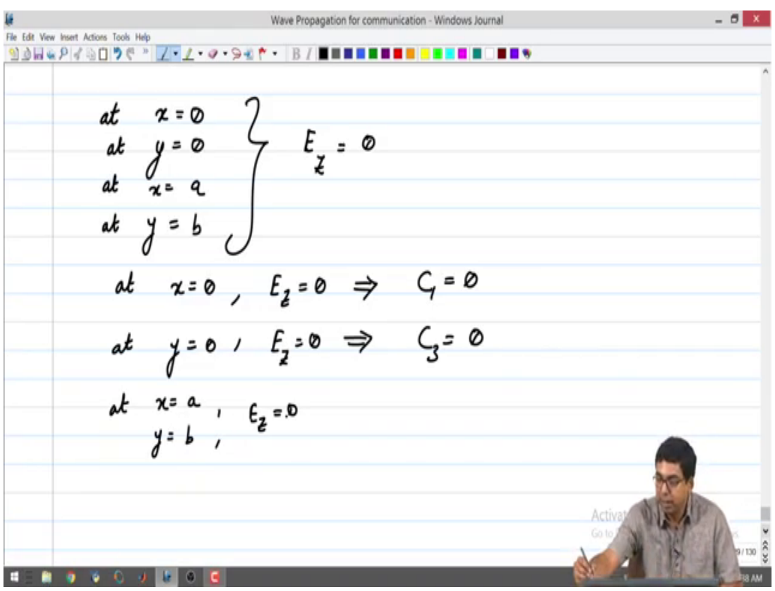

At say let us say that a one side of the X direction of your waveguide. So, I am having two parallel plates the X direction all right two parallel plates in the Y direction. So, I can say that if one of the plates is placed at x equal to 0 the other plate to say. Now at x is equal to according to the and the bottom plate it has y equal to 0 and y equal to a I know that on all these four facets according to the boundary conditions the electric field has to become identically 0 right. So, I can say that at x equal to 0 at y equal to 0 at x equal to at y equal to b right.

I have Ez becoming equal to 0 ok. So, we will start with looking at what happens with x equal to 0. Now, x equal to 0 right should make the electric field equal to 0 right. Now I look at the expressions for the functions Y and Z they have got no dependence on x at all right. So, it cannot affect the constant C4, C3, C5, C6 is already 0. So, there is no way at X equal to 0 these two are going to be affected. So, something has to happen here in the first expression for x is equal to 0.

Now, I look at X is equal to 0. So, I substitute in X(x) all right as x equal to 0. I notice that immediately the second term on the right side vanishes. And then when X is equal to 0 I have cos you know 0 and I have 1 and C1 is coming into the picture, but that has to be equal to 0 then right.

So, under these boundary conditions C1 has to become equal to 0 otherwise X equal to 0 you would you will still end up with a constant electric field right. So, we can say that at x equal to 0 Ez equal to 0 we will mean that your C1 is equal to 0.

Similarly, now we can take each of these conditions at y equal to 0. y equal to 0 means that if you go back to the descriptions of the functions. Capital X is not dependent on Y Capital Z it is not dependent on Y all right. So, these two are going to be present then at Y equal to 0. This particular term has to become equal to 0 to satisfy the boundary condition right Y equal to 0, once again you substitute Y equal to 0 this term is gone. So, there is no problem with this term and if I substitute Y equal to 0, C3 capital Y is equal to C3. Well, this has to become equal to 0 for me to satisfy the boundary condition at that location right.

So, we can say that now y equal to 0, Ez equal to 0 simply means that C3 is equal to 0, that is good all right many constants are getting wiped out because of the boundary conditions. I have then at x equal to a y equal to b Ez is equal to 0 once again right.

x equal to a y equal to b extra cannot you know change anything with respect to z function z all right. So, I have to look for something in these expressions that will make them 0 all right. Now, already I know that C1, C3 are 0's ok. C6 is 0 to make things simpler why do not we write down the electric field expression first and then we will start looking at it right.

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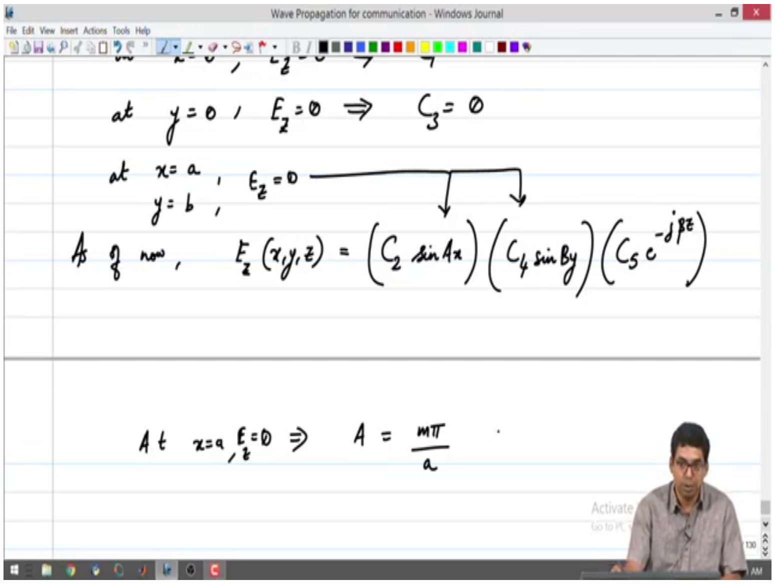

Ok. So, I am having some a C1 is 0. So, C two term is coming into the picture $C_2 Sin Ax$ all right ok. Now, I having

$$E_z(x, y, z) = (C_2 Sin Ax)(C_4 Sin By)(C_5 e^{-j\beta z})$$

that is what we are having now right. Here is what we are having and here we want to make use of x equal to a y equal to b electric field going to 0. I know that this boundary condition cannot affect the last term because that is dependent only on the z coordinate all right it has to affect only the first two.

Now, at x equal to if I want to make something, the x equal to a cannot affect the second term in this it can affect only the first term all right. So, $C_2 Sin Ax$ has to become equal to 0 at x equal to a. So, substitute x equal to a looks like $C_2 Sin Ax$ all right capital A small a is equal to 0. Just means that A has to be you know of the form this will look like this has to be some mπ divided by a right.

Then only if you multiply with x equal to a x is equal to a you multiply these two will get cancel and you will get mπ on the numerator all right $Sin(m\pi) = 0$. So, that satisfies the boundary condition. So, I can just say that

$$A = \frac{m\pi}{a}$$

So, that is a solution.

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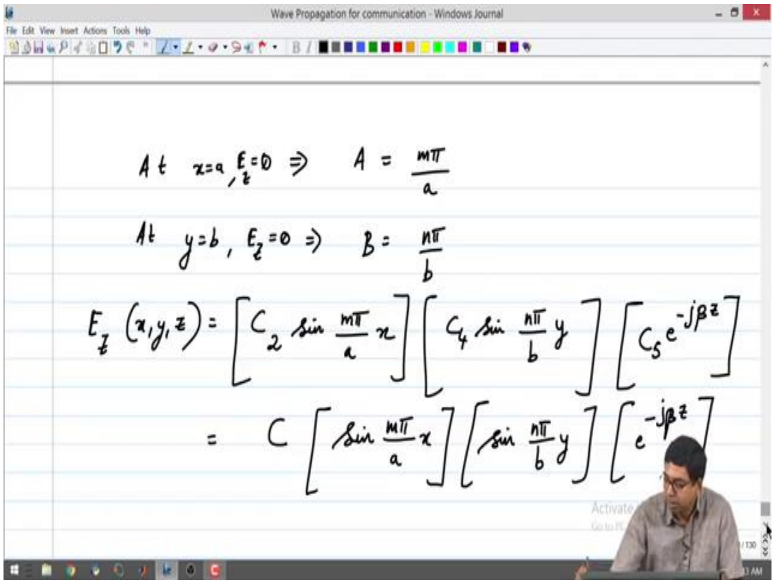

Similarly, at y equal to b once again the electric field is 0 this means that we will have another integer right the most general form

$$B = \frac{n\pi}{b}$$

So, there is no constraint and all that right ok.

So, now we have something to start analyzing can say that the electric field which is a function of x, y and z is a starting to look like

$$E_z(x, y, z) = (C_2 Sin(\frac{m\pi}{a})x)(C_4 Sin(\frac{n\pi}{b})y)(C_5 e^{-j\beta z})$$

This is what the electric field looks like I have three constants all of them are going to be present I do not see any need for these individual constants because I do not see a way to push them to 0 all right. So, I can multiply C2, C4, C5 and make that one constant all right. So, I can simply write this as

$$E_z(x, y, z) = C\left[Sin\left(\frac{m\pi}{a}\right)x\right]\left[Sin\left(\frac{n\pi}{b}\right)y\right][e^{-j\beta z}]$$

So, clearly we understand now that in the Z direction I am having a forward travelling wave and in the x and the Y direction depending upon the position x ok you are going to be having some fixed values of electric field coming into the picture right and depending in the Y direction depending upon the position. I mean the Y direction depending upon the position y all right and the separation between your two plates b you are going to be having some fixed patterns coming into your electric field right.

So, it will give you some patterns and depending upon m and n numbers you will be either having full half cycle right or you will be having two full half cycles right they may be out of phase with each other they will be having three half cycles extra in X direction in Y direction also you will be having a single half cycle double half cycle extra for the electric field.

So, now I can start to form some picture all right of what could happen, but before that let us have a look at some of the consequences already and rule out a few things right. Since, I have sin multiplied with sin multiplied with some $e^{-j\beta z}$ I am interested in knowing under what condition I will have no electric field ok.

So, the condition where I will have no electric field is either if

$$Sin\left(\frac{m\pi}{a}\right) = 0$$

$$Sin\left(\frac{n\pi}{b}\right) = 0$$

right which will mean that for all x and all y those two terms has to become equal to 0. For all x and for all y if they have to become equal to 0 the only way is π cannot become 0 ok. So, m could be 0 and n could be 0 if m is equal to 0 and n is equal to 0 you will end up having no electric field at all ok. So, m equal to 0 and n equal to 0 are not possible all right in this kind of a standing wave pattern that you are considering and a traveling wave in the Z direction m equal to 0 and n equal to 0 means that you have no electric field at all.

Student: But sir this only means that Ez is 0 not the other components so.

No I think that is not the case at all. So, we will just focus on Ez for now when we see the eigenvalues we will be able to follow it much better, but m equal to 0 n equal to 0 means that you do not have an electric field throughout the x and y transversal directions of the waveguide there is no spatial derivative of Ez present ok.

Because it is all 0 there is no spatial derivative. So, if you take del cross you will not get anything with respect to time right. So, Ez becoming equal to 0 the X and Y direction is not considered to be anything because it does not give you a spatially varying electric field right and then you cannot take simply del cross and then you can find out a magnetic field that varies with respect to time.

So, you will end up getting no electric fields that is spatially varying no magnetic fields that is time varying. Then you come back to the next curl equation you will get nothing again right because there is no electric field at all instants of time you cannot have spatially varying magnetic field also. So, it is not just only Ez it is the other components that you have considered also will be pushed because of the two curl equations ok.

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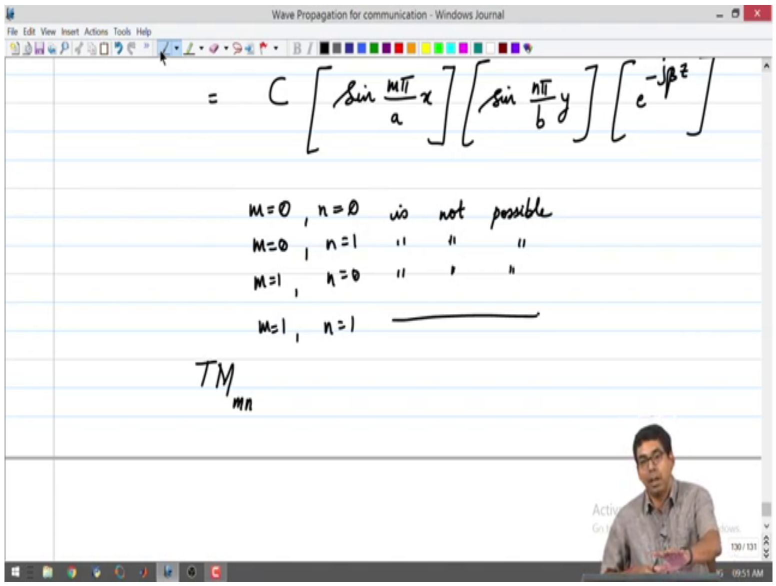

So, in this scenario m equal to 0 n equal to 0 is a not possible. Or is something which m equal to 0 n equal to 0 simply corresponds to having no fields inside of that region of interest at all ok. So, then the question can be asked what is the minimum value of positive number right for m and n ok which will hold you know some valid electric field which will give you a traveling wave in the Z direction ok. So, suppose I say that let us you know keep m equal to 0 then let us make n equal to 1 does not solve the issue.

Because still I will end up with the 0 electric field and m is equal to 1, n equal to 0 is still not possible right. Then I start to look at m equal to 1 n equal to 1 yes then you start getting some electric field patterns ok. So, there is some there are some conditions even after you derived the solution by eliminating the constants all right and m and n are going to tell you whether the electric field is going to be present or not we already know now know that m equal to 0 or n equal to 0 is not going to give you a presence of any electromagnetic field inside of this waveguide ok.

M equal to 1 n equal to 1 is the absolute minimum you could always increase m equal to 2 n equal to 2 extra. So, this should mean something in the case of parallel plate capacitor we saw that the m corresponded to the number of half cycles you will have in that direction right for the electric field it means the same thing over here as you go from one plate to another plate if you find that you are.

You will start with the electric field equal to 0 on one plate because of the boundary condition if you go to the center of the wave guide if you notice that you are having a peak value of the electric field and then you are going to 0 electric field, that means, that you have encountered 1 peak in the electric field, that means, that you have you have m equal to 1.

Same way you consider the plates in the vertical direction you go from one plate to another plate and try keep measuring the electric fields z component at all these points when you are going from one plate to another plate you will start at 0 because it is a plate it is a conducting plate electric field is 0 and then as you go into the waveguide a dimension what you will notice is you will encounter 1 peak.

So, you will be having some profile which looks like going from 0 all the way to a peak and to a 0 in this direction 0 all the way to a peak in this direction. So, you have to take a product of these two then you will end up getting a spot ok to look like a spot ok and the spot will have some sinusoidal profile in 1 direction sinusoidal profile in the other direction all right.

So, m equal to 2 will mean that you have 2 half cycles and n equal to 1 will mean that you have 1 half cycle. Then you can start forming different kinds of standing wave patterns right. So, these m and n are then used to represent what kind of a mode or a standing wave pattern you are talking about in this case, we are talking about TM all right that is what. So, usually people indicate the suffix like this right TMmn right either with the brackets without the brackets right.

Without the brackets is more common I guess right. So, which will just say to you exactly what kind of a standing wave pattern in the x and y directions a person is actually mentioning right. So, I think this should give some basis as to how one can solve for more it is provided you already know what to expect. So, in many of the research problems also we know what to expect then we go to the wave equation we see if we substitute everything if it makes sense all right does not violate any physics then we say that it is a valid solution ok.

So, in research it will be very common that you expect something to happen. So, you put all the forms of X(x), Y(y), Z(z) and then you try to make the constants equal to 0 and then you see if there is any absurdity present, there is no absurdity present then you say that it is a valid solution. So, a solution says for example, I want a wave doing something weird all right I force some form of the general solution and I force some boundary conditions consistent with the physics and if I see that it make sense and it does not violate anything then it is a valid solution right.

Now, one of the things that we have to do still is to use a computer to solve for this and try to get a better feel all right because we have not talked about anything related to eigenvalues and eigenvectors extra over here right. So, we have to talk about them we will do it in the following classes ok.

So, for now we will stop here.