Transmission lines and electromagnetic waves
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Lecture – 24

Plane Waves at Oblique Incidence – III

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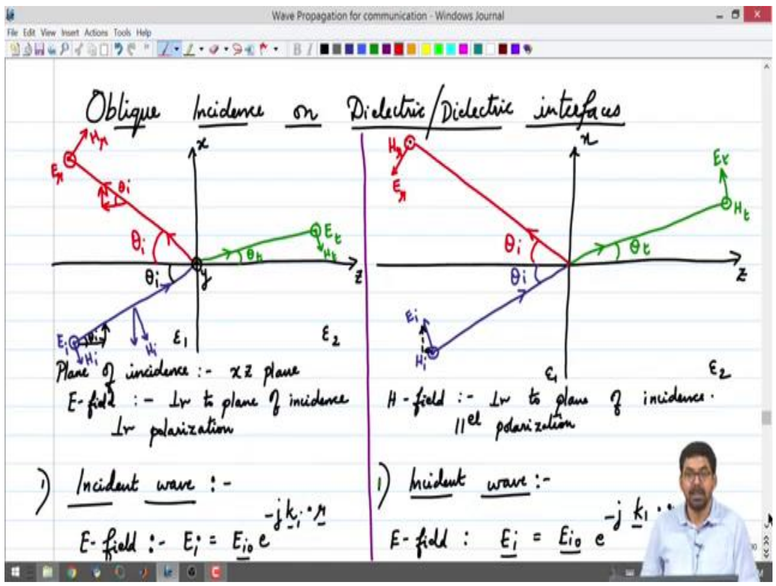

Ok, I think we will get started. Quick review of what we have done before, we have talked about dielectric-dielectric interfaces and we have talked about different polarization configurations. So, the case where the E-field is perpendicular to the plane of incidence is known as the perpendicular polarization and the case where the magnetic field is perpendicular to the plane of incidence is known as the parallel polarization.

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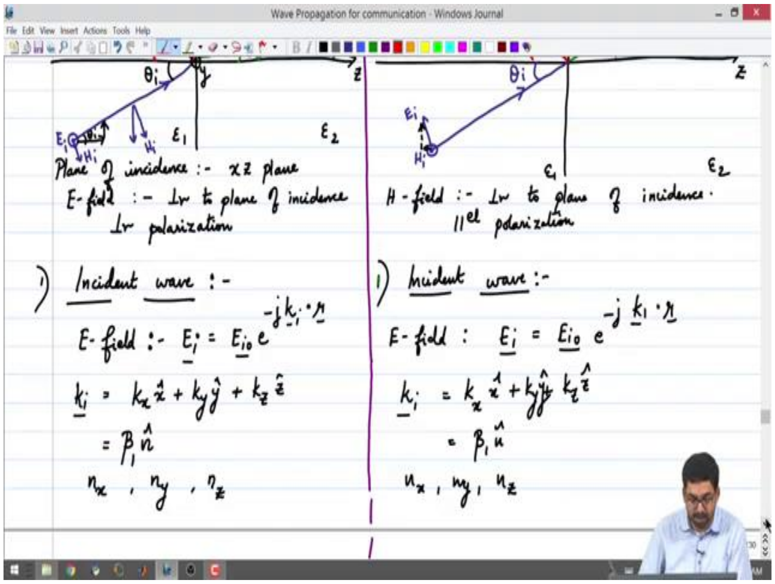

And we have seen how to write the incident, transmitted, reflected wave expressions ok. We found that for both the polarizations the form of the expressions for the E-field and the H-field were identical, ok.

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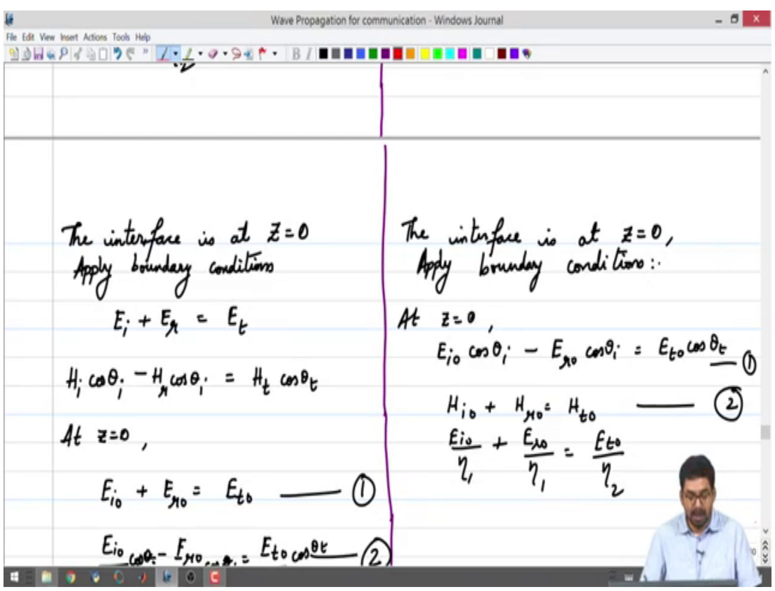

The details are definitely different, the reason why the details relating to the transmission and reflection coefficients are different are simply because the boundary conditions are different for different polarizations. In one case where the electric field was out of the plane of incidence everything was tangential. So,

$$E_i + E_r = E_t$$

But for the other case we had

$$\frac{E_i}{\eta_1} Cos\theta_i - \frac{E_r}{\eta_1} Cos\theta_i = \frac{E_t}{\eta_2} Cos\theta_t$$

So, the magnetic field boundary condition is also different for both polarizations.

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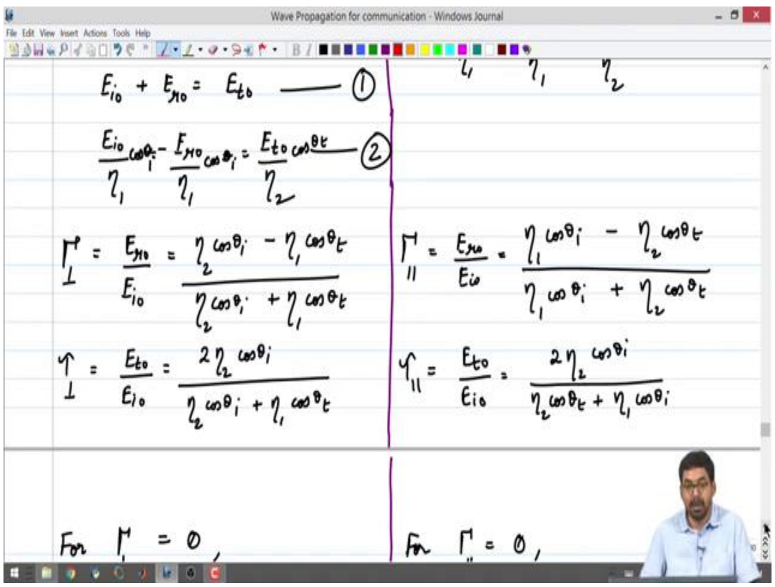

And because of this, we could get the values or we could get the expressions for the reflection and transmission coefficient for the 2 polarizations. There are subtle differences between the 2.

For the perpendicular polarization when we talk about electric field reflection coefficient, $1 + \Gamma_{\perp} = T_{\perp}$, but for the parallel polarization case $1 + \Gamma_{\parallel} = T_{\parallel}$. We will have to use some other expression, ok.

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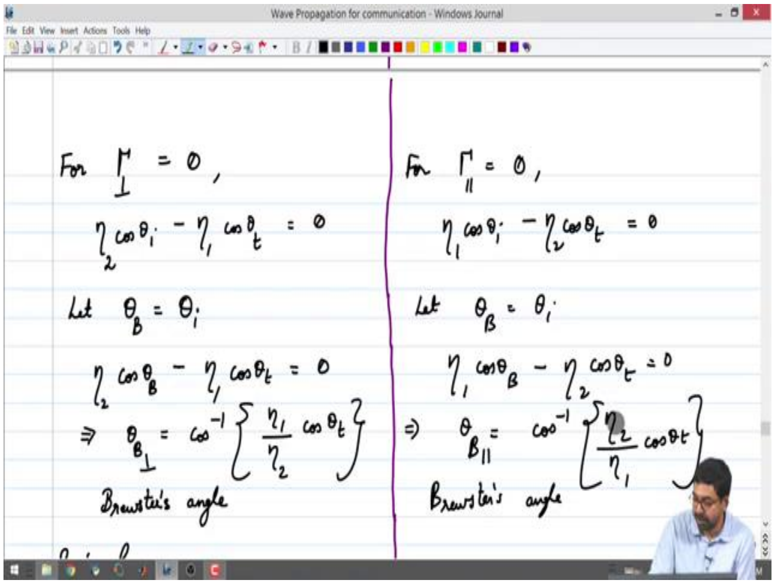

We also saw that in order to make the reflection coefficient 0 in both the polarizations, there is a possibility of using different incident angles for a given material configuration alright. So, the general form is you know for the perpendicular polarization, for the yeah for the perpendicular polarization you will have

$$\theta_{B_\perp} = Cos^{-1}(\frac{\eta_1}{\eta_2} Cos\theta_t)$$

and for the parallel polarization

$$\theta_{B_\parallel} = Cos^{-1}(\frac{\eta_2}{\eta_1} Cos\theta_t)$$

So, it's just a reversal of the intrinsic impedances.

You remember this much and if you are able to calculate cos θ_t from the given problem then you could calculate the Brewster's angle in no time, alright. But the Brewster's angle for different polarization is different, that means that in order for you to get a reflection coefficient 0, alright, you have to understand that it would be possible only for one polarization alright

And for the other polarization that angle of incidence where you get zero reflection is going to be different. So, impedance matching or having no reflections in electromagnetic waves with arbitrary polarizations is not easy unlike transmission lines, ok and it requires tremendous effort.

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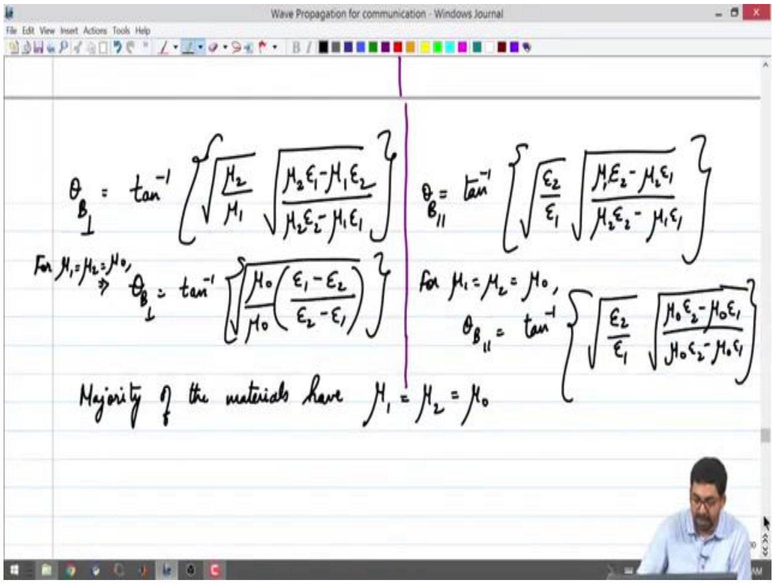

And we also saw that in general cases alright, the Brewster's angle expression was that right, however, one could say that the majority of the materials are non-magnetic alright. If that is the case, you could substitute for μ_1, μ_2 as μ_0, and then you will observe that for the perpendicular polarization, the case for Brewster's angle is not easy to get alright.

And in fact, in the case of non-magnetic material there does not exist a Brewster's angle for perpendicular polarization. However, for the parallel polarization you do have a real angel that you are getting alright, that is a detail alright.

But the most generic form is what we saw before alright

$$\theta_{B_\perp} = Cos^{-1}(\frac{\eta_1}{\eta_2}Cos\theta_t)$$

Now, we have to proceed and patch up a few things, ok. Respecting our understanding the first and foremost thing that we can do is look at the reflection and transmission coefficient expressions.

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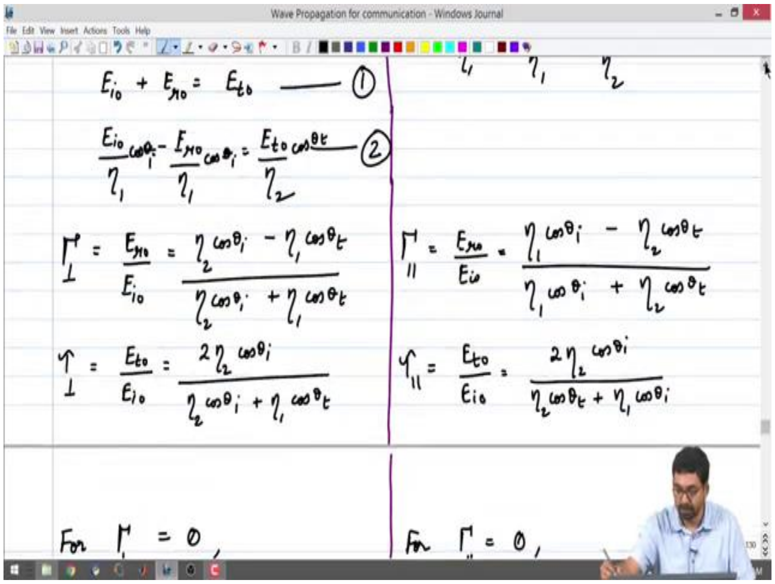

The simplest thing one can do with say perpendicular polarization right is take the expression for the $\Gamma_{\perp}$, substitute incident angle θ_i to be equal to 0, ok. If you have incident angle to be equal to 0, then it becomes normal incidence on the second medium right. Which also means that the transmitted angle has to be equal to 0.

So cosθ_i and cos θ_t will both be equal to 1 and then you will end up getting

$$\Gamma_{\perp} = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

Immediately, we have to notice that $\Gamma_{\perp}$ and $\Gamma_{\parallel}$ do not seem to be the same at normal incidence for the two polarizations, alright.

So, casually if you substitute θ a and θ_t to be equal to 0 degrees, you will notice that $\Gamma_{\perp}$ is negative of $\Gamma_{\parallel}$, alright. Now, this raises some confusion, alright. Why is it that in one case a reflection coefficient is negative in the other?

The simple interpretation that we have to look is it's not the problem with the reflection coefficient calculation, it is the way we have set up the problem itself, alright. So, if you go back to the diagrams that we have drawn, you have to understand that in the first case, we made a very clear assertion that the electric field directions do not flip alright.

So, the incident electric field was pointing out of plane, the reflected electric field is also pointing out of plane, the transmitted electric field is also pointing out of plane and all of them are oriented out of plane in the same direction.

So, we made an assertion that the fields do not flip the sign as per as E-field is concerned. But the way we have drawn the right-hand side figure, we have said that the E-field will flip it's direction ok. So, we have made the assertion that the E-field is going to flip it's direction in response to the change in the k alright.

So, the reason why that sign change is coming in is not because of some arbitrariness in the setup, it's because we have made the electric field flip it's direction in the parallel polarization case alright. So, there is a difference, but this should not be looked into very very seriously alright.

For normal incidence, the reflection coefficient is

$$\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

Now suppose, you have your setup in such a way that you have managed to flip the direction of the electric field while doing the analysis then you will end up with a minus sign ok.

So, this is some tiny thing that you will have to keep in mind if you directly substitute, you may get a difference of you know a 1 minus 1 sign between the two reflection coefficients. So, you have to be a little little careful in interpreting that ok.

The other thing is, let us go a little bit further and build on these 2, right. Let us take the Snell's law ok and so far in order to calculate θ_t we have used Snell's law ok, when $Sin\theta = n_2 Sin\theta_t$, ok.

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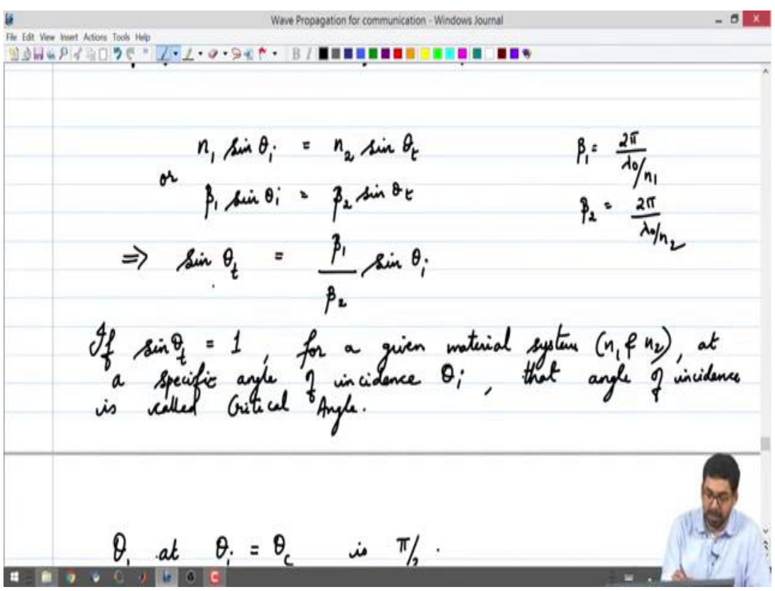

So, we will start with Snell's law,

$$n_1 Sin\theta_i = n_2 Sin\theta_t$$

You could also write this down in a different way, I mean, a bit we already did this before alright. You could also use β there is no issue

$$\beta_1 = \frac{2\pi}{\lambda/n_1} \ \& \ \beta_2 = \frac{2\pi}{\lambda/n_2}$$

You could also write this down as

$$\beta_1 Sin\theta_i = \beta_2 Sin\theta_t$$

Both are Snell's law, ok . And a one of the things that we have done in all the prior derivations is that we have used wherever we had sinθ_t or cosθ_t in our reflection coefficient calculations, we had cos θ_t coming into the picture.

Whenever we had some cos θ_t we tried to substitute it a in terms of θ_i, to reduce the number of variables right. So, we could say that you know,

$$Sn\theta_t = \frac{\beta_1}{\beta_2} Sin\theta_i$$

Now is it possible for $\theta_t = \frac{\pi}{2}$? Or is it possible for sin θ_t to be greater than 1? Consequently cos θ_t to become imaginary alright. So, we have to see that particular condition alright. Now if you tune your incident angle to be in such a way that sin θ_t is equal to 1. So, the left-hand side is equal to one that angle of incidence is known as the critical angle for a given material interface system ok.

So, if sin θ_t becomes equal to 1, with indices n1 and n2 alright, at a specific angle of incidence , ok. That angle of incidence it’s called a critical angle ok ok.

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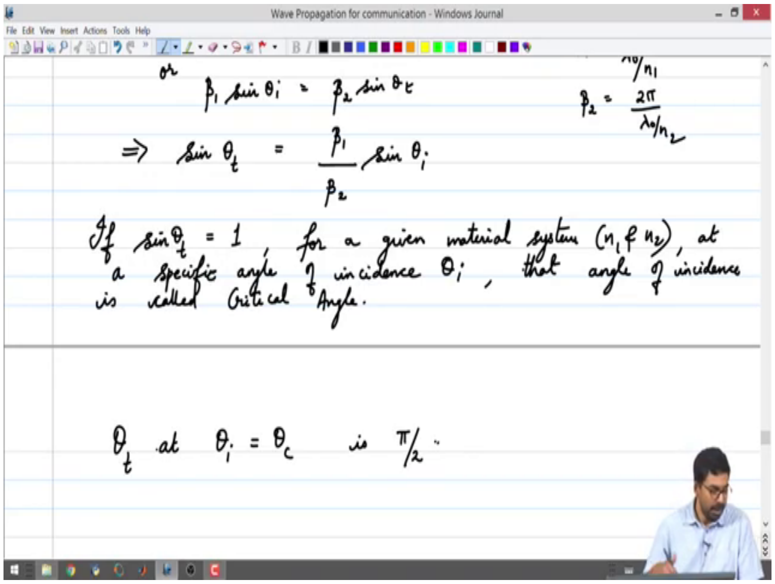

So, this also implies that θ_t alright at θ_i equal to θ_c it's $\frac{\pi}{2}$ or 90 degrees with respect to the normal ok .

You could also have for example, θ_i to be greater than this angle alright, and you would still notice that your θ_t becomes very tough to define right, and you know you have to look at what happens to the transmission and reflection coefficients in these cases, ok.

And one of the things that we could do is look at this specifically with respect to our expressions for the Γ and T that we have got in the previous case for the 2 polarization. We can look at just the reflection coefficients and try to make some inferences as to what is happening right.

So, we had $\Gamma_{\parallel}$ and $\Gamma_{\perp}$, ok. Now, we know that this angle is the critical angle alright, you could also have incident angle to be higher than that alright going all the way to the $\frac{\pi}{2}$ at the input itself ok which means that, at angles higher than this critical angle, ok sin θ_t is going to appear to be greater than 1 ok or in other terms

$$\frac{\beta_1}{\beta_2} Sin\theta_i \gg 1$$

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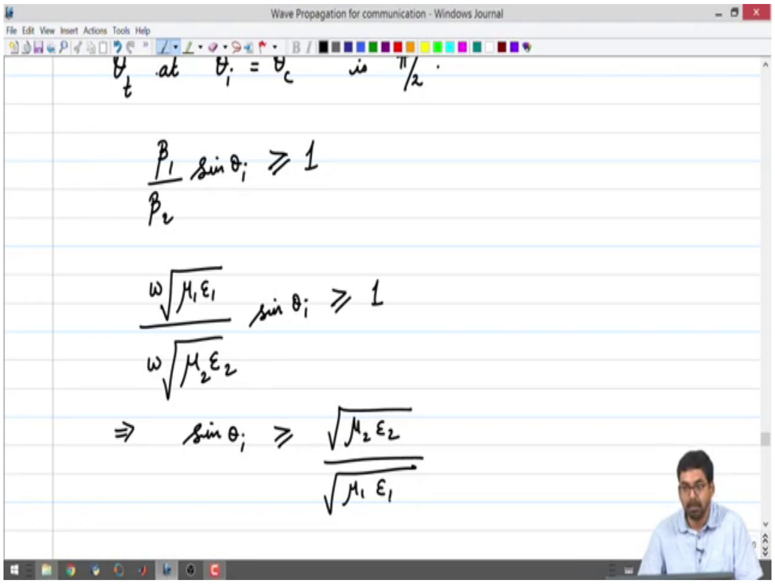

These are the specific cases that we are seeing. First thing that we have to determine is whether this is going to be possible, ok. So,

$$\beta_1 = \omega\sqrt{\mu_1\epsilon_1}$$

So, then we can say that

$$Sin\theta_i \geq \frac{\sqrt{\mu_2\epsilon_2}}{\sqrt{\mu_1\epsilon_1}}$$

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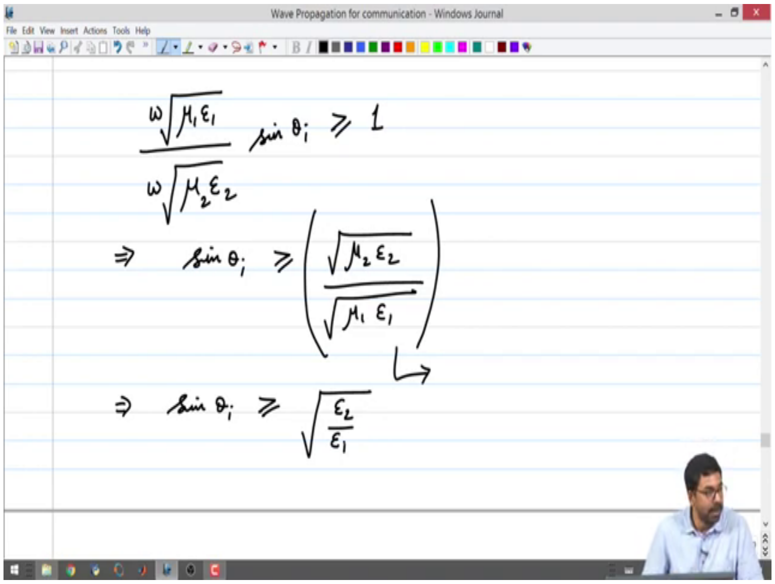

Now, let us look at this right-hand side term a little bit more closely. Let us say that in order to make my analysis a little simpler ok, in order to make my analysis a little simpler, I can always say that the media I am considering are non magnetic media. So, I can substitute μ_1 equal to μ_2 equal to 0, ok.

You can substitute μ_1 equal to μ_2 is equal to 0 a μ_0, alright. So, I just end up with the μ_0 getting cancelled on the numerator and denominator and I have

$$Sin\theta_i \geq \frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1}}$$

So, I just need to be ok.

So as far as the incident angle goes, we have control when you are doing the experiment you have control over the incident angle. Let us assume that you have control over the incident angle you can go from 0 degrees to 90 degrees. That is how you will rotate your incident beam with respect to the interface alright.

There is no way for you to make your θ_i greater than 90 degrees, alright. So, this means that the left-hand side of this inequality does have a limit, and has to be less than 1 ok. So, if the left-hand side has to be less than 1 alright, then we have some conditions coming on the right-hand side ok. This means that your right-hand side also has to be less than one alright.

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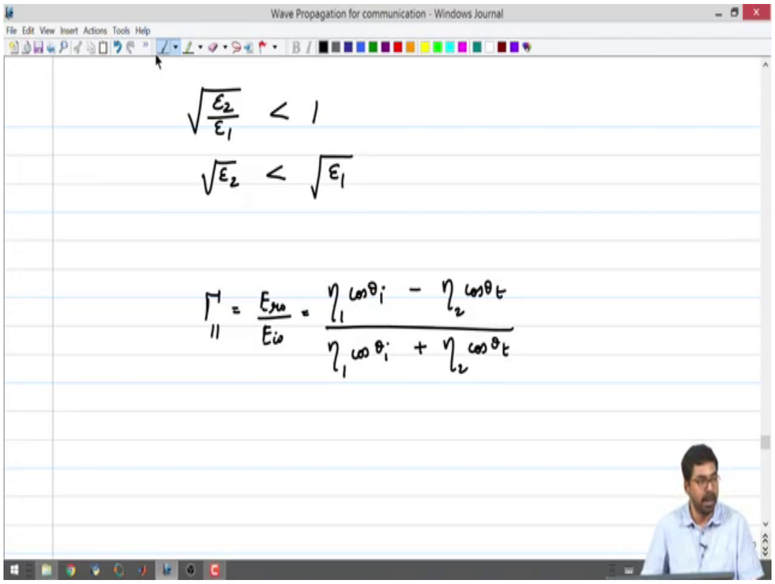

So, you can say that, ok. I am not going to make it equal to 1, equal to 1 means that you are just having the same material on 2 sides the interface no longer exists. So, the only case where you have 2 different media and the interface still exists is not having that equality over the.

So,

$$\sqrt{\epsilon_2} < \sqrt{\epsilon_1}$$

. So, if this condition is satisfied, right. If you have 2 media the first medium is having higher permittivity, the second medium is having lower permittivity then there is a chance that you are going to be ending up with θ_t, ok. Not being clearly defined.

That is sin θ_t will look like it is greater than 1 according to the Snell's law, we already know that if sin θ_t looks like it is going to be greater than 1 according to Snell's law, if you try to calculate cos θ_t using 1 minus square root I mean square root of 1 minus sin squared theta you are going to get a pure imaginary number, you are not going to be able to determine what is the angle of transmission in these cases ok.

So, first we have established that it is possible to have a sin θ_t to be greater than 1 just using simple Snell's law, the way to achieve that is by using your second medium to be rarer than your first medium, and there does exist an angle where your sin θ_t will go greater than 1 and cos θ_t will become purely imaginary.

So, that possibility still exists ok. So, it is not an impractical case where we are saying that θ_t will become $\frac{\pi}{2}$ and you know sin θ_t will become greater than 1. So, according to these expressions it is possible. But what does that physically mean? Something that we have to see ok.

So, if this is the case, we have to go back to our expressions for Γ ok. So, I am going to just go back to our expressions for gamma. So, I had $\Gamma_{\parallel}$, ok I will just go and copy and paste this expression ok.

So, I have $\Gamma_{\parallel}$ is equal to

$$\Gamma_{\parallel} = \frac{E_{r0}}{E_{i0}} = \frac{\eta_1 Cos\theta_i - \eta_2 Cos\theta_t}{\eta_1 Cos\theta_i + \eta_2 Cos\theta_t}$$

This is the expression that I have from the previous class, ok. And I want to take the scenario where the sin θ_t is precisely equal to 1, I will start from there and then I will try to figure out what will happen if sin θ_t becomes greater than 1.

So, at precisely when sin θ_t is equal to 1, we call the incident angle to be the critical angle, alright. So, if sin θ_t is equal to 1 and it becomes greater for higher angles of incidence, then, writing down cos θ_t becomes a difficulty over here right.

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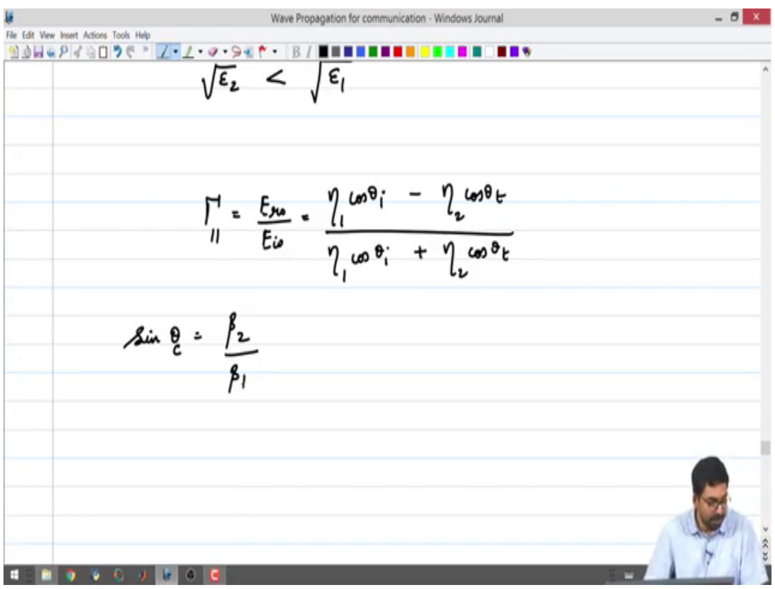

So, what one can do is

sin θ_c just to remember is $\frac{\beta_2}{\beta_1}$ or n2/n1 alright. And if this quantity becomes, if you have incident angles to be higher than this angle, then we are going to be having some θ_t which is not going to be possible for you to get real angles with cos right. So, if this is the case what can we do with respect to writing the angles inside?

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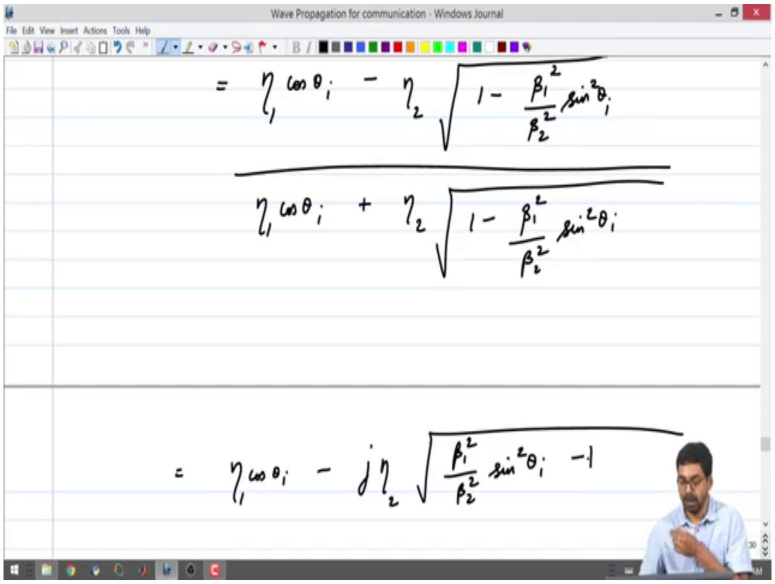

So,

$$= \frac{\eta_1 Cos\theta_i - \eta_2\sqrt{1-\frac{\beta_1^2}{\beta_2^2}Sin^2\theta_i}}{\eta_1 Cos\theta_i + \eta_2\sqrt{1-\frac{\beta_1^2}{\beta_2^2}Sin^2\theta_i}}$$

$$= \frac{\eta_1 Cos\theta_i - j\eta_2\sqrt{\frac{\beta_1^2}{\beta_2^2}Sin^2\theta_i - 1}}{\eta_1 Cos\theta_i + j\eta_2\sqrt{\frac{\beta_1^2}{\beta_2^2}Sin^2\theta_i - 1}}$$

you could just write it down like this right. So that you have the term inside your square root is positive ok.

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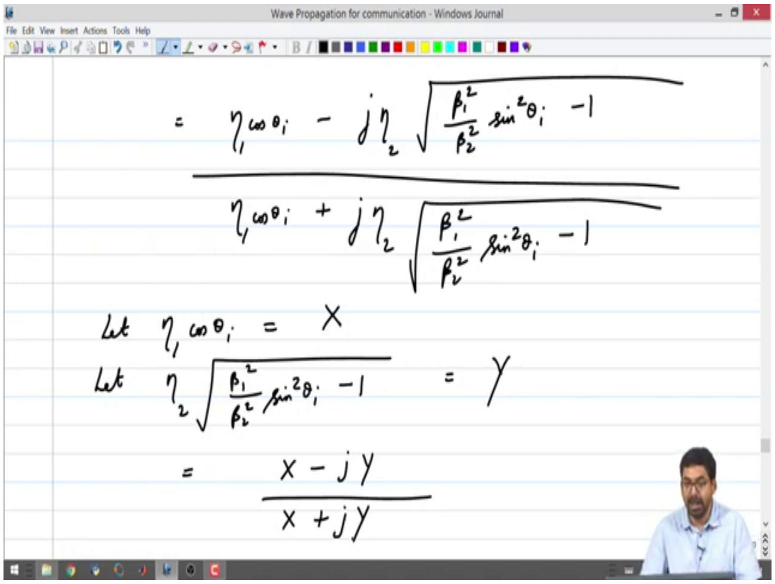

There is a reason for doing this alright, and the reason is because I want to make some inferences from the value that I would ascertain which is coming from here, alright.

One of the things that I know now clearly looking at this is to make this expression a little simple, I could say that let $\eta_1 \cos\theta_i$ be given a variable name x, ok. Let $\eta_2\sqrt{\frac{\beta_1^2}{\beta_2^2}Sin^2\theta_i - 1}$ be given the variable name y then, I can write this formula in a much simpler way, ok.

So, the formula would now look like

$$= \frac{x - jy}{x + jy}$$

So the reason for bringing that minus 1 out and then writing it in terms of j is to write it in the form of a ratio of 2 complex numbers that are conjugates of each other ok.

And there is a reason to do this again, I know that the magnitude of the complex number is given by the square root of x square plus y square, the magnitude or the absolute value of the complex number of its conjugate is also the square root of x square plus y square. So, it is clear to me that this is a ratio of 2 complex numbers of the same magnitude, ok.

So, I can say that this ratio as far as magnitude goes is equal to 1 ok. So, this is the purpose for rewriting it in the you know flip form and taking j outside. So, that I can make this assertion that magnitude of $\Gamma_{\parallel}$ the way we have written is equal to 1, ok .

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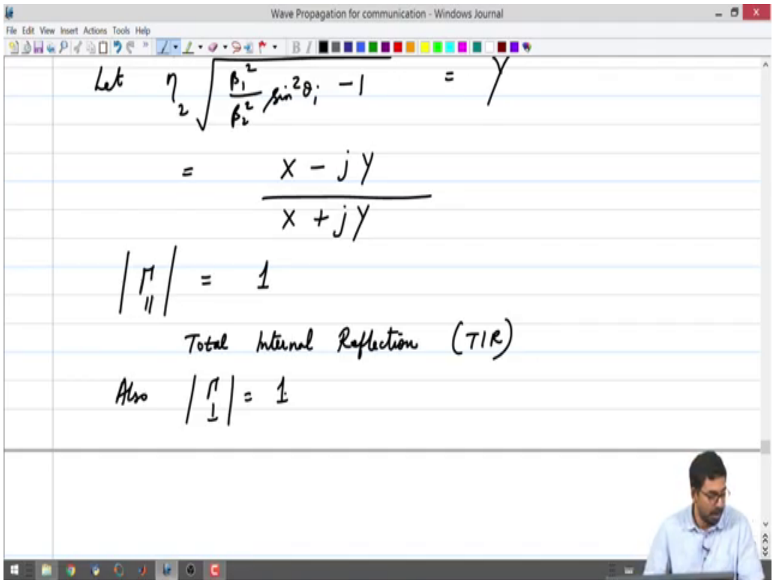

Once I have determined that magnitude of $\Gamma_{\parallel}$ is equal to 1, I know that no electromagnetic wave is being transmitted all of it is being reflected, alright. So, now I can say that at the interface between the 2 media when I have a denser medium followed by a rarer medium, if I adjust my incident angle in such a way that a you know sin θ_t is equal to 1 or higher alright, what I would end up getting is, magnitude of the Γ to be equal to 1, alright.

Notice that I have written clearly magnitude of $\Gamma_{\parallel}$ is equal to 1, I am not talking anything about it's phase here. But, all I know is whatever I am sending in is going to be reflected from the interface, right. So, this is true for all angles where cos θ_t is imaginary. Which means that all angles where θ_i is greater than theta c. Here, the magnitude of your reflection coefficient is going to remain 1, ok.

So, the angles at which this happens correspond to a scenario which is known as total internal reflection. So in short form, the people make it look like TIR, alright.

Total internal reflection is the phenomenon where you launch an electromagnetic wave from a denser medium to a rarer medium and you have your incident angle to be greater than equal to critical angle, then you will notice that in your reflection and transmission coefficients for different polarization, you will notice that the reflection coefficient becomes equal to 1, please for the parallel polarization that is what we have seen.

For the perpendicular polarization also it becomes the magnitude equal to 1 right. So let me just complete this by saying also, irrespective of the polarization that you are using, ok, the entire electric field is going to be reflected there is nothing you can do about it, ok. So, the interface between a dielectric and another dielectric could act as a perfect reflector ok.

And the question can be, what is the angle of reflection? We already know that the angle of reflection is going to be equal to the angle of incidence right. So that is something to remember right.

So now that you know what is this total internal reflection and it is possible? There are 2 extreme conditions we have seen. In the previous class, we saw the conditions where Γ is equal to 0. Γ equal to 0 was known as the Brewster's angle.

And in this class, we are seeing the extreme opposite where Γ is equal to 1, alright. This is known as TIR alright, and generally the wave propagation where you have transmitted, reflected is in between these 2, ok. So, you have seen the 2 extremes and you can have anything in between the 2. So, you can have a reflection coefficient going between 0 and 1 alright that is it right. So, those are the boundaries of what you can see.

But there is only 1 more detail that we will have to see, ok. Just like we have seen the 2 extremes with respect to the reflection coefficient, we also have to understand the way we have formulated the problem, we have also seen 2 extremes that is the electric field is either completely perpendicular to the plane or it's completely lying in the plane of incidence. Those are also 2 extremes alright.

In reality, you may have polarizations where the electric field has a component that is parallel, and has a component that is perpendicular to the interface. In all these conditions, you will have to break the electric field into 2 components. You have to project the electric field to the plane, take the component of the electric field along the plane and then you will have to project it perpendicular to the plane and take a component perpendicular to the plane.

You may have to calculate $\Gamma_{\parallel}$, $\Gamma_{\perp}$ for the single case and then we know that the electric fields overlap and it can be super post alright, it follows the theory of superposition it's a linear term you can add electric fields in a quantity.

So that means, that you will start taking $\Gamma_{\parallel}$, $\Gamma_{\perp}$ et cetera, and then calculating the reflected electric field for both these polarizations and then you will be adding them up together to get the net electric field that is being reflected, ok.

So, that is one one thing that we have to remember. Many of the problems, you may have some polarization which is neith$E_{r_{\parallel}}$ nor perpendicular. In those cases, you will have to you know, project your electric field into the plane, take the infield component and then out-of-plane and take the out of plane component and then you will have to find out $\Gamma_{\parallel}$, $\Gamma_{\perp}$.

So in a given problem, you may end up using both the polarizations to figure out what is the net effect. In doing that, we can immediately say that if that is the case and if we look at the expressions that we have, we already know that $\Gamma_{\perp}$ and $\Gamma_{\parallel}$ are not equal, right. They are not equal, they are different which means, that you could have different $\Gamma_{\perp}$, $\Gamma_{\parallel}$, alright. So, you have to be very accurate in the calculations that you make, and also the inferences that you will make, ok. So, let us take some conditions and let us start to look at what is going to happen, ok.

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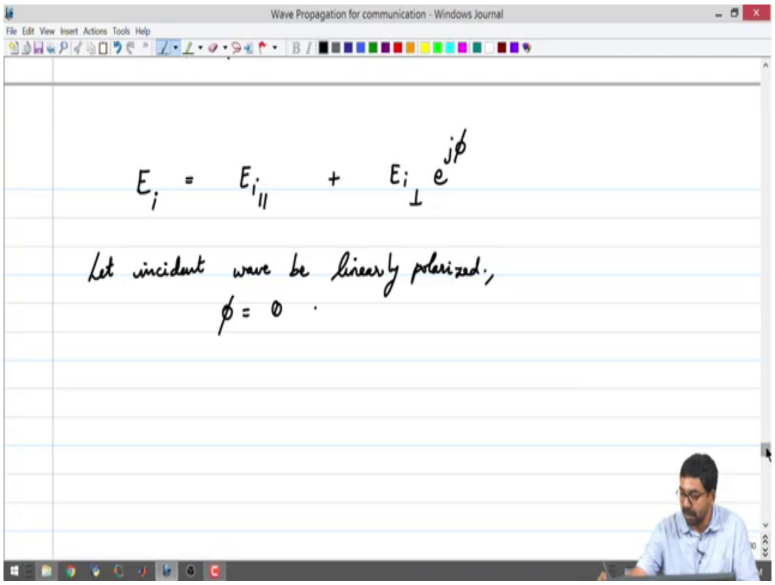

The first thing that we will do is we will say that any electric field, ok. With respect to this plane of incidence, can be divided into $E_{i_{\parallel}}$ and it can be divided into some $E_{i_{\perp}}$.

Now, in the case of E m waves when we had begun this, and we were talking about the differences between the E m waves and the transmission lines we were talking about polarization and then we talked about linear polarization, circular polarization and elliptical polarization, ok.

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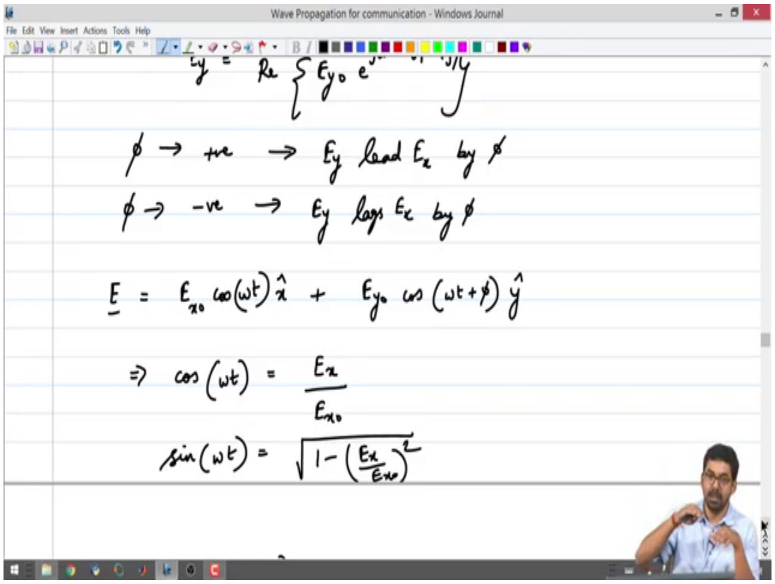

In those lectures ok, one of the things that we were noticing was you could have Ex and Ey, ok. They could have different values or they could have different magnitudes and there could also be a phase difference between Ex and Ey. We also saw those using an animation later on, alright.

So you will be having Ex going this way, Ey going the other way. But they need not be in phase alright, however, if they are in phase and if their values are different, then you will end up having a linear polarized light.

So, let us take our current scenario where we have calculated all these reflection coefficients, let us say that the incident light is linearly polarized, ok. Immediately, the question comes I mean, we have written

$$E_i = E_{i,\parallel} + E_{i,\perp}$$

Should we also be more generic and say that $E_{i_\perp}$ and $E_{i_\parallel}$ Can there be a phase difference between them? Just like Ex and Ey could have phase differences between them in the simulations that we saw before, here also it is quite possible that you can have a phase difference between them generally, ok.

But if you are talking about linearly polarized light, then we are talking about this phi becoming equal to 0, then it becomes a specific case where

$$E_i = E_{i,\parallel} + E_{i,\perp}$$

and you do not have any phase difference between $E_{i_\parallel}$ and $E_{i,\perp}$, ok.

So if it is linearly polarized, just make phi equal to 0. Ok. You will be having Ei is equal to $E_{i_{\parallel}}$ plus $E_{i_{\perp}}$, then you will take the $E_{i_{\parallel}}$ component, you will be doing the calculations for the reflection coefficient, you will be taking the $E_{i_{\perp}}$ component and you will be calculating the reflection coefficient.

In both these cases, you will be able to calculate the reflected wave Er and then once you get $E_{r_{\parallel}}$ $E_{i_{\perp}}$, you will just add the 2 to get the net reflected electric field in medium number 1, ok.

So, this is the procedure if you had a linearly polarized wave. Now, while doing this, there are other things that we have to consider, ok. If you look at the expressions of transmission and reflection coefficients that we are having, these are real numbers ok, $\Gamma_{\perp}$, $\Gamma_{\parallel}$ are all real numbers. There is no need for them to be complex, ok.

So, if this is the case for a linearly polarized wave to go and hit the interface and then bounce back alright, there would not be any arbitrary phase imparted to you're an incident wave alright, it is because your reflection coefficients are real, alright.

Which means that you would not have any changes in phase coming out of this, so, if your incident electric field was a linearly polarized and if you split that into parallel and perpendicular components, each of them is going to be having a reflection coefficient that is real, alright and then you add them together alright, you will end up with no manipulation of phase between them.

So, you would not be giving only parallel polarization some phase, perpendicular polarization no phase and all that will not happen, they will still remain in phase ok.

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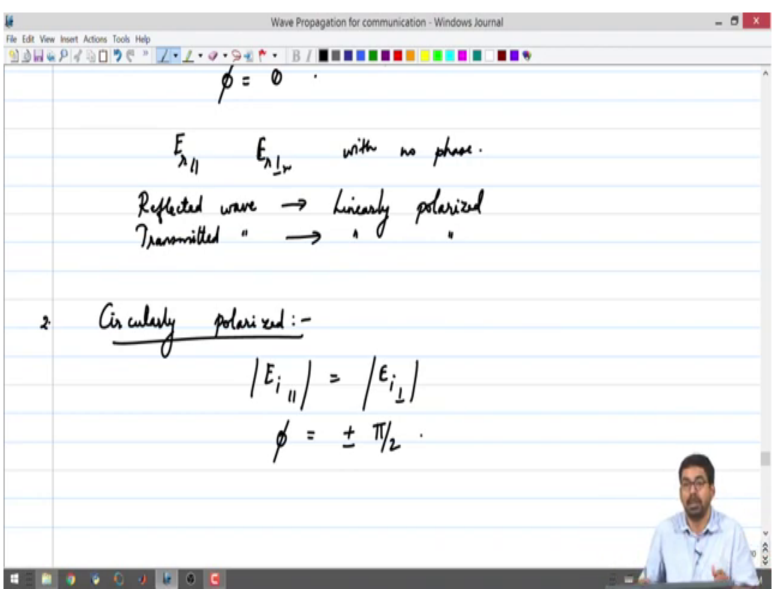

So, which means that $E_{i_\parallel}$ a, $E_{i_\perp}$ in the case of linearly polarized wave, will lead to $E_{r_\parallel}$ $E_{r_\perp}$ with no phase. So, if you have to add them up together, you will immediately realise that Er is also going to be linearly polarized, but, $E_{r_\parallel}$ value is not the same as $E_{r_\perp}$ value. That is the reflection coefficients are not identical.

So, the linearly polarized wave could undergo a change in angle at which it is polarized simply because the Y component or the perpendicular component is different from the parallel component, that is all. But it will remain linearly polarized.

So, linearly polarized wave hitting the interface, the transmitted light will be linearly polarized, reflected wave will also be linearly polarized, however, the angle at which it is polarized, could change simply because the reflection coefficients and the transmission coefficients for the both polarizations are not identical. So, it will stretch one component more than the other ok.

So, a linearly polarized wave will remain linearly polarized in this material system, right? So, reflected wave, linearly polarized, transmitted wave, it's also linearly polarized right. So, then we can take another scenario if your incident wave is circularly polarized, ok.

So, for a circularly polarized wave in the prior lectures we had seen that mod Ex was equal to mod Ey and the phase difference of plus $\frac{\pi}{2}$ or minus $\frac{\pi}{2}$ was present between these 2. If it was + $\frac{\pi}{2}$ you would have one-handed circular polarization. Maybe, it is right or left, alright.

And if it is the opposite - $\frac{\pi}{2}$, you will be having the opposite evolution of polarization or we call it left or where it's the opposite one is right-handed another one is left-hand circular polarization.

So, here we can say that equivalently here, a your $E_{i_\parallel}$ alright, is the same as a your $E_{i_\perp}$ magnitude, that is how you define circularly polarized with respect to an interface, and there is a phase difference between the 2 and it could be either + $\frac{\pi}{2}$ or - $\frac{\pi}{2}$ depending upon whether you are having a left or right circular polarized wave, ok.

So, this would be the condition for having a circularly polarized wave hitting your media interface, ok. Now once again, we know that the reflection and the transmission coefficients are real ok, so, they do not change the phase they just change the values of your reflected and a transmitted waves peak, ok.

They do not change any arbitrary phase between these 2 components. Which means that the reflected and the transmitted waves are going to exhibit the same phase difference between the 2 components alright.

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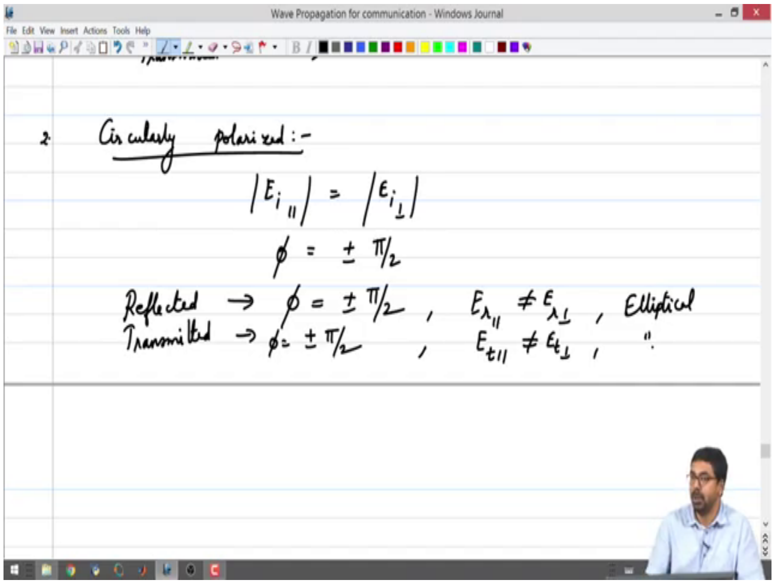

So, for the reflected case your phi will you know still remain at $\pm\frac{\pi}{2}$ for the transmitted case .

$$\phi = \pm\frac{\pi}{2}$$

Now, we also know on top of that just like in the linearly polarized case, $\Gamma_{\parallel}$ is not equal to $\Gamma_{\perp}$.

Reflection coefficient values are not equal, which means that whether if you had $E_{i_{\parallel}}$ and $E_{i_{\perp}}$ to be equal $E_{r_{\parallel}}$ and $E_{r_{\perp}}$ may not be equal.

So what happens is, you will end up with $E_{r_{\parallel}}$ not being equal to $E_{r_{\perp}}$, but they still have a phase of $\frac{\pi}{2}$ between them and this corresponds to an elliptical polarization , ok.

The same thing happens in the case of transmitted waves, the transmission coefficients are not equal ok. So, which means that your $E_{t_{\parallel}}$ $E_{t_{\parallel}}$ need not be equal to $E_{t_{\perp}}$, once again, becomes elliptical , ok.

Now there is also another tiny detail that you will still have to remember. We have said that $E_{r_{\parallel}}$ is not equal to $E_{r_{\perp}}$. That generally means elliptical. it's also possible that one of them is 0.

Because you you have a case where you have Brewster's angle occurring in 1 polarization, alright, where you have Γ equal to 0 for that polarization. But in a normal non-magnetic material Γ for the other polarization is not 0.

So, it is perfectly possible that you get only one polarized component, the other polarized component is not present in the reflected case. So it's perfectly possible that your input was circularly polarized and the reflected is only linearly polarized, ok.

So, there are cases where multiple things can happen, but one thing is for sure, you cannot have reflected and transmitted to be circular. The most generic case they look like an elliptical, but it is possible that this elliptical gets down to being linear in one of these components.

So, the circularly polarized light there are many things you will have to look at the media interfaces, calculate the coefficients and calculate you know the values of the reflected fields and then you have to decide what the polarization is going to be. But in general, it has to be elliptical. It could you know, become linear under certain conditions depending upon your value of $\Gamma_{\parallel}$ and $\Gamma_{\perp}$ ok.

So, this is the thing. So, what have we seen now? We have seen that the magnitude of the electric field is changing upon reflection and transmission, we have also seen that the polarization of your EM wave is changing depending upon your medium configuration. The only thing that we are certain is there is no arbitrary phase change between components because your a Γ and a T are actually real numbers.

So, these are the things that we have seen and we have also seen some extremes. Brewster's angle corresponds to Γ is equal to 0, and we have also seen the opposite case which is total internal reflection where Γ is equal to 1, ok.

So, anything is possible using a dielectric-dielectric interface, and we also know that if you make the angle of incidence to be equal to 0, you will get a reflection coefficient to look like transmission line reflection coefficients.

However, here you have 2 polarizations and in the way you have constructed the problem, you have flipped the electric field. So, one polarization will have a negative reflection coefficient of the other.

So, you have to be careful in interpreting these things, alright. With this most of the topics related to dielectric-dielectric interfaces are over, ok. The only thing that we need now is to start looking at dielectric metal or dielectric conductor interfaces alright, and then we will have to see how to make use of these things in some settings to send guided light et cetera.

But before that what I want to do is, I want to go back to the programming part. Because it has been a while since we did some programming, ok. I want to take a couple of these things and just show you what a plane wave is alright, by solving Maxwell's equations, we have seen only graphs; we have not actually seen waves that are bound in 2 spatial dimensions. We have seen only in 1 spatial dimension.

So, the next class I want you to bring your computer again with octave. We will take the simplest case where we have del cross e, del cross h, but, instead of having E in X direction, H in Y direction, all that, we will try to see whether we can make a two-dimensional analysis. That is, we will try to see what is a point source? What is a spherical wave-front? What is a plane wave-front? Et cetera and get a more visual feeling.

This part of the simulations will be slow, ok. Because it has, the matrix sizes will become where it was 100 now it will be 100 by 100 ok. So, it will be slow, but we will do some portions in class, some portions I will ask you to do for your own information back home ok, because it is slow alright.

So, that is the only thing. So, next class I request you to bring your computers with octave loaded. We will go over some visualization for these plane waves, possibly total internal reflection or Brewster's angle also ok. So, we will stop here.