

Transmission lines and electromagnetic waves
Prof. Ananth Krishnan
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture – 25
Octave Simulation of Perpendicular Polarisation - I

Ok, we will get started all right. So, I think today what we are going to be starting is to look at ways of simulating the perpendicular and parallel polarization structures ok. We have done only 1 d propagation in space for the transmission lines and for the plane waves right. So, today we are going to start with a slightly more complicated polarization setup, that is, we have already seen perpendicular and parallel polarization. So, I will be going over the perpendicular polarization in full length from start to finish for parallel polarization, I hope that you will be able to make a program on your own right.

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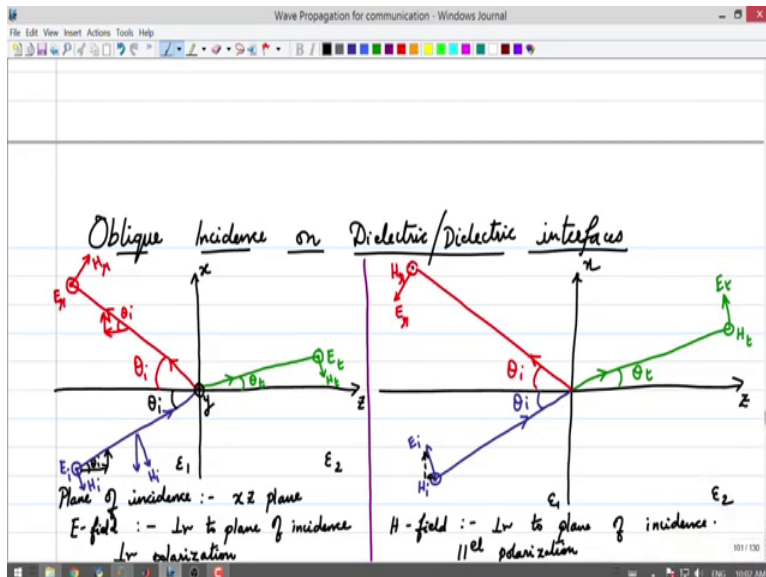
Perpendicular polarization (simulation setup):-

1) H_x, E_y, H_z xz plane \rightarrow propagation
 k_x, k_z

2) $\nabla \times \underline{H} = \epsilon \frac{\partial \underline{E}}{\partial t}$

So, I will begin. So, I will go back to the schematic that we have drawn before.

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So, we had an xz plane all right, and the y axis was pointing out of that plane, the electric field was having a component in the y direction and the magnetic field was having x and z components ok. The direction of propagation is in the xz plane, the y direction there is no propagation. These are the basic things that we are seeing from the oblique incidence on dielectric dielectric interfaces specifically for perpendicular polarization ok.

So, now we are going to take the same thing and we are going to try to arrive at some equations that we can use in octave to start modeling these concepts ok. Before beginning, let us make some small notes over here all right. So, I am having H_x , E_y , H_z ok. These are the 3 components that we have for that particular polarization ok. Other than that, the direction of propagation is in the xz plane ok. This means that your \widehat{k}_x under z component ok.

So, the other way that we can look at it is when we are talking about modeling, usually we have to start with the curl equations ok. So, previously we have done for E_x , H_y , K_z all right now, it is a little bit more involved that is all right. So, I will write down the first curl equation that we are going to write in different forms ok.

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LHS $\nabla \times \underline{H} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial z} \\ H_x & 0 & H_z \end{vmatrix}$

$= \hat{i}(0) - \hat{j}\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right) + \hat{k}(0)$

$= \hat{j}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)$

So, we begin by expanding the left hand side. So, I have i, j and k ok. Now, since the propagation is only in the xz plane $\frac{\partial}{\partial x}$, this becomes 0 and $\frac{\partial}{\partial z}$ ok, there is no K y. So, that means that you do not have $\frac{\partial}{\partial y}$ ok. And H is having x and z components. So, I will write down Hx, 0, Hz ok.

$$\begin{aligned} \nabla \times H &= |\hat{i} \hat{j} \hat{k} \frac{\partial}{\partial x} \quad 0 \quad \frac{\partial}{\partial z} \quad H_x \quad 0 \quad H_z| \\ &= \hat{i}(0) - \hat{j}\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right) + \hat{k}(0) \\ &= \hat{j}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right) \end{aligned}$$

So, now that I have a left hand side I am able to notice that, if I use this configuration with Hx and Hz for a plane of propagation xz plane. I am getting del cross H to have only a j vector component, which should tell you that the right hand side should have only a j vector component. So, I already know that my right hand side is epsilon $\frac{\partial E}{\partial t}$, E has only a y component right. So, that will be its matching right.

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RHS

$$\epsilon \frac{\partial E}{\partial t} = \epsilon \frac{\partial E_y}{\partial t} \hat{j}$$
$$\epsilon \frac{\partial E_y}{\partial t} \cong \epsilon \frac{E_y(x, z, t + \Delta t) - E_y(x, z, t)}{\Delta t}$$

LHS in difference form,

$$\frac{\partial H_x}{\partial z} \cong \frac{H_x(x, z, t) - H_x(x, z - \Delta z, t)}{\Delta z}$$

So, let us go ahead and write down the right hand side also right ok ok. So, I have

$$\epsilon \frac{\partial E}{\partial t} = \epsilon \frac{\partial E_y}{\partial t} \hat{j}$$

Now, having expanded this we are going to start writing down the finite difference form all right and identifying what is the unknown, bringing it to the left hand side here we know that we have a quantity with the time derivative. So, it should be abundantly clear that the time derivative means that the unknown quantity for this equation is the value of that variable at the next instant of time. So, you have to use forward difference for time all right.

So, we will start writing down, writing this down the difference form

$$\epsilon \frac{\partial E_y}{\partial t} \approx \epsilon \frac{E_y(x, z, t + \Delta t) - E_y(x, z, t)}{\Delta t}$$

So, I have

$$\frac{\partial H_x}{\partial z} \approx \epsilon \frac{H_x(x, z, t) - H_x(x, z - \Delta z, t)}{\Delta z}$$

So, this becomes H_x , it is a function of x , z and time t . So, now I need to make a spatial difference of the magnetic field right. So, I will simply use say forward difference for one kind, backward difference for the other kind, it is the same thing that we have been doing in the past programs. If you want to avoid problems related to only forward or backward difference

For example, in the prior programs, if you switch the order in which you take the difference or if you make both of them to be forward difference extra. You would have had instability issues. If you do not want to deal with them, you can always convert all of them to central difference. But, since we already have a base program before, I am just moving on the same manner ok.

$$\frac{\partial H_x}{\partial z} \approx \epsilon \frac{H_x(x, z, t) - H_x(x, z - \Delta z, t)}{\Delta z}$$

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LHS in difference form,

(i) $\frac{\partial H_x}{\partial z} \approx \frac{H_x(x, z, t) - H_x(x, z - \Delta z, t)}{\Delta z}$

(ii) $\frac{\partial H_z}{\partial x} \approx \frac{H_z(x, z, t) - H_z(x - \Delta x, z, t)}{\Delta x}$

LHS = RHS & Bringing unknown quantity to the left,

The other term, that we have is

$$\frac{\partial H_z}{\partial x} \approx \epsilon \frac{H_z(x, z, t) - H_z(x - \Delta x, z, t)}{\Delta x}$$

This would be the difference form. So, we have to add I mean we have to take the left hand side, right hand side, equate, push one unknown quantity to the left side and have all the other quantities to the right side so ok. So, in this case, I will just write down the expression that we will get alright ok.

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$$\underline{E_y(x, z, t + \Delta t)} = E_y(x, z, t) + \frac{\Delta t}{\epsilon \Delta} \left[H_x(x, z, t) - H_x(x, z - \Delta z, t) - H_z(x, z, t) + H_z(x - \Delta x, z, t) \right]$$

So, I am having

$$\begin{aligned} E_y(x, z, t + \Delta t) &= E_y(x, z, t) + \frac{\Delta t}{\epsilon \Delta} [H_x(x, z, t) - H_x(x, z - \Delta z, t) - H_z(x, z, t) \\ &\quad + H_z(x - \Delta x, z, t)] \end{aligned}$$

E_y as a function of x and z and the future value of E_y . It is going to depend on the current value of E_y plus Δt divided by ϵ . Let us see now all right, what we can do. Now, if you look at the denominator for $\frac{\partial H_x}{\partial z}$ and $\frac{\partial H_z}{\partial x}$ ok. The first a bullet point here has the denominator of Δz and the other one has a denominator of Δx all right, and you need to take a difference between the two quantities. It will become easier, if the denominators are identical ok.

So, this just means that given a xz plane, you will discretize it in such a way that, Δx is going to be equal to Δz . So, you will divide the region into a smaller number of squares all right. So, we can always say let Δz be equal to Δx ok, and we can make that as some variable called just delta ok.

So, in our program and in the way we are approaching delta means, both Δx and Δz ok. So, then it becomes easier to take some common factors outside. So, I can have delta t divided by epsilon delta ok. And then, I can write down this term inside of that ok. So, I have

$$\begin{aligned}
 E_y(x, z, t + \Delta t) &= E_y(x, z, t) + \frac{\Delta t}{\epsilon \Delta} [H_x(x, z, t) - H_x(x, z - \Delta z, t) - H_z(x, z, t) \\
 &\quad + H_z(x - \Delta x, z, t)]
 \end{aligned}$$

So, I can write down the difference form of that here right. So, I will be having H_x of ok. So, to get the electric field at the next instant of time, I will need to know the present value of the electric field all right.

And magnetic fields at different positions at the same instant of time and then, I have to code this expression in the program for managing this particular curl equation ok. Now the other thing that we have to do is, go to the other curl equation that we have right now.

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$$2) \quad \nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

$$= -\mu \frac{\partial \underline{H}}{\partial t}$$

LHS

$$\nabla \times \underline{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix}$$

$$= \hat{i} \left(-\frac{\partial E_y}{\partial z} \right) - \hat{j} (0) + \hat{k} \left(\frac{\partial E_y}{\partial x} \right)$$

So, we have the other curl equation

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

So, here we are having ideal dielectric. So, we are not considering the j ok or the displacement current in the case of the first curl equation and here we are just

$$\text{saying } \nabla \times E = -\frac{\partial B}{\partial t}$$

ok.

So, you could also write this term as

$$= -\mu \frac{\partial H}{\partial t}$$

Once again, we have to expand the left hand side $\frac{\partial}{\partial x}$. Since, there is no propagation in the y direction that becomes 0. And I have 0, E_y , 0 because, electric field has only one component right. Now, I have to write down the determinant so I am having

$$\nabla \times E = |\hat{i} \hat{j} \hat{k} \quad \frac{\partial}{\partial x} \quad 0 \quad \frac{\partial}{\partial z} \quad 0 \quad E_y \quad H \quad 0_z|$$

$$= \hat{i} \left(-\frac{\partial E_y}{\partial z} \right) - \hat{j} (0) + \hat{k} \left(\frac{\partial E_y}{\partial x} \right)$$

Now clearly it is a I mean it is having 2 a unit vectors right i and k, should give you an indication that the right hand side which is a time derivative should also have i and k that means, you are talking about Hx and Hz. So, we are on the correct path right.

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RHS

$$-\mu \frac{\partial H}{\partial t} = -\mu \frac{\partial H_x}{\partial t} \hat{i} - \mu \frac{\partial H_z}{\partial t} \hat{k}$$

LHS = RHS & Equating i components,

$$-\frac{\partial E_y}{\partial z} = -\mu \frac{\partial H_x}{\partial t}$$

$$\Rightarrow \frac{H_x(x, z, t + \Delta t) - H_x(x, z, t)}{\Delta t} = \frac{1}{\mu \Delta} \left\{ \begin{array}{l} E_y(x, z + \Delta z, t) \\ -E_y(x, z, t) \end{array} \right\}$$

So, once again right hand side, now, this can also be written as

$$-\mu \frac{\partial H}{\partial t} = -\mu \frac{\partial H_x}{\partial t} \hat{i} - \mu \frac{\partial H_z}{\partial t} \hat{k}$$

Once you do that the LHS and RHS all you need to do is, you need to take only the unit vector i equate the coefficients and then take k vector equate the coefficients ok. So, the first thing you can do is LHS equal to RHS and equating only the i component all right, so, you will have $-\frac{\partial E_y}{\partial z} = -\mu \frac{\partial H_x}{\partial t}$ ok $-\frac{\partial E_y}{\partial z} = -\mu \frac{\partial H_x}{\partial t}$ all right.

So, I can always try to figure out what is unknown over here. I have a time derivative on the right hand side which means that I have to represent that as a forward time

derivative. So, this I will be having at H_x which is a function of x , z and the future right ok.

$$\frac{H_x(x, z, t + \Delta t) - H_x(x, z, t)}{\Delta t} = \frac{1}{\mu \Delta} \{E_y(x, z + \Delta z, t) - E_y(x, z, t)\}$$

And then, we use a difference for the electric field right ok. Now, the unknown quantity is the first quantity over here, all the remaining terms go to the right side and that gives you a second equation for finding out H_x at the future instant. So, the objective is you had some H_x , E_y , H_z . You have to find out the value of each of these components at the next instant of time all right, provided you have the values of all the other components of the current instant of time this is the objective. So, you have to find out updated equations for all the three quantities that you are interested in, H_x , E_y , H_z .

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$$\frac{H_x(x, z, t + \Delta t) - H_x(x, z, t)}{\Delta t} = \frac{1}{\mu \Delta} \{E_y(x, z + \Delta z, t) - E_y(x, z, t)\}$$

LHS = RHS ← Equating k components,

$$\frac{\partial E_y}{\partial x} = -\mu \frac{\partial H_z}{\partial t}$$

$$H_z(x, z, t + \Delta t) = H_z(x, z, t) - \frac{1}{\mu \Delta t} \{ \dots \}$$

So, we have done E_y . Now, we have H_x , the only thing remaining is H_z ok. So, you can always say, equate the left and the right hand side and equate k components right, you will end up with another equation ok. So, you have

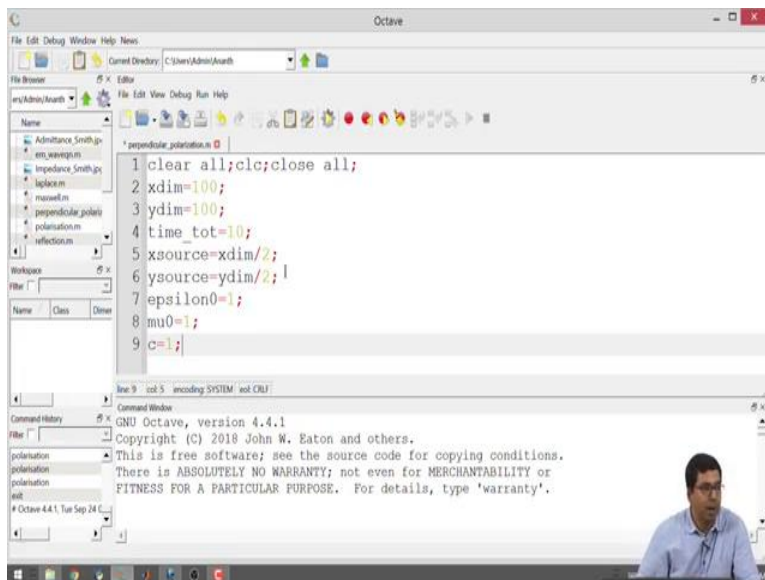
$$\frac{\partial E_y}{\partial x} = -\mu \frac{\partial H_z}{\partial t}$$

is not very different from the previous equation that we have written just there is a minus sign coming into the picture but, other than that it is a very clear the unknown quantity is going to be

$$H_x(x, z, t + \Delta t) = H_x(x, z, t) - \frac{1}{\mu\Delta} \{E_y(x, z + \Delta z, t) - E_y(x, z, t)\}$$

So, I will be having some expression for finding out the electric field derivative with respect to x all right, that is it all right. So, now you have 3 equations. Each of them is telling you what the future value of that particular component is going to be right. So, we stop with this. Now, we go to the program part all right. We fire up octave and we try to write this program now all right and then, we will start fiddling around with the program to understand a few more concepts ok ok.

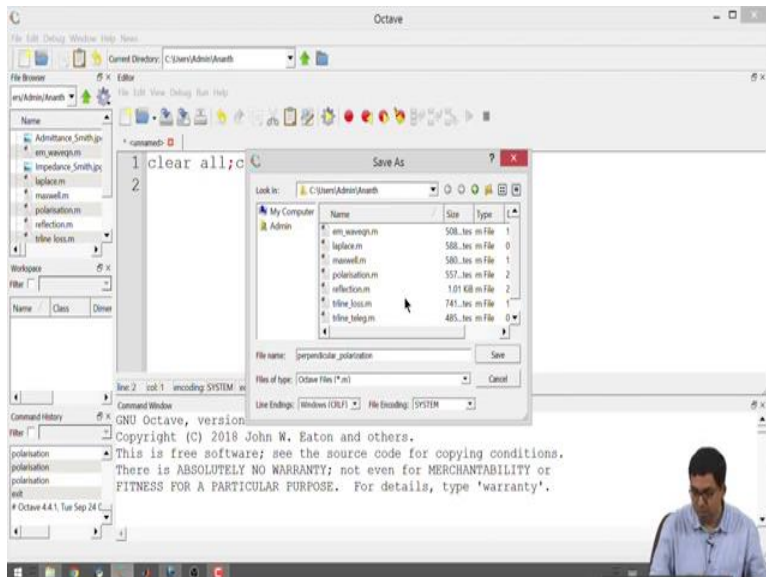
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```
1 clear all;clc;close all;
2 xdim=100;
3 ydim=100;
4 time_tot=10;
5 xsource=xdim/2;
6 ysource=ydim/2; l
7 epsilon0=1;
8 mu0=1;
9 c=1;
```

GNU Octave, version 4.4.1
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FITNESS FOR A PARTICULAR PURPOSE. For details, type 'warranty'.

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So, I want to start with some of the dimensions for the space that we are considering. Let us consider that you have 100 units in the x and 100 units in the y. So, it is a square region. You can consider this to be having 100 Δx and 100 Δz all right. So, you can have x dimension is 100 ok ok. So, I have a space which is 100 by 100 ok.

And I want to start with some total time for which I want to write this program. I will start with some small values to check if my programs work correctly and then, I will improve the program as and when we go ok. So, in order to solve the 2 curl equations, there is one additional thing that you will need: you will need to define the position of a source of this electromagnetic wave all right.

In the case of a transmission line, you would have used voltage at a specific place that was equal to 1 or 2 extra. In the case of this 1 d propagation, we would have used a source of electric field at a particular position needed to have a source ok. So now, what I am going to do is, I am going to make it a little bit more generic. So, I am going to say the x position of the source right so, I am saying that it will be right in the middle.

In future, if I want to change the position of the source, I will alter these variables. Instead I want to go to the main part of the program. I will just alter these variables. So, here x source corresponds to a single point in the x direction and y source

corresponds to a single point in the y direction. So, ok the way we are writing y is actually the x in our perpendicular polarization diagram and this axis was z but, I think that visualization is ok.

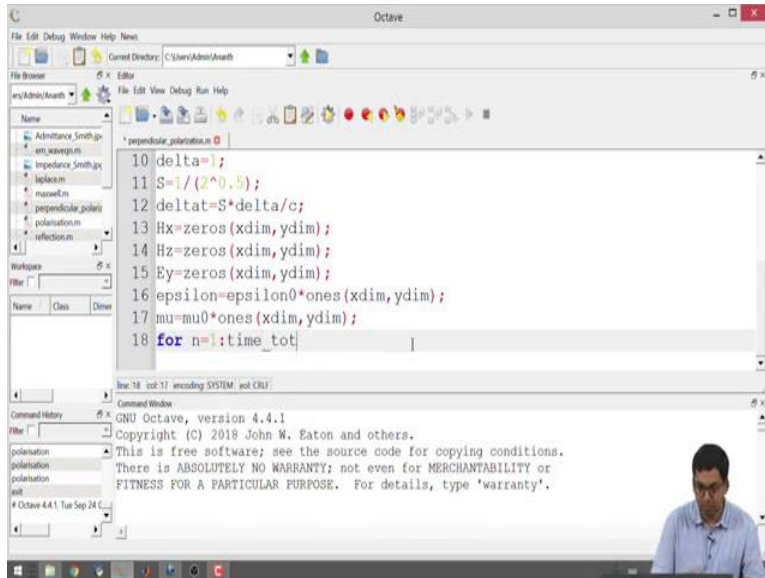
So, we are drawing a we are trying to make a source at a single point right. So, we have to see what is going to happen and how we can make some inferences from there and then slowly we will begin to expand to see what is a plane wave and what is a wavefront extra right. So, one of the things that we have to do is define some values for permittivity and permeability. Since I do not like to use the absolute values of the permittivity because it is very small it tends to the negative 12 may get approximated as 0 and I may not get any output at all.

So, I like to use only the relative permittivity. In most of the practical scenarios, I I see that people use only the refractive index ok which is $\frac{1}{\sqrt{\mu_r \epsilon_r}}$ all right. So, whatever phenomenon you want to model, you can use the vacuum permittivity to be equal to 1. Vacuum velocity to be equal to 1 and then for the other material, you use the corresponding ϵ_r and μ_r . You may want to say for example, model the change in velocity, change in wavelength extra alright.

So, most of the cases this is sufficient but, if you want to go further and get a real accurate value for the velocity extra you may have to use no $8.854 * 10^{-12}$ all right and the permeability has to become $4.5 * 10^{-7}$ right. I do not like using very small numbers so I will use only the value 1 right for vacuum. Correspondingly, what this means is the velocity right in vacuum is made as some 1 right.

So, whatever velocity we are getting we can normalize it to the vacuum velocity ok. I want to divide the space into a number of Δx and Δz . So, I have a 100 by 100 space, I want to divide it into some smaller regions and we have called that variable as delta.

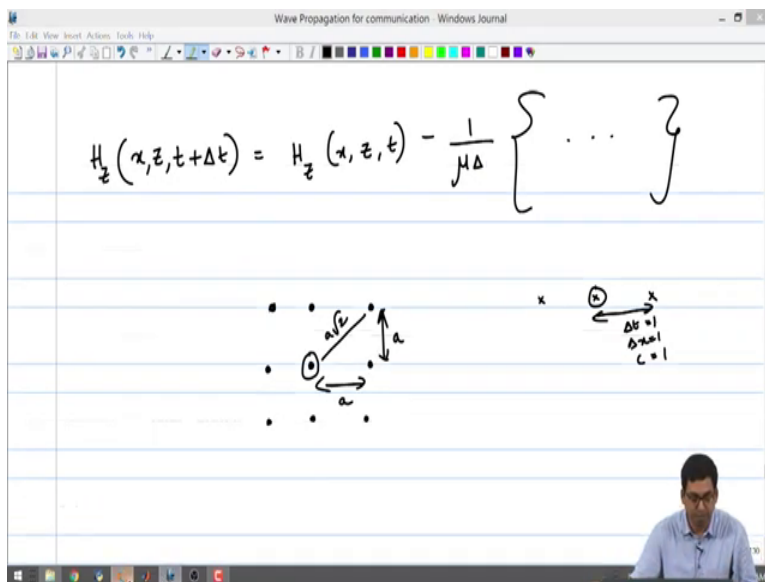
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```
10 delta=1;
11 S=1/(2^0.5);
12 deltat=S*delta/c;
13 Hx=zeros(xdim,ydim);
14 Hz=zeros(xdim,ydim);
15 Ey=zeros(xdim,ydim);
16 epsilon=epsilon0*ones(xdim,ydim);
17 mu=mu0*ones(xdim,ydim);
18 for n=1:time_tot
```

So, I just want to make it as delta is equal to 1. Now, here there is a catch ok delta t, previously, when we are done all the simulations we had also used delta t equal to 1 ok. We had used delta t equal to 1 in our previous programs. Here we have to be a little bit careful all right.

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$$H_z(x, z, t + \Delta t) = H_z(x, z, t) - \frac{1}{\mu \Delta} \left\{ \dots \right\}$$

The whiteboard also contains two diagrams. The left diagram shows a grid of points with a central point labeled '0'. A horizontal arrow labeled 'a' points to the right, and a vertical arrow labeled 'a' points upwards. A diagonal arrow labeled 'a/c' points from the center to the top-right point. The right diagram shows a coordinate system with x and z axes. A circular arrow indicates a direction of rotation. Below the axes, the parameters are listed: $\Delta t = 1$, $\Delta x = 1$, and $c = 1$.

Because, delta t equal to 1 can lead to instabilities in the program all right. I would not go into a large amount of details for this because there is an advanced course

for you to take computational electromagnetics all right but, the premise of the argument is like this ok. Suppose, I take a few grid points in my space, suppose, I take a few grid points in my space and let us say that I put a source over here all right in this point and right.

Now, at the next instant of time the idea is it has to go to the next grid point so that you can see the propagation in your simulation all right. So, what essentially that Maxwell's equation is saying is it will move one grid space in one unit time that you have taken then only you will be able to see what is going on all right. So, here our velocity is equal to 1 all right. So, it should go one grid point all right in one time step.

Previously in the case, a what whatever we have done before in the case of transmission line or in the case of 1 d propagation the grid points where along a line ok. So, for this electric field to move to the next point all it all you have to do was you know a model it in such a way that your Δt is 1, Δx is equal to 1 and c is equal to 1. So, it will move to the next grid point in one unit time step ok

Now, in this case you have a two dimensional space. So, a little bit more care has to be taken ok. Suppose you have a point source, the point source usually gives off circular wave fronts or circular wave fronts. So, if you have a point source of an electromagnetic wave it is going to give circular wave fronts ok. Then, you have to think about what all can happen all right, the electric field can go from here to the next point to the right, it can go to the next point to the left, it can go to the point on the top, bottom extra.

But, it can also go in these directions all right, and can also go in these directions. Now, how you define the nearest neighbor is a complicated question to ask ok. Now, suppose I want this program to remain stable ok the premise is you have to consider this region all right. The velocity will be maximum according to the program in this region because in one unit time step you want the wave front to go from here to there to capture this direction propagation ok.

Otherwise you would not capture it. In one time step, the wave front all right has to go in this direction and it travels a longer distance. So, if your side is going to be marked by a this is also a, this distance is a root 2. It travels a longer distance at the same Δt ok. So, you have to consider the highest velocity in your system possible and then figure out what your Δt would be like right.

So, in this case we have to consider this to be the maximum velocity because in Δt travels a $\sqrt{2}$ distance but, in the other directions all right for a time Δt travels a distance only of a . So, in these directions all right in the Cartesian directions x and y right your velocity is only divided by Δt . But in the diagonal direction, it is a $\sqrt{2}$ divided by Δt . Those things come into the picture. More details will be given to you in the higher level courses ok.

And, if you are already you should have a feeling if you are going to go for a three dimensional space, then you have to consider the body diagonal alright because the velocity along that will be the highest. So, depending upon the dimensionality of the problem, the choice of Δx and Δt should be taken care of right. But, it is not very very difficult in this case right. We will define a Δt to be equal to say Δx divided by c and we have to factor this $\sqrt{2}$ somewhere. So, I am going to introduce factor $1/\sqrt{2}$. This is the only catch in this entire program all right.

And, I would not go into more details about this because, this is a topic that is dealt very extensively in the other course computational electromagnetics where you will be studying about core and friedrich lei stability factor all right. But for now, I think this is good enough right you have to account for the nearest neighbor that is all ok. Now, you can go on with the remaining parts of the program. H_x is now going to be two dimensional. It is going to have some zeros in all the space that we are considering right. H_z is also going to look like zeros everywhere.

E_y starting it off with zeros ok. And on top of that, I want to be able to use this program to model dielectric dielectric interfaces just like what we have done in the theoretical parts, which means that, when I am talking about a space grid and a I I want to talk about dielectric dielectric interface, I want to have the power in the program to change the permittivity at different places ok.

So, ϵ_0 should be actually a matrix. We have put ϵ_0 equal to 1 a μ_0 equal to 1 prior in the program but, what I want to do is I want to create a matrix right. So, what I am doing here is ϵ_0 multiplied by ones of x_{dim} comma y_{dim} . So, ϵ_0 becomes a matrix with dimension x_{dim} by y_{dim} and it has ones everywhere. We are starting with pure vacuum, later on, if I want to change permittivity only on the right half or only on the top, only on the bottom all right.

If I want to create multiple interfaces, I can always manipulate this epsilon ok. One actually creates a matrix with dimensions x_{dim} , y_{dim} with all the elements being equal to 1 ok. Remember that you cannot put an epsilon equal to 0 because, 1 by a square root epsilon naught mu naught is the velocity it will become infinite the whole theory will fail right.

Similarly, you can also have mu is equal to mu0 times x_{dim} comma y_{dim} ok. Now, all we are left to do is start to write the loop. So, I am going to have a main loop. Is there any issue? Ok. This part becomes if there is an issue?

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```

17 mu=mu0*ones(xdim,ydim);
18 for n=:time_tot
19   for i=:xdim-1
20     for j=:ydim-1
21       Hx(i,j)=Hx(i,j)+(deltat/(delta*mu(i,j)))*(Ey(i,j))-Ey(i,j));
22       Hz(i,j)=Hz(i,j)-(deltat/(delta*mu(i,j)))*(Ey(i+1,j))-Ey(i,j));
23     endfor
24   endfor
25 endfor
26 endfor
  
```

Student: (Refer Time: 34:31) x_{dim} and z_{dim} .

Yeah You can change it to z dim. Originally, I had written it in the x and y direction the way I see on the screen but, actually the x in the in our derivation this direction was positive z this direction was a positive x. So, we can always change it. It is just a detail, nothing changes. I think after writing the program, we will have control of it and we can change. I think that is easy, right ?

So, now I have a time loop and then inside this I need to have some updates for all points in space ok. So, I am going to have it equal to I will just make it 1 to x_{dim} minus 1. For j equal to 1 to ok. I am going to take the expressions that we have got

in the finite difference form. All we need to do is we need to update the magnetic field and then we need to update the electric field.

So, I will start with a magnetic field update. So, I will just say

$$H_x(i, j) = H_x(i, j) + (\text{deltat}/(\text{delta} * \mu(i, j))) * E_y(i, j + 1) - E_y(i, j)$$

This is what an update equation for Hx would look like right. It is a lengthy expression but, if you are able to work it out then, there is no issue at all. It is just plugging the expression over here right.

Then we have to go for

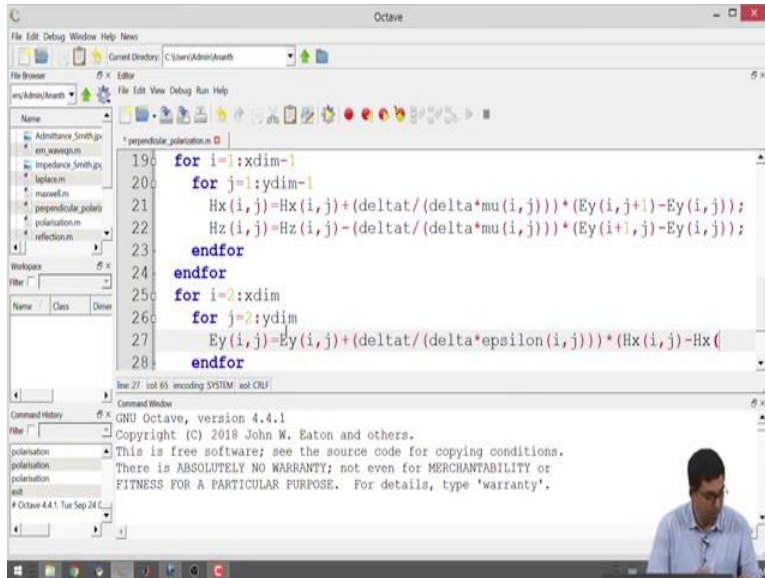
$$H_z(i, j) = H_z(i, j) - (\text{deltat}/(\text{delta} * \mu(i, j))) * E_y(i + 1, j) - E_y(i, j)$$

So, I will just copy paste this part. So, it is a forward difference for $\frac{\partial E_y}{\partial x}$ right. So, in the first line I have j plus 1 and j, here I have i plus 1 and i right, that is the only difference of course, Hx and Hz ok. Now this loop will update the magnetic field all right at the current instant of time for all points in the space that we are considering right.

So, once we have done that because the way we have written our prior codes was having forward difference in part, backward difference in another part extra. We have to write another space loop because, here you see the for example I have $\frac{\partial E_y}{\partial z}$ goes to j plus 1 and j. So, the highest spatial position is j plus 1. So, that is why we are having xdim minus 1 and ydim minus 1. So, that you do not go array index out of bounds all right.

But in the other part, we will be having a j minus 1 or i minus 1. So, you do not want to go out of bounds. So, you need to write another loop right.

(Refer Slide Time: 38:56)

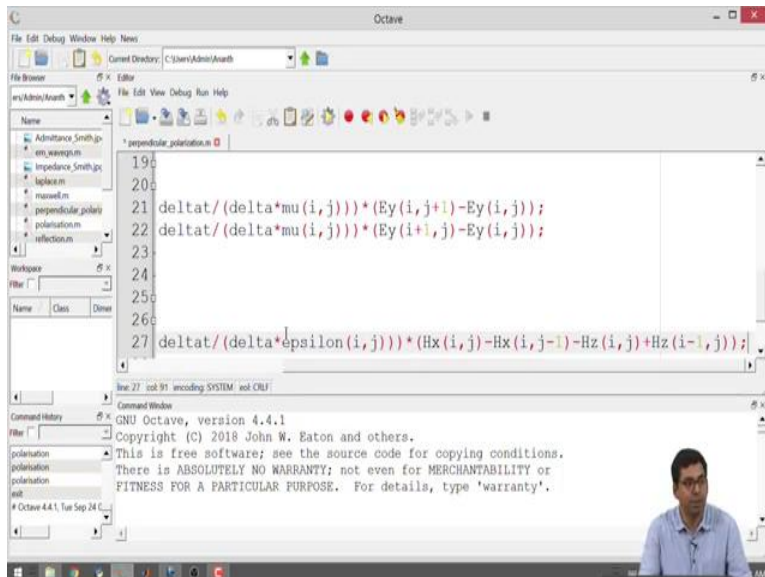


```
190 for i=1:xdim-1
200   for j=1:ydim-1
21     Hx(i,j)=Hx(i,j)+(deltat/(delta*mu(i,j)))*(Ey(i,j+1)-Ey(i,j));
22     Hz(i,j)=Hz(i,j)-(deltat/(delta*mu(i,j)))*(Ey(i+1,j)-Ey(i,j));
23   endfor
24 endfor
250 for i=2:xdim
260   for j=2:ydim
27     Ey(i,j)=Ey(i,j)+(deltat/(delta*epsilon(i,j)))*(Hx(i,j)-Hx(i-1,j))-
28     Hz(i,j)+Hz(i-1,j));
```

So, I will have it equal to 2 to xdim. For j equal to 2 to ydim right and then, I will write down and update the equation for Ey ok.

$$Ey(i,j) = Ey(i,j) - (deltat/(delta * epsilon(i,j))) * (Hx(i,j) - Hx(i,j - 1) - Hz(i,j) + Hz(i - 1,j)).$$

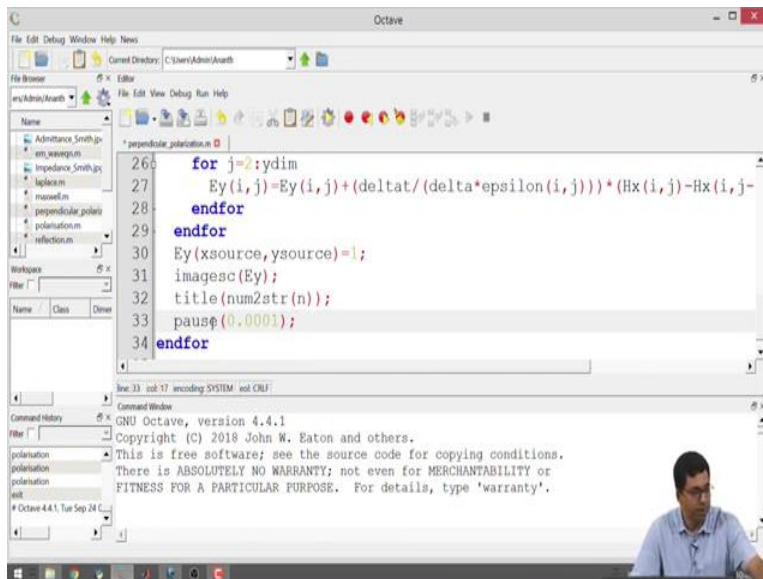
(Refer Slide Time: 40:03)



```
21 deltat/(delta*mu(i,j))*(Ey(i,j+1)-Ey(i,j));
22 deltat/(delta*mu(i,j))*(Ey(i+1,j)-Ey(i,j));
27 deltat/(delta*epsilon(i,j))*(Hx(i,j)-Hx(i,j-1)-Hz(i,j)+Hz(i-1,j));
```

But, there are no tricks or anything. I mean whatever you have written with your derivation is the exact same thing written in the different form. But, those are the tough parts, that is it I mean all your update equations are done for H_x , H_z and E_y ok. And once you are able to do this, that means, that you have written the curl equations accurately all you need to do is now define the source value, define some plotting commands and you are done with the program alright.

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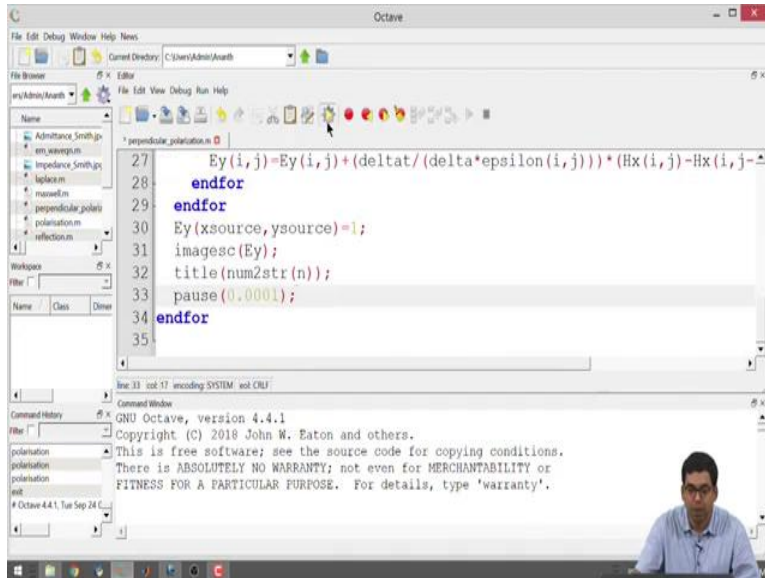
```
266 for j=2:ydlim
27   Ey(i,j)=Ey(i,j)+(deltat/(delta*epsilon(i,j)))*(Hx(i,j)-Hx(i,j-
28   endfor
29 endfor
30 Ey(xsource,ysource)=1;
31 imagesc(Ey);
32 title(num2str(n));
33 pause(0.0001);
34 endfor
```

GNU Octave, version 4.4.1
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There is ABSOLUTELY NO WARRANTY; not even for MERCHANTABILITY or
FITNESS FOR A PARTICULAR PURPOSE. For details, type 'warranty'.

So, let us go ahead and do that. So, I want to have a source at x source comma y source and I am just going to make it 1. So, I am going to have a point source of 1 volt per meter at the center of the simulation region right. And I want to make some plot all right. So, I will just do `imagesc`. I want to make a plot of E_y first right ok, and I want to give it some title ok The title that I am putting here is the current time instant for which it is all. So, that I can estimate how slow or how fast it is extra right.

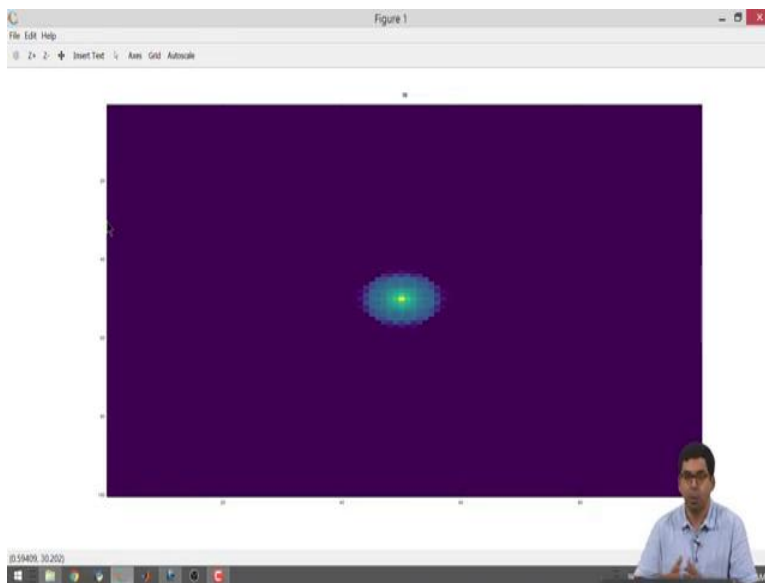
And I know that, if I have this plot commands within the for loop before it plots, the computer will try to calculate the next a you know arithmetic calculations so you need to have a small pause command over here ok. So, almost everything is done in this program and the only thing that we need to now do is run all right ok.

(Refer Slide Time: 42:48)



So, I am going to hitting run right ok now I see that I have a point source at the center even though our grid is I mean it is supposed to be a square region because the monitor is is you know having space in a rectangular manner it is plotting as a rectangular so, we will fix that right.

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(Refer Slide Time: 43:13)

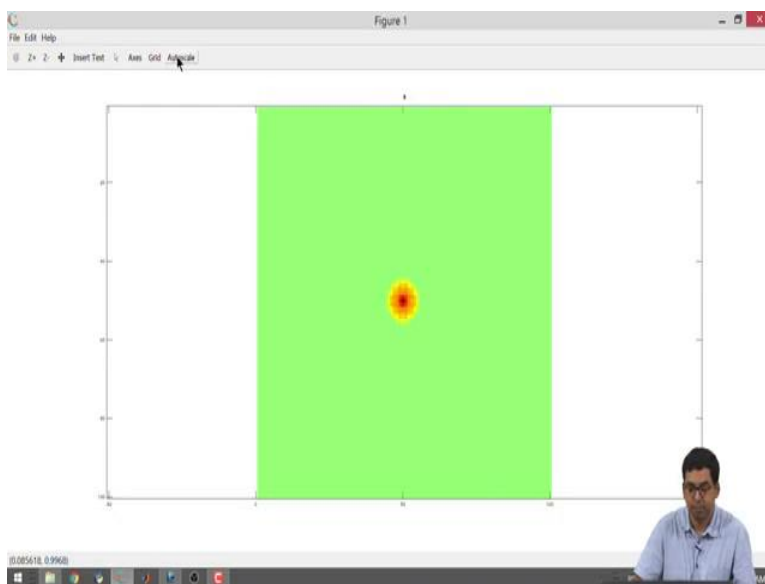
```

Octave
File Edit Debug Window Help News
Current Directory: C:\Users\Admin\Asarth
File Browser
Name
perpendicular_polarization.m
27 Ey(i,j)=Ey(i,j)+(deltat/(delta*epsilon(i,j)))*(Hx(i,j)-Hx(i,j-1)
28   endfor
29   endfor
30 Ey(xsource,ysource)=1;
31 imagesc(Ey,[-1,1]);colormap('jet');axis('equal');
32 title(num2str(n));
33 pause(0.0001);
34   endfor
35
Command Window
line: 31 col: 20 encoding: SYSTEM col: CR LF
# Octave 4.4.1, Tue Sep 24 C
perpendicular_polarization
perpendicular_polarization

```

So, I do not like the color it is showing right, so I use a jet and I also want to be between something and something. So, just modifying the images c command to go between minus 1 and plus 1 small details that is it, even if you do not have these parts it is fine.

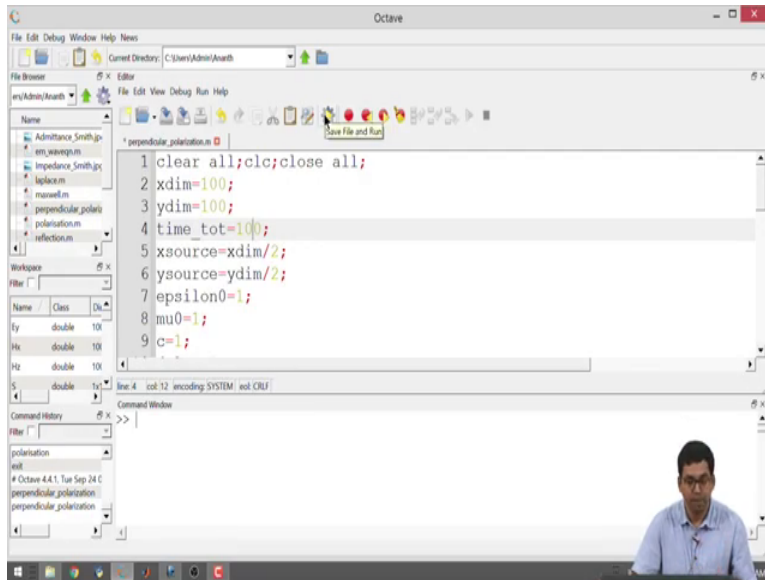
(Refer Slide Time: 43:44)



Once again the axis is 0 to 100 here, 0 to 100 here, because of the way the image commands are used. We can always use another command axis equal to make it look like a regular, graph. So, 10 times a step, I had a point source of electric field all right and it is giving out circular wave fronts ok. On some of your computers this

may be very slow ok. So, we will fix that in the next class to get to a little better option.

(Refer Slide Time: 44:20)

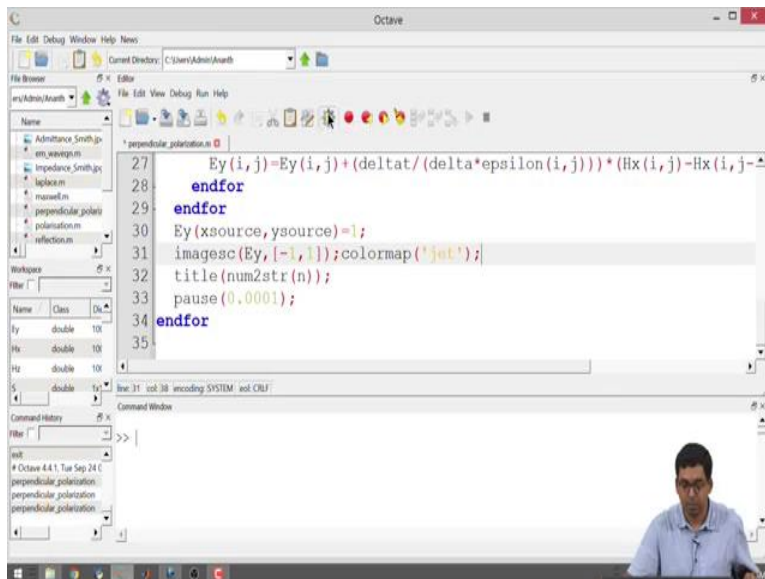


The screenshot shows the Octave software interface. The main editor window contains the following code:

```
1 clear all;clc;close all;
2 xdim=100;
3 ydim=100;
4 time_tot=100;
5 xsource=xdim/2;
6 ysource=ydim/2;
7 epsilon0=1;
8 mu0=1;
9 c=1;
```

The interface also shows a file browser on the left with a tree view of files, a workspace window with a table of variables, and a command window at the bottom.

(Refer Slide Time: 44:44)



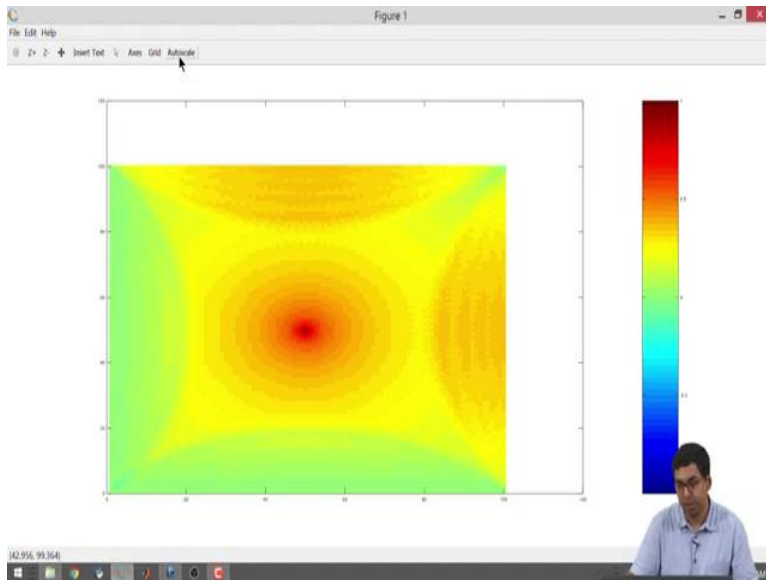
The screenshot shows the Octave software interface with the code updated to include a loop and plotting:

```
27 Ey(i,j)=Ey(i,j)+(deltat/(delta*epsilon(i,j)))*(Hx(i,j)-Hx(i,j-1));
28 endfor
29 endfor
30 Ey(xsource,ysource)=1;
31 imagesc(Ey,[-1,1]);colormap('jet');
32 title(num2str(n));
33 pause(0.0001);
34 endfor
35
```

The interface also shows the same file browser and workspace window as in the previous screenshot.

But for now, I will just increase the total amount of time to say 100 right and I will run this ok oh god hm ok.

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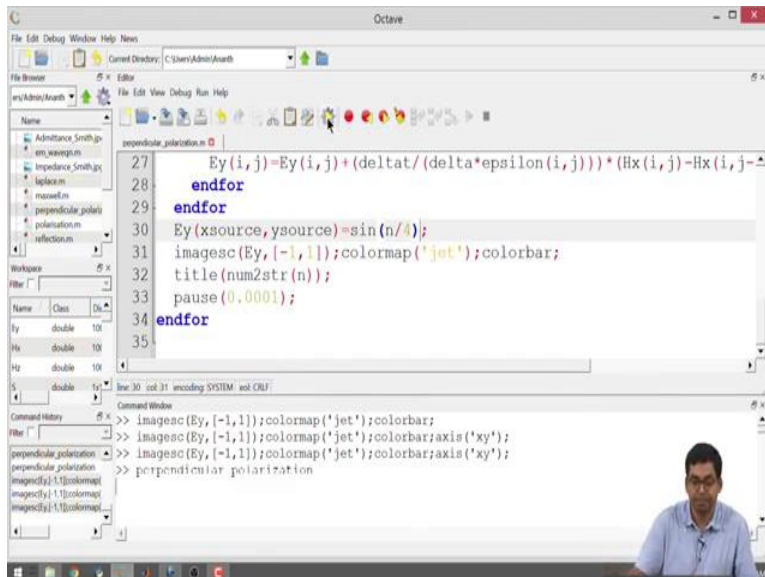
We can see that as time progresses, the source is going outward. I mean you are having the electric fields going outward. Since it is a point source, it is giving out a circular wave front ok and we are plotting y value of the electric field.

The other thing to notice over here is, in the center you have a bright red dot as it goes outwards it is becoming more and more yellowish all right. It is because of the color scale right. Now, one of the things is the yellow part is actually having less electric field magnitude than the red part. So, it is telling you that as it travels out, the energy is getting distributed across a larger region and the peak value of the electric field that you are having going out is actually reducing all right.

So, now it will hit the boundaries ok. Now, we will start some weird things. I mean start to see some weird things right. So, you are having reflections from the boundaries ok, and clearly we can figure out a few things the left and the top boundaries are doing something different from the right and the bottom boundaries ok there is a pattern all right.

In the left and the top, it looks like the electric field is decreasing, the net electric field is decreasing, something hit, something got reflected, the superposition of the 2 is becoming green over there again. Actually if you have the color bar then it would have become easy to ok.

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```
Octave
File Edit Debug Window Help News
Current Directory: C:\Users\Admin\Ananth
File Browser
Name
Adminance_Smith.p
en_waveequm
Impedance_Smith.p
logdim
moulin
perpendicular_polar
perpendicular_polar
reflection.m
Workspace
Filter
Name Class Dn
ly double 100
Hx double 100
Hz double 100
S double 100
perpendicular_polarization.m
27 Ey(i,j)=Ey(i,j)+(deltat/(delta*epsilon(i,j)))*(Hx(i,j)-Hx(i,j-d
28   endfor
29   endfor
30 Ey(xsource,ysource)=sin(n/4);
31 imagesc(Ey,[-1,1]);colormap('jet');colorbar;
32 title(num2str(n));
33 pause(0.0001);
34   endfor
35
Command Window
>> imagesc(Ey,[-1,1]);colormap('jet');colorbar;
>> imagesc(Ey,[-1,1]);colormap('jet');colorbar;axis('xy');
>> imagesc(Ey,[-1,1]);colormap('jet');colorbar;axis('xy');
>> perpendicular_polarization
```

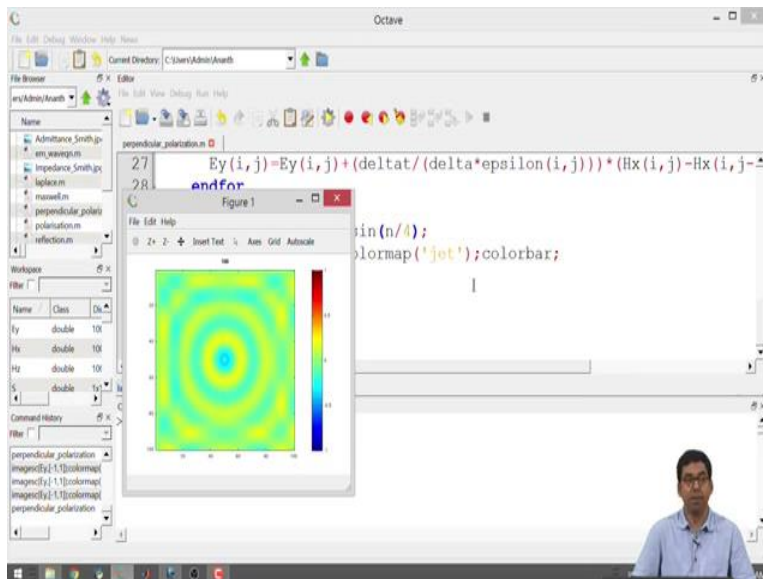
So, I have the color bar. It also makes the axis x y so that we can ok this is easy ok all right. So, I have left and bottom ok. So, this is going from 0 to 100 on both sides which is easier for me to interpret because, when I am writing a code I do not want to see different axis, 0 to 100 on the x, 0 to 100 on this axis. I have a source right in the middle where it was giving out circular wave fronts, the wave hit the boundaries some portion of it got reflected everywhere.

Almost all portions should have been reflected because we have not handled any boundary conditions by default, we have left it to something, we have to see what is going on all right. Here and here, looks like something is happening that is different from the top and the right hand side right. So, clearly this is the effect of boundaries and I can guess that in this place, the electric field is flipping its direction upon you know reflection and then adding up to the one that is going and hitting the boundary. So, it is creating some negative or destructive interference right.

And you are having a net zero electric field coming into the picture. Here maybe it is not changing its sign. So, I can guess what kind of boundary conditions I have. If the electric field is flipping its direction means that you have a perfect electric conductor. So, I can guess that I have a perfect electric conductor on the left and on the bottom, and I have a perfect magnetic conductor on the right and on the top because the electric field did not flip. If I plot Hx, Hz I will be able to figure out what is going on with the other two boundaries.

So, I get a rough idea just like in the case of transmission lines, you had an open circuit on one side, a short circuit on the other side extra all right. Here, it is the same thing that is happening. Now, since you have a base program, there are a variety of things that one can do. I can manipulate the source. I can say that let the source be say you know a sinusoid of frequency say n by 4 , I mean or I mean time period I am changing ok.

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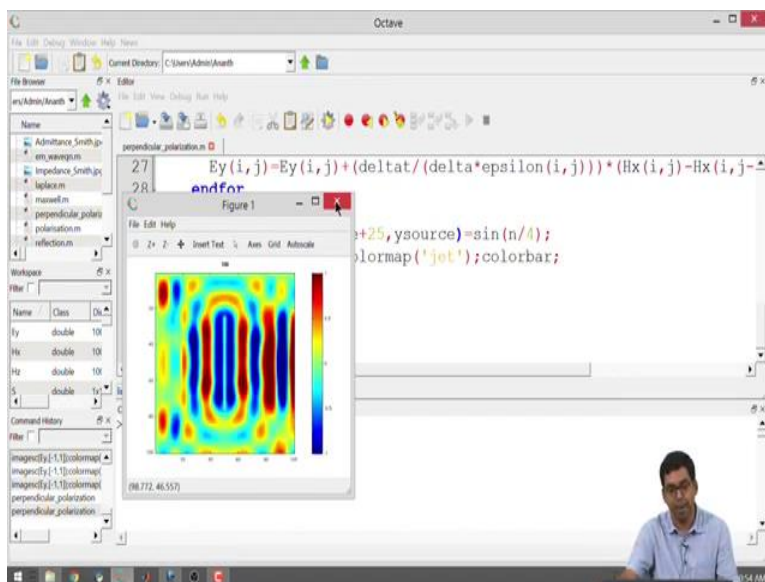
With respect to time, it is producing a positive electric field and a negative electric field. Positive electric field corresponds to the red color, the negative one corresponds to the blue color and as it goes out, you will see that the color is fading a little bit because the energy is getting distributed throughout the wave front ok. When it was starting the wave front was so small, the energy was only in the small place, as it goes out, the energy is distributed everywhere, so it becomes lesser and less intense.

Then it hits the boundaries and then you start seeing strange things happening ok. Now, this is perpendicular polarization. The only thing that is different here compared to what we had done in the class is we had plane waves with the clear direction of travel all right. Here, we are talking about wave fronts that are circular all right. So, one will have to fix it, in order to fix it there are some ways right either

you can make your simulation region really large and wait for the circular wave front to look like a plane wave front at some faraway place all right.

Or, you could start manipulating the source for example, I could say that you know I'm just ok. So, what I have done here is instead of having a single point ok I am having x source minus 25 to x source plus 25, that means that I am going to have a line which is going to be emitting an electric field corresponding to $\sin n$ over 4. So when I will run this, you will be able to figure out exactly what I have done.

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So, if I want to create a plane wave front, all I did was take a series of point sources at different places along the line and then made them all emit with the same phase that is it. So, I am getting a line shaped wave front coming to the sides ok right but still, I am having something going out here right. It is a natural consequence you cannot do anything about it ok. So, now we have the base program for perpendicular polarization to be ready all right.

Now, there are a few things that we have to do more all right. First, we have to see if we can make this program a little bit faster so that we can run a lot more experiments ok. So, we already know how to do that we can vectorize the for loops so, in the next class we will vectorize the for loops, make it a little bit faster, we will see the comparison 1 to 1, and then, we will start playing with material interfaces all right, and then we will see whether whatever we have got in our analytical

results are we able to make some additional visualizations over here and draw some more inferences ok.

We are going to put some interfaces and start to see what is happening all right. So, this is how one would simulate Maxwell's equations in two dimensions, and this is the limit to which we can see in this course because three dimensions the computer will become really, really, really slow ok. So, we will stop here. We will meet in the next class which is tomorrow. Please bring your computer again.

And we will be making a faster version of this and will be putting interfaces and trying to draw inferences ok. And for parallel polarization, after we have gone through this whole exercise my belief is that you should be able to write your own program all it needs is changing the update equations that is that is about it.

So, I will stop here.