Transmission lines and electromagnetic waves
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Lecture – 28
Parallel Plate Waveguide

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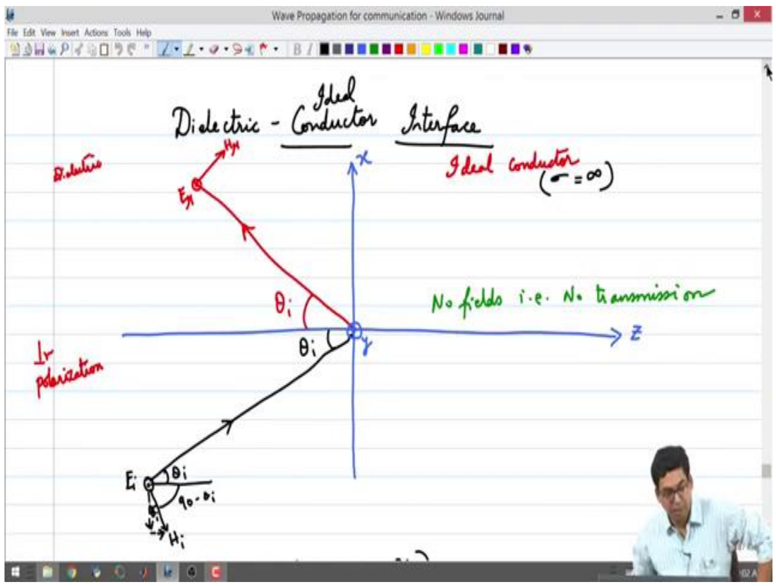

All right, we will get started ok just to go with what we saw last class, there was an interface between a dielectric and an ideal conductor ok. And we have used the same coordinate system has what we had used for dielectric interfaces. There is an electromagnetic wave being launched right. This is a plane wave, it is being launched to the interface and in this case since fields do not exist of an ideal conductor you do not have any transmission coefficient to be calculated. So, we attempted to calculate the reflection coefficient.

We made an assumption that the electric field does not flip it sign and the magnetic field flips it sign sin order to form the right handed triad ok. And we proceeded to write down the expressions for the incident fields which was identical to the case when you had the dielectric interface right.

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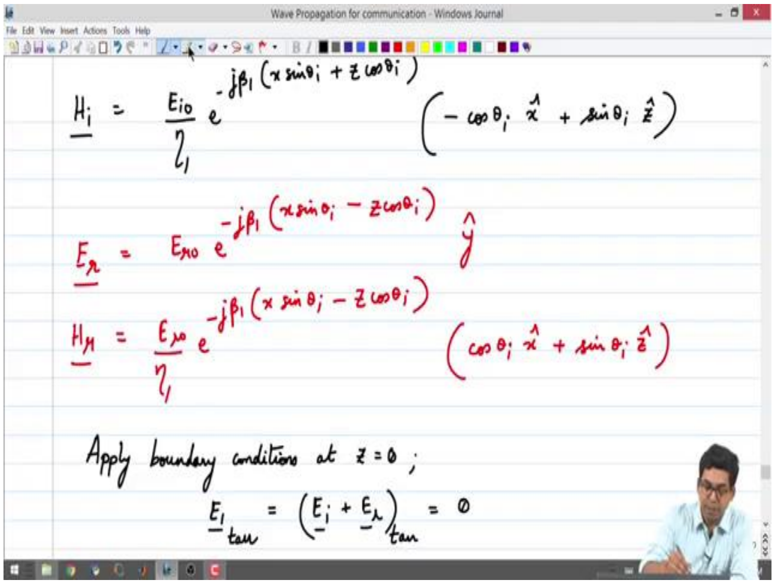

Once we wrote that down, we wrote down the expressions for the reflected field in the way that we have assumed right and we applied some boundary conditions on the interface ok. So, tangential components of E field continuous normal components of H fields continuous right.

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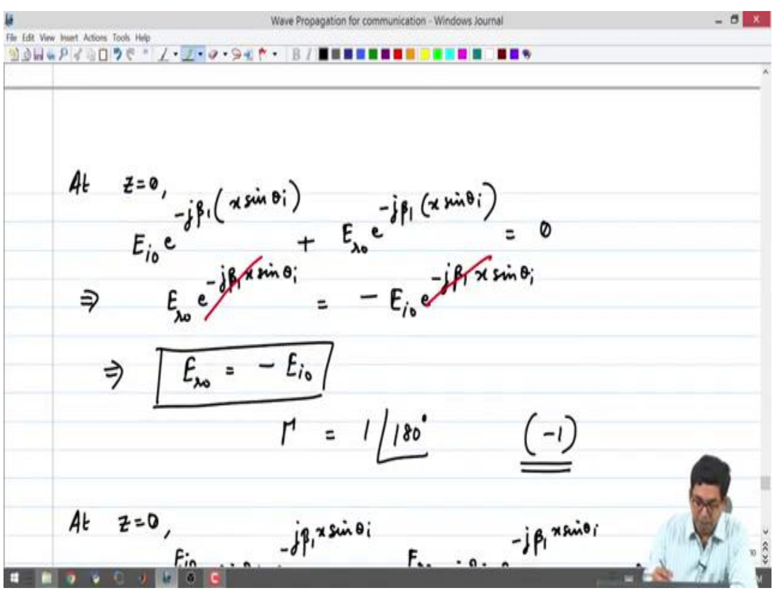

And once we did that, we were able to obtain that the E_{r0} or the reflected electric fields is going to be flipped in sign all right with respect to E_{i0}. So, even though we made an assumption that the electric fields are pointing in the same direction. The calculation reveals that compared to the direction that we have assumed the electric field actually flips it is direction right.

So, one of the things to remember is that the e field will flip it is direction by 180 degrees when it hits a hard metal ideal conductor interface right not from a dielectric. So, the reflection coefficient can be now written as one at an angle of 180 degrees. The equivalent condition for the transmission line was the case of a short circuit right where you had voltage reflection coefficient to be equal to minus 1 right. And we proceeded to write down the total fields in medium number 1, if there is a reflection on the interface.

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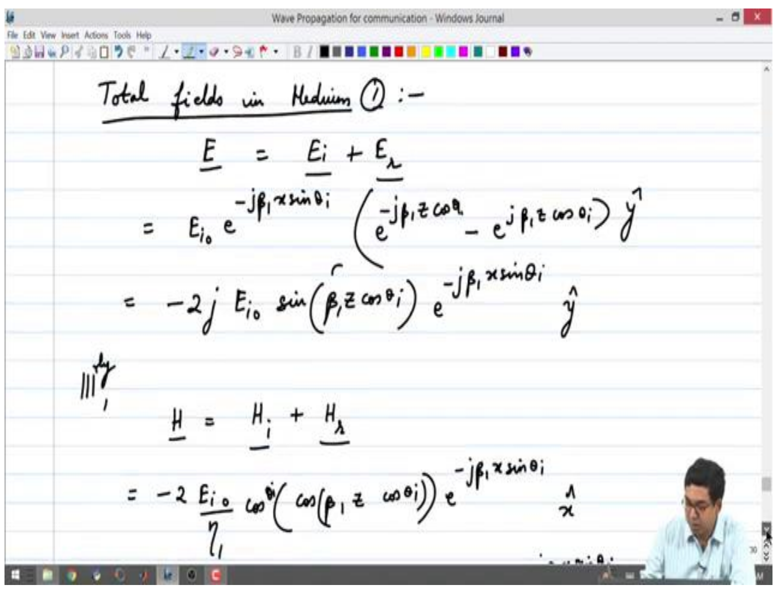

And once we finished writing these expressions, we wrote a simple computer program all right where we were able to see standing waves in the z direction all right. And a traveling wave in the x direction right. We had done it for a one of the angles, but you could change the program to use any θ_i and you would be getting different values of you know standing wave, different standing wave patterns depending upon a variables that you are using right. So, now, we are going to extend the same concept, but a little bit further right.

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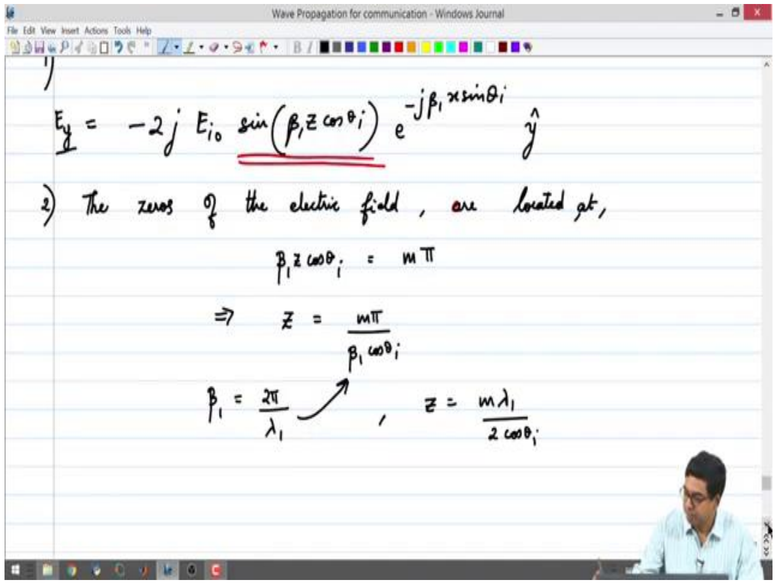

$$E = -2jE_{io}Sin(\beta_1 zCos\theta_i)e^{-j\beta_1(xSin\theta_i)}$$

So, I am going to start by writing down the expression for the total field, electric field that we had prior in the prior class alright ok. It was electric field y direction right I think we are using the usual additions right. So, this was my total electric field in medium number 1 ok. Now let us look at this a little bit more closely and try to see what else can we infer from this right.

Well, the first thing is that the zeros of the electric field ok are going to be present corresponding to the term here right. So, I have

$$E_y = -2jE_{io}Sin(\beta_1 zCos\theta_i)e^{-j\beta_1(xSin\theta_i)}$$

a $Sin(\beta_1 zCos\theta_i)$ over here as z changes β_1 is a constant for a given medium. Suppose, you say that in your problem z that theta is fixed, angle of incidence is fixed, then depending upon the distance you move away from that conductor you are going to be having some zeros and these are going to be periodic ok. So, the zeroes of the electric field can be located all right.

So, you can have $\beta_1 zCos\theta_i$ ok, it is going to be equal to sum mπ. This is the argument of the sinusoid right. Whenever it is going to be m pi, we are going to be having zeros along the z direction ok. So, now, that we know where the zeros would be ok we can also directly say at which positions z you will be having zeroes of the electric field right.

So, you can always write this down as

If you want to make it a little bit more intuitive, you can always write down

$$\beta_1 = \frac{2\pi}{\lambda_1}$$

Where λ_1 is the wave length in the medium and if you substitute for β_1 over here right. You will be having

$$z = \frac{m\pi}{2Cos\theta_i}$$

This is the location at which you are electric fields will completely vanish ok. What does this mean? Right.

So, if we go back to the configuration that we have drawn right. If we travel in this case along the negative z direction, right. So, if we travel away from this in this format right. There are fixed places where the electric field is going to be 0 ok. Now electric field equal to 0 can be interpreted in multiple ways, but we will look at it in a simple case from the example that we have taken itself.

We know that inside an ideal conductor, there are no fields all right and the electric field is equal to 0 right, in this case the electric field is tangential right. So, that means that, your boundary condition that needs to be satisfied right is

$$E_{1_{tan}} = (E_i + E_r)_{tan}$$

there is no transmitted electric field right.

Which means that, the line all right the constant lines in z direction where the electric fields are looking as 0 can be thought of as a surface of a conductor itself ok. It is as though you have placed an ideal conductor at some locations. So, electric field equal to 0 means that you do not have a potential gradient across that region and you can always say that you have placed a conductor in that region right.

So, it is an imaginary plate, if you keep it there the electric fields would have disappeared to 0. So, it also means that suppose I do place a second conductor ok and it will satisfy the boundary conditions that we have written originally. It will satisfy the expressions for the electric fields that we have written prior all right. So, it will appear as though nothing has changed in the system except that you have place an actual conductor on the left side ok.

So, you could always take a semi-infinite conductor all right and you can place it on the left side it is an ideal conductor and that would not change the expressions for the electric or the magnetic fields inside this region ok. So, one thing that we can start now looking at is what happens when you have multiple conductors coming into the picture.

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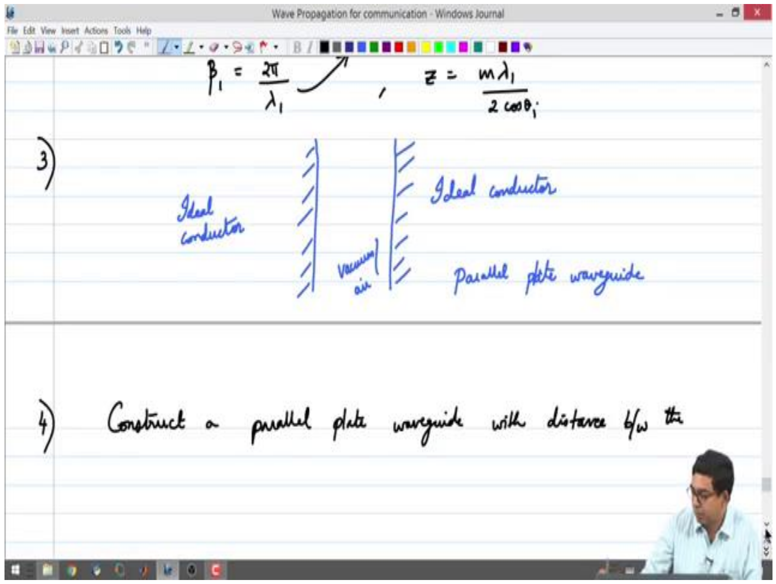

So, the schematic now would look like. So, I have an ideal conductor on the right, an ideal conductor on the left side on both of these surfaces the boundary conditions are satisfied and you will notice that you have not perturbed the fields in any way ok. Now what could the result be? Ok.

We already saw from the previous class that you are not having an energy transfer along this horizontal direction which was the case of a you do not have transfer of energy into the conductor at all right. Also, we noticed that since you are launching the field in this manner all right it goes bounces off the interface and travels upwards.

Suppose I placed a second conductor somewhere over here this way would travel further nothing would be transmitted into the ideal conductor and it would get reflected back all right. And you will notice that there will be some net travel in the positive x direction ok. So, you will be having some net travel in the positive x direction which is consistent with what we have seen in the x direction, it was a traveling wave all right. And in the z direction it was a standing wave pattern all right.

And since this is a traveling wave in the positive x direction, in this configuration if I placed a second plate on the left side there would be complete reflection from that plate and there would still be a net travel in the positive x direction exactly following the expressions that we have written before right.

The consequence of this is that the wave is going to be completely confined between these two plates ok. So, it is not going to be spreading everywhere, but the electromagnetic wave is going to be completely confined between these two plates all right and if the two plates are going to a large distance or the large x direction. The wave is the wave is also going to travel bouncing back and forth between these two plates and reaching another side.

Another way of thought is, if you did launch a wave on one end of a parallel plate configuration all right. You will notice the wave emerging out of this parallel plate configuration on the other side all right, without diverging, without going out of these region at all right. So, since this wave is completely being guided from a source to an end all right by merely using two parallel plates such a configuration is known as a parallel plate waveguide ok.

This is known as a parallel plate waveguide. So, one could also think of a what else can you do with parallel plate waveguide, other than having them straight. Well, because you are making these out of ideal conductors, there is no requirement on you to make it completely straight. For example, you could take this curve it all right and make it go you know 90 degrees to the other side and the it will bounce back and forth and it will find it is way out on the other side right.

So, you can take the wave and you can guide it in the way you want all right. There are some extreme conditions that you may have to think about later. But for now, if you had these two parallel plates, the electromagnetic wave is going to be between them all right and the net direction of travel is going to be in this direction unless you bend the plates in somewhere. For example, if you bend the plates like this and then back and then back then you will be starting to get the wave going in the horizontal direction extra.

So, you can play with the way the wave will actually travel and the net energy is going from one side to the other end of the parallel plate configuration. So, these a this is the premise of what is known as a parallel plate waveguide in equivalent in optics is you just consider that do you have two mirrors perfectly plane mirrors all right. Positioned parallel with each other. You will just shine a beam of laser on one side it is going to bounce back and forth and go to the other side all right. That is an analogy right it is not going to work in the same way right, but it is just an analogy all right.

Now one of the things a that we have to start looking at is what is the difference between directly shooting an electromagnetic beam from one side to the other side and actually doing this kind of a configuration, right. Well, the first thing that I notice is the propagation is not directly in between these two plates, that is you do not have the energy transfer directly going like this, but instead the wave is undergoing multiple reflections all right and then there is an effective moment in the x direction. Now if you have a casual observer placed outside of this waveguide configuration.

The observer will notice that a plane wave has been launched at some instant of time and they have note down that time all right. Then, they note down the instant at which the wave emerges out of this parallel plate configuration on the other side ok. They can measure the length of this parallel plate configuration all right.

Then they try to find out the distance or the length all right, length divided by the time it took for the wave to come from one side to the other side and they will notice that an even though in between you had air for a for an observer who is outside all right. The velocity is actually small ok compare to homogeneous air right.

So, this is the first thing that we have to realize the consequence is that you are going to have propagation that is slower than in a homogeneous dielectric medium. Because, it does not travel, it does not follow the path of the you know least distance between point A and point B it is bouncing multiple times and it is reaching on the other side.

So, you can call these kinds of configurations as delays all right. So, you are taking a signal and you are passing it through this configuration at a specific angle, it will be delayed by. So, much time when it reaches the other side. So, in a simple scenario all right a wave guide can be used as a delay element. Why would one use delay element is a you know there are a lot of factors involved?

For example, if you are having some two signals and you want to make a logical operation out of the manned OR, NOT, AND or extra all right. You may want to, you may get one signal on one of the pins input pins first and you want to wait to make the decision after the second signal also arrives at the second pin. Which means that a you want to delay one signal the first I mean, the signal with respect to the other. So, that both of them are arriving at the same time extra.

So, the purpose of that delay there are numerous purposes for which people can use, but compared to a homogeneous medium if this is vacuum or air, compared to the scenario where you do not have these two plates for an observer from outside this system, the velocity in this will appear to be smaller than the velocity in homogeneous vacuum.

This is the first thing right. So, you cannot have waveguide like this right which will give you velocity equal to that of a you know bulk homogeneous medium right. This is the first thing that we have to notice right. The second thing is, immediately I have used a an incident angle of θ_i, in the previous class to denote the angle of incidents. We already know that, if a you know θ_i is going to be small, if θ_i is going to be large right.

If θ_i is going to be 0 degrees, which means that I am not having an effective moment going in the positive x direction at all. I am going to be having the wave merely bouncing between these two plates and there is going to be no travel. For an observer, who is outside all right, it means that, there is an infinite delay ok or the velocity has become 0 ok.

If you change the angle, if you keep increasing the angle for an observer outside, the velocity will keep increasing all right. So, this means that in this case the velocity is not fixed the velocity does depend upon the angle of initial lunch into the system and for an observer who is outside. Depending upon the angle of launch the velocity is going to be different. That means, that there is a possibility of constructing variable time delay by merely changing the angle of launch ok.

So, it is not a fixed delay, it could be a variable delay right. It could go all the way from infinite delay to almost no delay compared to a homogeneous medium if you launch it is straight in between these two plates right. So, it is a delay element all right, but it is a variable delay element if you do have control over the angle of launch of the electromagnetic wave ok.

Now, a one could also think about this in a connected way right. So, let us say that as an engineer, you will construct this configuration first and then you will start doing some analysis ok.

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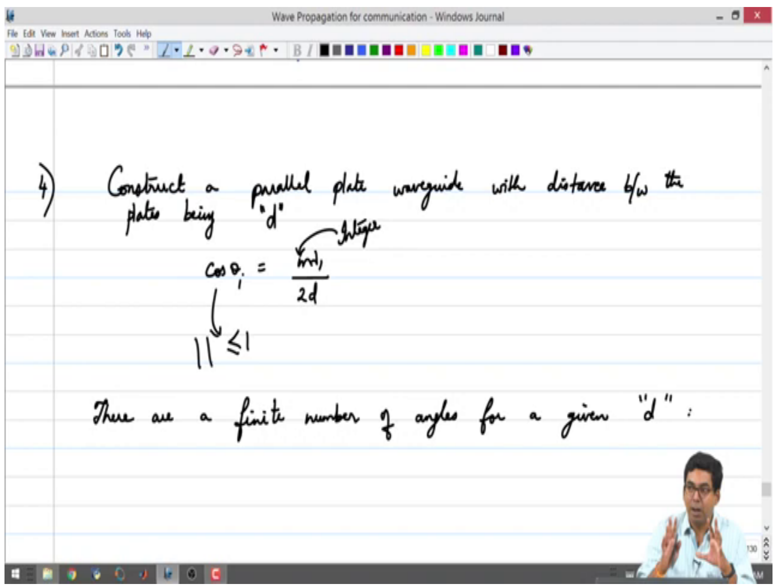

So, you can say that, you can construct a parallel plate with distance between the plates being, say denoted by d because, it looks like a parallel plate capacitor right. So, distance between the capacitor, we usually call it with the letter d. So, you can just use d all right ok.

Now is it true that at all angles ok, is it true that at all angles, the boundary conditions will be satisfied just like what we were talking now right we are saying that as the angle changes ok as the angle changes, there will be no perturbation what is so, ever smoothly. Is it a really true? Ok. So, the condition is a we are looking for zeros of the electric field as long as you do not alter the zeros of the electric field and you have place the plate exactly at the zeroes of the electric field. What we are arguing is correct ok.

But the condition for zeroes of the electric field was a you know it is

$$Cos\theta_i = \frac{m\lambda_1}{2d}$$

this is what we had written right. z is of course, the distance between the two plates which has been replaced to d all right and

$$Cos\theta_i = \frac{m\lambda_1}{2d}$$

This condition has to be satisfied for you to have zeros located in the manner which we derived before right.

Which means that we have to look at this a little bit more carefully. Is there specific angles at which you can launch? The answer it seems to be so right.

$$Cos\theta_i = \frac{m\lambda_1}{2d}$$

So, let us take that rosy picture and repaint it again all right. Originally, we said that we have a parallel plate it will be a source of a delay it is a variable source of delay depending upon the angle of launch, but it is not going to be a smooth variation of delay by no means all right.

Because m is an integer ok, m is an integer all right and we know that cosθ_i cannot be greater than 1 so, this has to be less than equal to 1 right, a the magnitude at least right. The absolute has to be less than equal to 1. So, there are constraints in the way you launch the electromagnetic wave. We cannot take any parallel plate configuration and just say that you will lost the wave and you will get a very perfectly controlled delayed version on the other side.

Looks like, there is some constraint on the angle ok. Now we have to look at this even more closely. Is there a constraint on the resolution of the angle or is there a constraint on something else all right all were trying to say is this cosθ_i magnitude has to be less than equal to 1 ok. Which means that already we are seeing that there are a finite number of angles for a given separation. Because, you cannot keep on increasing m to,3,4 5,6 suppose, d is given all right, λ_1 is given ok.

You can choose values of m which will make cosθ_i less than equal to 1

$$|Cos\theta_i| \leq 1$$

You cannot simply choose m equal to 10000, 20000 extra all right and the $\frac{m\lambda_1}{2d}$, the modulus has becomes greater than 1.

So, this place is some constraints. You cannot have any choice of m, you can have only finite choices of m for a given constructed parallel plate waveguide with separation d ok. Since there is only integer values of m being assumed over here, there are very limited angles at which you can launch and actually not perturb the system and get an outcome outside ok.

So, it is at delay, but it has constraints, you cannot launch at any angle and expect the electromagnetic wave to arrive at any I mean any configuration for a given configuration there are finite number of angles as long as this angles satisfies the condition at the absolute value of

$$|Cos\theta_i| \leq 1$$

$$\frac{m\lambda_1}{2d} \leq 1$$

Only then you are going to be having some waveguide like properties ok.

So, first of all we notice that the velocity is smaller, second we notice that it can act as a delay element, third it is not a fully controllable delay element. It has got its quirks, there are only finite number of angles at which you can launch ok. What else? Ok, let us look at the total field expression once again ok.

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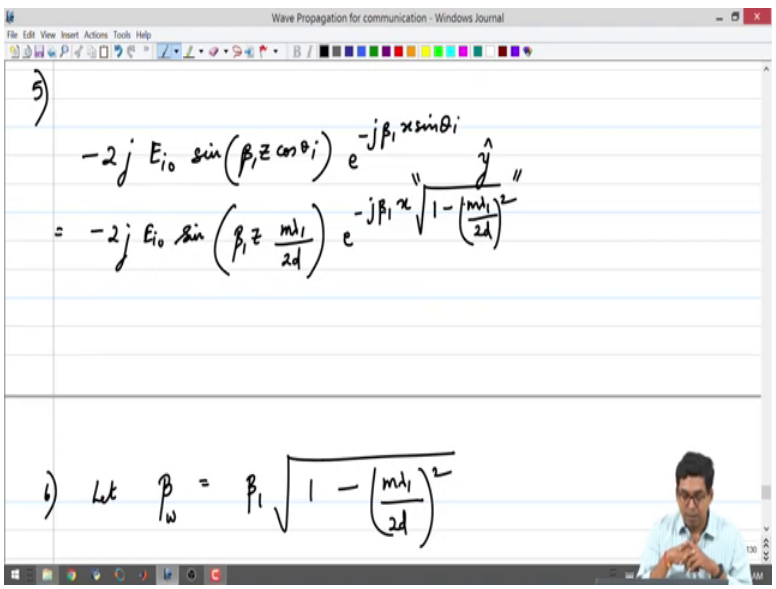

So, I am going to take the total field expression once again ok going to take the total field expression

$$E = -2jE_{io}Sin(\beta_1 zCos\theta_i)e^{-j\beta_1(xSin\theta_i)}$$

and all I am going to do is a substitute for $\cos\theta_i$ ok. I know that $\cos\theta_i$ the way we have written here is $\frac{m\lambda_1}{2d}$ all right. Of course, the choice of m has to be guided by the principle that modulus of $Cos\theta_i$ is less than is equal to 1, but it is $\frac{m\lambda_1}{2d}$ right.

So, I want to make some substitutions for

$$Cos\theta_i = \frac{m\lambda_1}{2d}$$

$$Sin\theta_i = \sqrt{1 - Cos^2\theta_i}$$

So, I just wanted to make that substitution in the total field expression. So, I can just say that this is

$$E = -2jE_{io}Sin(\beta_1 zCos\theta_i)e^{-j\beta_1\left(x\sqrt{1-\left(\frac{m\lambda_1}{2d}\right)^2}\right)}$$

so I have just taken some constraints that I have with respect to $Cos\theta_i$. There is an equality there $\frac{m\lambda_1}{2d}$ and the choice of m of course, is guided by the fact that $|\cos\theta_i| \leq 1$. But suppose, I did make that choice the total fields have to follow this particular relationship all right.

Now there are a lot of things happening over here. First of all, we know that this is an expression which tells you that there is a propagating wave in x standing wave in a z direction right. On top of that the pattern depends upon the position that is the distance between the two plates d and the number m that you can choose ok.

So, there are too many parameters happening over here, but let us just have a look at the x direction first. As I have $e^{-j\beta_1\left(x\sqrt{1-\left(\frac{m\lambda_1}{2d}\right)^2}\right)}$. Let us look at this first ok. Suppose, I were to create a new medium to understand this right. Let us say that by taking an original medium to be β_1.

So, I say said that vacuum or air had an electromagnetic wave launched into width of wavelength λ_1 all right and the β_1 in that medium is

$$\beta_1 = 2\pi/\lambda_1$$

in that medium. So, since it is vacuum there is no change over the $2\pi/\lambda_1$ right, but when I am putting two plates all right. It looks like a phase constant that is different from β_1 right. It is $\beta_1\left(x\sqrt{1-\left(\frac{m\lambda_1}{2d}\right)^2}\right)$ right.

So, previously would have just had $e^{-j\beta_1 x}$, but now you are having some s$\sqrt{1-\left(\frac{m\lambda_1}{2d}\right)^2}$. So, this parameter is coming into the picture because you have added some two plates with a distance d between them all right. And your choice m is such that $|\cos\theta_i| \leq 1$.

So, we can always say that, you can create what is known as a β_w, w is just for a wave guide ok. It is a wave guide, it has the same homogeneous medium β_1, but in between two plates separated by a distance d. It seems to have a different phase constant if you define β_w to look like

$$\beta_w = \frac{\beta_1}{\sqrt{1 - \left(\frac{m\lambda_1}{2d}\right)^2}}$$

If I do this then, I can write down the exponential as $e^{-j\beta_1 x}$ right right. It looks like a homogeneous medium with a phase constant of β_w right. So, there are a few things once again that we need to look into ok.

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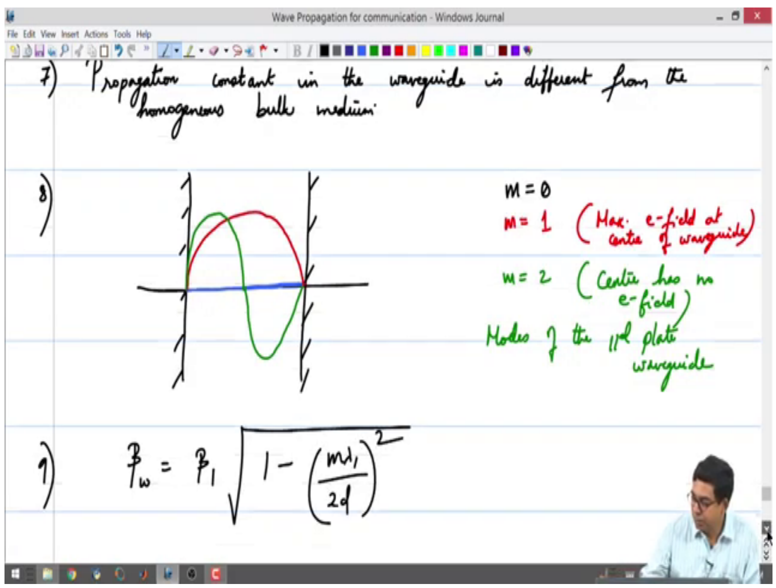

The first of all, the propagation constant is different from the homogeneous bulk medium. It is definitely different β_w is not equal to β_1, it could be if you choose m is equal to 0 ok.

Could be if m is equals to 0 or d is equal to infinity. That is a second plate is not even you know placed all right. So, it reduces to a single you know a plate configuration right ok. Now a what would this mean? right ok. So, by placing two plates in a homogeneous medium, a lot of things changes. First of all, the effective velocity has changed, phase constant has changed. If the phase constant has changed, we know the formula for phase constant which means that the effective wave length inside of this configuration has also changed all right.

The angle of launch previously you had no restriction you could launch wherever you want all right by putting the second plate, in order to satisfy the boundary conditions, you have to launch it at certain angles only. For you to get a traveling wave in the in the x direction and have all the boundary conditions to be satisfied in the z direction right.

So, a lot of restrictions have come into the place right, but what does all this mean? This means that a let us take the parallel plate configuration that we have drawn ok and look at it a little bit more closely ok. Suppose, my choice is a m equal to 0, I just want to draw the total fields ok. I just want to draw the total fields and what I want to do is, I want to go from one plate to another plate and I want to draw the relative total fields. That is, I wanted to say on the left side it is an interface it is going to be 0, on the right side it is going to be 0 in the middle what is going to be happening is what I want to make a plot out of.

So, I can draw a line like this and I want to draw the magnitude of the or the amplitude of the total fields right. So, for m equal to 0 right so, you can go back to the expression for the total fields that we have written m equal to 0 ok would mean that the sin is equal to 0 and consequently you have no electric fields being present at all ok. m equal to 0 is that is a not a very viable option there are no electric fields a present inside the two conductors at all.

So, if I were to draw this all right, I would be having just a flat line like this where that is this edge corresponds to 0 it has to satisfy the boundary condition. Boundary condition is that at the interface between the dielectric and the conductor you have 0 electric fields in between it is governed by that expression for the total field it is 0 ok. No electric fields for m equal to 0. So, there is a constraint ok, first of all.

Let us have a look at the same thing, let us say m is equal to 1 ok. I go back to my expression, I plug in m equal to 1 all right. So, m equal to 1 will make this $\lambda_1/2d$. So, I have β_1z* $\lambda_1/2d$ right. Now what I am trying to plot is with respect to the z direction what is happening to the electric field. So, I will keep all the other parameters constant, vary only the z. To go from one plate to another plate and I will try to plot what is going to be happening.

I know that at the edges the fields are going to be 0, in between I can see that as z increases right, there will be a pattern that is evolving all right. And this pattern is something that we have already seen without m right in the previous class when we tried to draw write a program and a try to the visualize the fields. We saw that there is a standing wave pattern all right in the z direction the nodes are fixed with respect to z direction you will clearly notice a standing wave pattern for the magnitude right.

So, here what you will end up seeing. You will end up seeing an electric field pattern like this at the edges of the dielectric conductor interface you will be having 0 fields corresponding to the boundary conditions. In the middle you will be having a maximum value of the electric field right ok. Now this gives some hope that there is going to be an electric field present within the medium all right.

And we also say that there is a maximum of the electric field present right in between the two conductors. So, the maximum of the electric field is present right at the center of the waveguide ok. So, can you say that ok. Let us now take a different scenario and up the value of m to 2 right. You go back to your total fields expression take the program keep all the things constant all right and try to vary only z ok.

You will notice that for m equal to 2, you will notice that your total electric field would vary like those as you go from one plate to another plate, the electric field will go to a peak then it will have a 0 crossing, then it will go to a negative peak and then it will end on the other plate. If you wanted to plot the magnitude then you would you look like you know two lobes in the positive axis, but here I want to make an emphasis that it does change sin right.

So, m equal to 2 corresponds to this kind of an electric field profile, we noticed that m equal to 0 had no peak, m equal to 1 had 1 peak, m equal to 2 had 2 peaks right. One in the positive and one in the negative x direction. So, also m equal 2 is a case where the center of the waveguide has no electric field ok.

So, we can also go one step further and say that the red color profile that we have drawn is symmetric about the center of the wave guide, that is if I take the waveguide folded into a half like this, the profile is going to be symmetric all right. Where as in the case of m equal to 2. If I were it to fold it is not really symmetric because one is having a peak in the upward direction and the other one is having a peak in the opposite direction right.

So, so, we can also say that this m equal to 1 it is a symmetric mode ok and m equal to 2, not a symmetric mode and asymmetric mode right. So, these are some observations that we can make. You can keep drawing for m equal to 3, m equal to 3, we will have three half cycles of the sinusoid. m equal to 4 will have 4 half cycles of the sinusoid extra. So, the m actually signifies, how many half cycles of the sinusoid you have within the parallel plate configuration. m equal to 0 has 0 half cycles, m equal to 1 has 1 half cycle, m equal to 2 has 2 half cycles all right half cycles is a full wave 2 half cycles is a full wave.

So, it keeps going like this so, the electric field can have these kinds of patterns. But what is interesting is that the change is not continuous that is you can have either m equal to 0, m equal to 1, m equal to 2 extra.

You can have either the electric field having the red color profile or the green color profile for example, in this case what we have drawn. You it does not go systematically from one profile to another profile slowly. You get there is an abrupt change either you have this or you have something else all right that is coming in because of the integer major of m that we are considering over here all right.

So, this means that the e field profiles are kind of discrete ok this is not a continuous change from one profile to another profile all right. And this is also a very important thing all right. Why do we need to know these shapes, because initially in the beginning of the lecture we talked about the waveguide being present between a transmitter and a receiver right? Suppose you did transmit all right.

Let us say that your receiver is going to be of a finite area compared to the waveguide all right, let us say that let us just for the sake of argument. We will make the area to be less than that of the waveguide ok. Normally, it is not so normally the receiver where I mean the detector will be much larger than your waveguide, but just for the sake of argument right. Let us say that I have a detector which is smaller in size all right.

And I want to be able to get the maximum information coming from the transmitter side then the question arises should I position my detector all right. Detector is something that will take the electromagnetic wave and convert it to electric current ok should I position it at the center of the waveguide? Should I position it at first quarter, second quarter, should I position it, where should I position my detector to get the maximum information that becomes a question all right.

What size should the detector be in order for me to get the maximum information? Of course, you have to be the entire length of the waveguide has to be covered all right. So, a lot of things can happen, if you want to a take a detector that smaller when people do this that also you need to know, when do people use tinier detectors in between the waveguide to you know get this.

Suppose the experiment is to figure out the mode profile itself. These are known as modes m equal to 0, m equal to 1 and equal to 2 are all known as modes. In order to know the profile of the modes the only way to know it is if you use a tiny detector and you move it from one wall of the waveguide to the other wall while you are measuring the current ok.

Then only you will notice that oh there is a maximum current at the center. So, this kind of a profile is a m equal to 1 profile you can say all right. If you move the tiny detective going from one side to the other side and you measure a positive peak a negative peak or you may negative peak I do not know how you will you measure. But you will be measuring 2 you know half cycles clearly or three half cycles clearly. Those cases you can say the mode e that it is having is corresponding to m equal to 2 or m equal to 3 extra.

So, these are known as modes of a waveguide all right. And there are no gradual changes between one profile to another profile all right. So, now, we have written β_w is

$$\beta_w = \frac{\beta_1}{\sqrt{1 - \left(\frac{m\lambda_1}{2d}\right)^2}}$$

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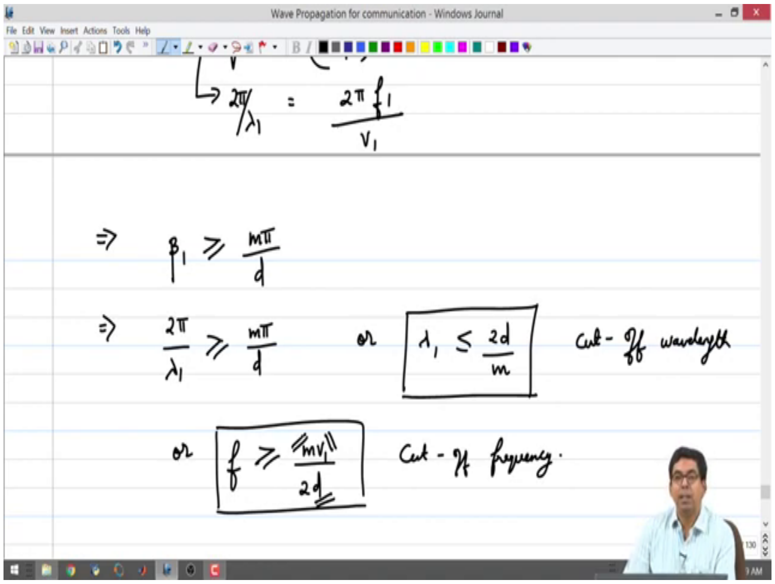

So, you can say this is corresponding to the homogeneous medium is $2\pi/\lambda_1$. If you want to write it in terms of frequency, you can write this as

$$\beta_1 = \frac{2\pi f_1}{v_1}$$

We assume it to be vacuum v1 will be replaced with c ok. Now I am having square root of 1 minus something square. Again the choice of m is limited ok.

If I make m abruptly large, what will happen is, I will end up with a square root of a negative number all right and then I will have j times something multiplied with β_1. And I have already $e^{-j\beta_1}$ something. That j will multiply with just this j and from being a quantity that tells you that there is a traveling wave it will look like an exponential decay ok.

So, there are constraints on how you can choose m, how you can choose d, how you can choose theta, all right. So, just looking at the term inside the square root I can say that for β_w to not become imaginary right.

$$\beta_1 \geq \frac{m\pi}{d}$$

This taking β_1 minus β_1 multiplied by $\lambda m_1/2d$ the whole square right to the root and then we are saying that one term has to be greater than the other term.

Square root has to be greater than equal to 0 so, all right. Equivalently you can also write this down in terms of wavelength right. So, you can say that this is

$$2\pi/\lambda_1 \geq \frac{m\pi}{d}$$

Now there are so many constraints right. Theta has to be chosen carefully, m has to be chosen carefully, d has to be chosen carefully, now lambda also has to be chosen really carefully,

$$\lambda_1 \leq \frac{2d}{m}$$

Also simply because, now homogeneous medium you just place two plates right.

If you did not do this, if you did not place it carefully, we notice that β_w will be become imaginary and there will be an exponential decay of your wave. For the wave to travel one side to the other side you need to meet all these criteria all right.

So, this

$$\lambda_1 \leq \frac{2d}{m}$$

means that, you have taken a waveguide, you have constructed a waveguide with the distance between the plates to be equal to d. You are considering a profile of the electric field corresponding to m and you have to make sure that the

$$\lambda_1 \leq \frac{2d}{m}$$

So, there is an upper wave length of the source that you can launch into a constructed configuration of a parallel plate wave guide all right. So, this is known as cutoff wave length. The term cutoff wave length is usually not very popular right, the signal processing community usually they talk about frequencies ok.

So, you can always convert this to frequency and say that, where v1 is the velocity in the homogeneous medium right. So, this becomes cutoff frequency. What does this mean? For a given choice of d all right, enclosing a medium with velocity v for a choice of the electric field profile m there exist a minimum frequency ok.

$$f \geq \frac{mv_1}{2d}$$

Only above this minimum frequency, you will not have exponential decay of your fields and the fields will survive all right. Below that frequency the fields will not survive in this configuration all right. What does this mean? This means that the wave guide is actually an element that in signal processing you would call it as high pass filter ok.

So, you supply of frequencies the wave guides will transmit only frequencies of above this cutoff frequency right. So, it is high pass filter by nature ok. So, the equivalent of a high pass filter in the case of transmission line or in the case of optics it is going to be a simple parallel plate wave guide right. So, you will have frequencies above this cutoff value being transmitted.

So, if you are aware of the system where you have multiple frequencies coming into the picture and you want to be make sure that you get only frequencies above something you want to say remove noise extra. You could construct something like this and make sure that only a signal is going through the assumption there is signal frequencies higher than your noise frequency right which is not the always the case, but it is a general assumption we can make ok.

So, now, we see that by placing a second plate in that medium you have imposed too many restrictions. Well, the consequence is that you will have electromagnetic wave going from one side to the other side of this configuration and also bent waves to anything with this configuration, but it does have some constraints and the constraints are you have to choose the frequency carefully, you have to choose the velocity, a you have to choose the a distance between the two plates carefully, you have to also choose the angle of launch carefully, you have to choose the profile of the electric field that you want and all of them have to be chosen carefully, then you will get a transmission. And inherently the waveguide is going to be a high pass filter ok. So, we will stop here. We will continue in the next class about this wave length.