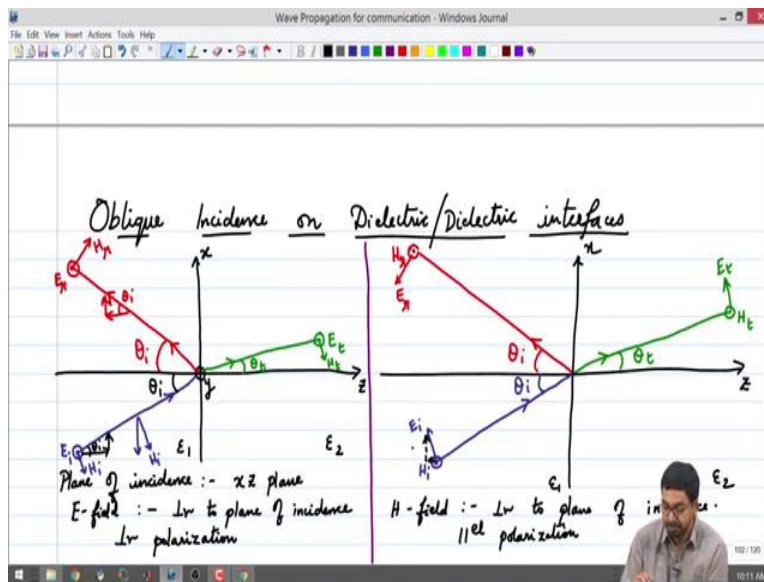


Transmission lines and electromagnetic waves  
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Lecture - 23  
Plane Waves at Oblique Incidence – II

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We will get started, just briefly go on what we had seen in the last class ok. So, we had divided the number of configurations into 2 for oblique incidents. We defined what is known as the plane of incidence. It's the 2 dimensional diagram that we draw on the paper all right, describing the interface and the direction of travel right.

And we saw 2 configurations for polarization which was possible one where the electric field is perpendicular to this plane of incidence and the other configuration was magnetic field perpendicular to the plane of incidence. And in the previous class we had derived the transmission and reflection coefficients for the electric field perpendicular to the plane of the incidence ok.

So, this is you know, the left hand side one is known as the perpendicular polarization. We had written the expressions for the incident wave, ok transmitted and reflected wave, and then we had applied boundary conditions.

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The interface is at  $z=0$   
Apply boundary conditions  
 $E_i + E_r = E_t$   
 $H_i \cos \theta_i - H_r \cos \theta_r = H_t \cos \theta_t$   
At  $z=0$ ,  
 $E_{i0} + E_{r0} = E_{t0}$  — ①  
 $\frac{E_{i0} \cos \theta_i - E_{r0} \cos \theta_r}{\eta_1} = \frac{E_{t0} \cos \theta_t}{\eta_2}$  — ②

Which means here we have a dielectric dielectric interface and we have applied tangential components of the E field and tangential components of the H fields to be continuous all right.

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$E_{i0} + E_{r0} = E_{t0}$  — ①  
 $\frac{E_{i0} \cos \theta_i - E_{r0} \cos \theta_r}{\eta_1} = \frac{E_{t0} \cos \theta_t}{\eta_2}$  — ②  
 $\Gamma_{\perp} = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 \cos \theta_i - \eta_1 \cos \theta_t}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t}$   
 $\tau_{\perp} = \frac{E_{t0}}{E_{i0}} = \frac{2 \eta_2 \cos \theta_i}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t}$

Once we had the 2 equations, we solved the 2 simultaneous equations for gamma and tau ok that is where we stopped, this class we will go over the other polarization also and then we will try to mark some inferences from these 2 derivations right, all right.

So, we will get started with parallel polarization here, the H field is perpendicular to the plane all right, and as in the previous lecture the assumption here is that the H field remains oriented in

the same manner for the reflected and transmitted the electric field will change it's direction depending upon k ok.

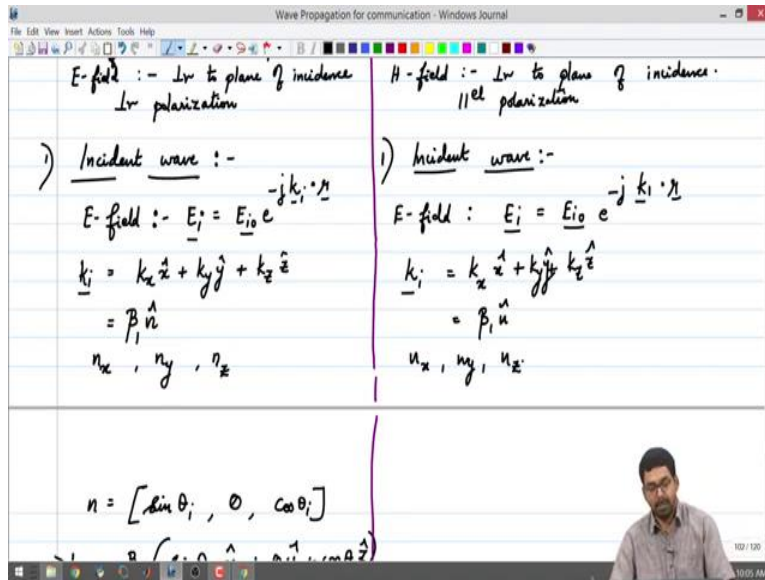
So, likewise we had marked E incident to be in one direction corresponding to the right handed triad, the direction of travel is towards this interface at the origin it gets some portion of it gets reflected and the reflected wave travels back, because the k vector is in different direction.

And the magnetic field is assumed to be in the same direction as the incident wave and the electric field has flipped ok. I think the way it has been drawn looks like a cross for the Hr so, i will just correct that a little bit ok.

So, the magnetic field is out of plane and the electric field is oriented in such a way to form a right handed triad, so the same thing happens for the transmitted ray all right. The only detail that we need to add is  $\theta_t$  can be different than  $\theta_i$  and if it is two different media with  $\epsilon_1$  and  $\epsilon_2$  permittivity  $\theta_t$  is different than  $\theta_i$  ok.

So, we will start with writing down the expressions for the incident wave, transmitted wave, reflected wave and the boundary conditions.

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Once again the electric field is going to have similar form right so, it is going to look like

$$E_i = E_{i0} e^{-jk \cdot r}$$

in medium one right, and k incident is going to be

$$\begin{aligned} k_i &= k_x \hat{x} + k_y \hat{y} + k_z \hat{z} \\ &= \beta_1 \hat{n} \end{aligned}$$

Which is identical to the left hand side case and  $\hat{n}$  will have components x y and z. Which can be marked as  $n_x, n_y, n_z$  all right, these are identical to the left hand side.

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Handwritten derivations from a slide:

$$n = [\sin \theta_i, 0, \cos \theta_i]$$

$$\Rightarrow \underline{k}_i = \beta_1 (\sin \theta_i \hat{x} + 0 \hat{y} + \cos \theta_i \hat{z}) \Rightarrow \underline{k}_i = \beta_1 \sin \theta_i \hat{x} + \beta_1 \cos \theta_i \hat{z}$$

$$\text{Now, } \underline{r} = x \hat{x} + y \hat{y} + z \hat{z}$$

$$\Rightarrow \underline{E}_i = \underline{E}_{i0} e^{-j \beta_1 (x \sin \theta_i + z \cos \theta_i)} \Rightarrow \underline{E}_i = \underline{E}_{i0} e^{-j \beta_1 (x \sin \theta_i + z \cos \theta_i)}$$

2) Reflected Wave :-

$$\underline{E}_r = \underline{E}_{r0} e^{-j \underline{k}_r \cdot \underline{r}}$$

$$\underline{k}_r = \beta \hat{n}$$

So, since everything is merely identical all right or everything is identical so far all right the components will look exactly the same  $[\sin(\theta_i), 0, \cos(\theta_i)]$ . Remember that we are resolving a vector over here the  $k$  vector between two different polarizations has not changed all right it is the same diagram it's going in the same direction, so practically there is no difference in the expression for the  $k$  all right.

So, here you can write that

$$\begin{aligned} k_i &= \beta_1 (\sin \theta_i \hat{x} + 0 \hat{y} + \cos \theta_i \hat{z}) \\ &= \beta_1 \sin \theta_i \hat{x} + \beta_1 \cos \theta_i \hat{z} \end{aligned}$$

Once again the only difference between the two is the way in  $E$  and  $H$  are oriented ok the expression will be  $E_i e^{-j \underline{k} \cdot \underline{r}}$

This is for a plane wave.  $\underline{r}$  is going to be position vector

$$\underline{r} = x \hat{x} + y \hat{y} + z \hat{z}$$

irrespective of the polarization.

And in the wave that we have drawn the  $k$  is oriented in the same direction in both the cases, only the direction of the electric field is different right. So, which means that the expression for the electric field is going to look identical ok ok ok. the only difference being the  $E_{i0}$  all right how it is oriented extra because, I have drawn a vector symbol below  $E_{i0}$  I think some care has to be taken later on, but for now the expression is going to be exactly the same as the other polarization.

Now since a this has been established that for the incident wave the form of the electric field expression is identical to the other polarization, it is very easy to say the same thing about the reflected and the transmitted fields also, because the  $k$  vector is going this way for the reflected a wave and the  $k$  vector is going the other way for the transmitted. So, the components of  $k$  for the reflected and transmitted will be identical to those cases in the other polarization also.

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$$\underline{E}_i = \underline{E}_{i0} e^{-j\beta_1(x \sin\theta_i - z \cos\theta_i)}$$

$$\underline{E}_r = \underline{E}_{r0} e^{-j\beta_1(x \sin\theta_r + z \cos\theta_r)}$$

$$\underline{E}_t = \underline{E}_{t0} e^{-j\beta_2(x \sin\theta_t + z \cos\theta_t)}$$

3) Transmitted wave :-  
 E-field :-  $\underline{E}_t = \underline{E}_{t0} e^{-j\beta_2(x \sin\theta_t + z \cos\theta_t)}$   
 $\underline{E}_t = \underline{E}_{t0} e^{-j\beta_2(x \sin\theta_t + z \cos\theta_t)}$

3)  $|H_i| = \frac{|E_i|}{\eta_1}$   
 $|H_r| = \frac{|E_r|}{\eta_1}$

So, directly one could write down the expressions for the electric field to look like what we had for the other polarization ok. And I am going ahead and writing down the expression for the transmitted electric field also right

student: sir plus

plus ok.

The next thing that we did in the previous lecture was write down the expression for magnetic field in very simple form all right, all we did was use Ohm's law to figure out the magnitude of the magnetic field and then write down boundary conditions.

Now electric field expressions are the same magnetic field expressions are also going to be the same because you are going to divide each of them by eta that is it all right.

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3)  $|H_i| = \frac{|E_i|}{\eta_1}$   
 $|H_r| = \frac{|E_r|}{\eta_1}$   
 $|H_t| = \frac{|E_t|}{\eta_2}$

The interface is at  $z=0$

So, it goes without saying that the form of the incident reflected and transmitted waves in both the polarizations are going to remain the same right.

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The interface is at  $z=0$   
Apply boundary conditions  
 $E_i + E_r = E_t$   
 $H_i \cos \theta_i - H_r \cos \theta_r = H_t \cos \theta_t$   
At  $z=0$ ,  
 $E_{i0} + E_{r0} = E_{t0}$  — ①  
 $\frac{E_{i0} \cos \theta_i - E_{r0} \cos \theta_r}{\eta_1} = \frac{E_{t0} \cos \theta_t}{\eta_2}$  — ②  
 $\Gamma = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 \cos \theta_i - \eta_1 \cos \theta_t}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t}$

The interface is at  $z=0$ ,  
Apply boundary conditions:  
At  $z=0$ ,  
 $E_{i0} \cos \theta_i - E_{r0} \cos \theta_r = E_{t0} \cos \theta_t$   
 $H_{i0} + H_{r0} = H_{t0}$

Then where is the difference coming from? All right the difference is in the boundary conditions all right that is why the difference comes. So, we will write down the boundary condition and then we will go ahead and see what the differences can be all right.

So, for both the polarizations the form of the electric field and the magnetic field expressions are identical the only change is coming due to the application of boundary conditions. So, once again the interface is at  $z$  equal to 0, and apply boundary conditions.

Previously, we had the electric field pointing out of this plane all right, which means that the entire electric field was tangential to the interface. So, we had written

$$E_i + E_r = E_t$$

Now we have to be a little careful because  $E$  is not pointing out of the plane so, that means, that we have to take the component of  $E_i$  which is tangential to the interface, and then we have to apply the boundary condition which is tangential  $E$  field is going to be continuous right.

So, this means that you will have to resolve this into two parts ok, one which is like this and the other one like this and you have to take the tangential part which is the vertical component over here, and you have to apply the boundary condition that is tangential components will be continuous at the interface ok.

So, I will go back the boundary condition here is at  $z = 0$  ok, I have  $E_{i0} \cos \theta_i$  ok, and the electric field flipped its direction while it is going back so, you will notice that the vector direction for the incident wave for the tangential component is opposite to that of the reflected components, so you have to take the difference between the two all right. Which is the same thing what we did for the magnetic field in the other polarization right ok.

And in the case of magnetic field in this configuration the magnetic field is pointing out of the plane so, whatever magnetic field you have is going to be tangential, so, the boundary condition corresponding to the magnetic field is simply going to be

$$H_{i0} + H_{r0} = H_{t0}$$

The plus sign is simply because the magnetic field does not flip its direction so the net magnetic field vector is still pointing out of the plane on the left side and on the right ok all right.

So, now once again you have a 2 equations the magnetic field can be written as

$$\frac{E_{i0}}{\eta_1} + \frac{E_{r0}}{\eta_1} =$$

So, that means, you will have 2 equations simultaneous equations and you will have  $E_{i0}$ ,  $E_{r0}$ ,  $E_{t0}$  to be calculated, since there are only 2 equations and 3 unknowns if you substitute, as in the previous case you can solve only for the ratio, so here we will denote the gamma by a suffix representing parallel polarization right.

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$H_i \cos \theta_i - H_r \cos \theta_i = H_t \cos \theta_t$   
 At  $z=0$ ,  
 $E_{i0} + E_{r0} = E_{t0}$  — (1)  
 $\frac{E_{i0} \cos \theta_i - E_{r0} \cos \theta_i}{\eta_1} = \frac{E_{t0} \cos \theta_t}{\eta_2}$  — (2)  
 $\Gamma_{\perp} = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 \cos \theta_i - \eta_1 \cos \theta_t}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t}$   
 $\Uparrow_{\perp} = \frac{E_{t0}}{E_{i0}} = \frac{2 \eta_2 \cos \theta_i}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t}$

$E_{i0} \cos \theta_i - E_{r0} \cos \theta_i = E_{t0} \cos \theta_t$  — (1)  
 $H_{i0} + H_{r0} = H_{t0}$  — (2)  
 $\frac{E_{i0}}{\eta_1} + \frac{E_{r0}}{\eta_1} = \frac{E_{t0}}{\eta_2}$   
 $\Gamma_{\parallel} = \frac{E_{r0}^{\parallel}}{E_{i0}^{\parallel}} = \frac{\eta_2 \cos \theta_t - \eta_1 \cos \theta_i}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i}$   
 $\Uparrow_{\parallel} = \frac{E_{t0}}{E_{i0}}$

So, this is  $\eta_1$  right reflected is the same ok. So, now you will have 2 equations and the unknowns that you can solve are only for the ratios, so the ratio is reflected to the incident right. In this case comes out to be

$$\Gamma_{\parallel} = \frac{E_{r0}}{E_{i0}} = \frac{\eta_1 \cos \theta_i - \eta_2 \cos \theta_t}{\eta_1 \cos \theta_i + \eta_2 \cos \theta_t}$$

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$\Gamma_{\perp} = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 \cos \theta_i - \eta_1 \cos \theta_t}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t}$   
 $\Uparrow_{\perp} = \frac{E_{t0}}{E_{i0}} = \frac{2 \eta_2 \cos \theta_i}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t}$

$\Gamma_{\parallel} = \frac{E_{r0}}{E_{i0}} = \frac{\eta_1 \cos \theta_i - \eta_2 \cos \theta_t}{\eta_1 \cos \theta_i + \eta_2 \cos \theta_t}$   
 $\Uparrow_{\parallel} = \frac{E_{t0}}{E_{i0}} = \frac{2 \eta_2 \cos \theta_i}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i}$



Once again you can solve for the other ratio also which would be  $E_{t0}$  divided by  $E_{i0}$ , and that turns out to be

$$T_{\parallel} = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

You do not need to memorize these expressions all right once again for the quizzes either these expressions will be given or you will be allowed to bring a formula sheet right, but the interpretations are important that is all ok.

So, now that we have seen the reflection on the transmission coefficient for both the polarizations, it's abundantly clear that there are some commonalities between the two there are also some distinctions between the two right. The commonalities if you look at the reflection coefficient for the perpendicular and the parallel polarizations, the first thing you notice is that the numerator has a minus b form that is there is a negative sign between two terms. Irrespective of whether you have perpendicular or parallel polarization all right.

Which means that it is possible that under some scenario the numerator vanishes the reflection coefficient is 0 and the entire wave is actually transmitted all right, so the entire wave is actually transmitted and the reflection coefficient is 0 all right ok.

So, let us take the numerator ok and just equate it to 0 all right and see what is the consequence or what could the material configuration be right. So, for gamma perpendicular to be equal to 0 the only way is the numerator has to become equal to 0 so, we can say that ok it's a ratio ok.

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The image shows a digital whiteboard with handwritten notes. The notes are divided into two columns by a vertical purple line. The left column is for perpendicular polarization and the right column is for parallel polarization. Both columns start with the condition for zero reflection coefficient,  $\Gamma = 0$ . For perpendicular polarization, the equation is  $\eta_2 \cos\theta_i - \eta_1 \cos\theta_t = 0$ . For parallel polarization, the equation is  $\eta_1 \cos\theta_i - \eta_2 \cos\theta_t = 0$ . Both columns then set  $\theta_B = \theta_i$  and solve for  $\theta_B$ . For perpendicular polarization, the result is  $\theta_{B\perp} = \cos^{-1} \left\{ \frac{\eta_1 \cos\theta_t}{\eta_2} \right\}$ , labeled as Brewster's angle. For parallel polarization, the result is  $\theta_{B\parallel} = \cos^{-1} \left\{ \frac{\eta_2 \cos\theta_t}{\eta_1} \right\}$ , also labeled as Brewster's angle. At the bottom left of the whiteboard, it says "Brewster's Law". At the bottom right, there is a small video inset of a man speaking.

For gamma perpendicular to be equal to 0 the only way is ok ok. Now we already know that  $\theta_t$  is dependent upon  $\theta_i$  and the way  $\theta_t$  is dependent upon  $\theta_i$  is given by Snell's law

$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

Which means that for a given material configurations with intrinsic impedance  $\eta_1$  and  $\eta_2$  it may be possible for us to adjust the angle  $\theta_i$  in such a way that  $\eta_2 \cos \theta_t$  becomes equal to  $\eta_1 \cos \theta_i$  and let us say that in an experiment we do not have control over everything the only thing we have control over is say the incident angle  $\theta_i$  all right.

And so let us say that for a particular angle  $\theta_i$  if this were to happen what would that  $\theta_t$  be ok So, since this is a special case we just say that  $\theta_t$  to denote that it is a special case let us say that a new variable all right, is equal to  $\theta_t$  ok theta suffix equal to  $\theta_t$ . So, this is the special angle at which this numerator becomes equal to 0 ok.

Then a what we can do is we can rewrite this equation as

$$\eta_2 \cos \theta_B - \eta_1 \cos \theta_t = 0$$

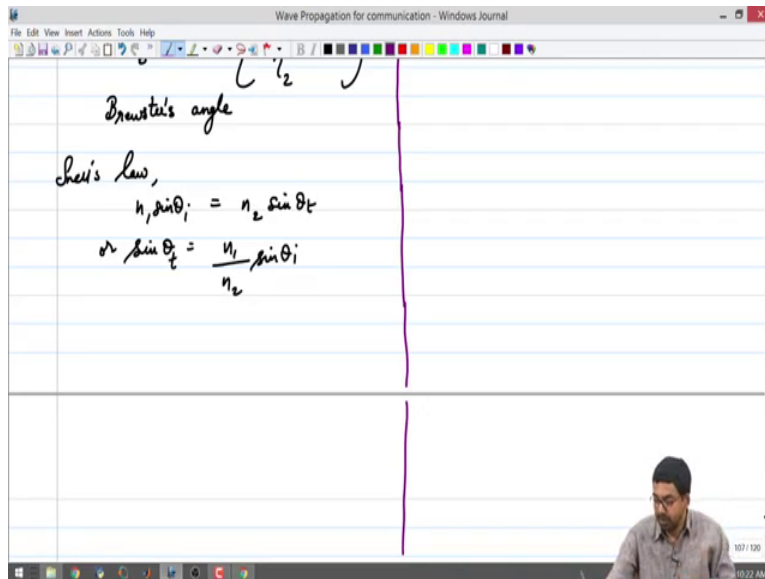
So, directly we can say that a

$$\theta_{B\perp} = \cos^{-1} \left( \frac{\eta_1}{\eta_2} \cos \theta_t \right)$$

So, this particular angle is known as Brewster's angle all right. So, this is known as Brewster's angle, however, there are some small caveats ok and we will deal with these caveats in a minute now ok.

Now, let us say that I have, I am going to start with Snell's law again so, I am having 2 x 2 variables over here, I am having  $\cos \theta_B$ ,  $\cos \theta_t$  so on the right hand side I have  $\theta_t$ , I want everything to be in terms of  $\theta_B$  ok, I want to get rid of this  $\theta_t$  because  $\theta_t$  is any way dependent upon  $\theta_B$  see it by Snell's law that is it alright. So, you could write this down in terms of a single variable  $\theta_B$  everywhere instead of having  $\theta_t$  on the right side.

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So, for doing that you could make use of Snell's law you can say that a right. So, we can say that

$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

$$\sin \theta_t = \frac{n_1}{n_2} \sin \theta_i$$

$$\cos \theta_B = \sqrt{1 - \frac{n_1^2}{n_2^2} \sin^2 \theta_B}$$

So, I could always do that instead of  $\theta_i$  it is a special angle I will call it as  $\theta_B$  and make a substitution in the prior case right.

Now there are also other ways to look into it in more detail in general form the Brewster's angle is just this much if you are given enough a you know input and if you are asked to find out the Brewster's angle you could find out from the first expression which is  $\cos^{-1}\left(\frac{n_1}{n_2} \cos \theta_i\right)$ .

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$$\beta_1 = \frac{2\pi}{\lambda_0/n_1} \quad , \quad \beta_2 = \frac{2\pi}{\lambda_0/n_2}$$
$$\Rightarrow \sin \theta_t = \frac{\beta_1}{\beta_2} \sin \theta_i$$
$$\cos \theta_t = \sqrt{1 - \frac{\beta_1^2}{\beta_2^2} \sin^2 \theta_i}$$

Because we know that phase constant in a medium

$$\beta_1 = \frac{2\pi}{\lambda/n_1}$$

in that medium so, you could always write this down as free space wavelength divided by  $n_1$  right remember that when the wave travels into a denser medium than vacuum the wavelength will become smaller similar to the case with transmission lines where we had seen on one side to be small,  $l_c$  on the other side was increased and the wavelength shrunk the velocity also reduced in the simulations right.

So, similarly the phase constant in a medium is written as  $2\pi/\lambda$  in that medium. So, it is  $\lambda/n$ . So, if you were to take  $\beta_2$  you will be having  $\frac{2\pi}{\lambda_0/n_2}$  so, this gives us some idea that  $n_1$  divided by  $n_2$  can also be written as a ratio of  $\beta_1/\beta_2$  ok.

So, this means that in Snell's law ok,

$$\cos \theta_B = \sqrt{1 - \frac{n_1^2}{n_2^2} \sin^2 \theta_B}$$

Now, one of the things that we will notice over here is  $n_1$  divided by  $n_2$  all right can also be written in other forms for example, you can write it as a ratio of permittivity and permeability  $\beta$  can also be written as the omega square root of  $\mu \setminus \epsilon$  right. So, you can always expand

these components into more and more constituents and try to look at it more deeply there is no there is nothing wrong with it right ok ok.

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$$\beta_1 = \omega \sqrt{\mu_1 \epsilon_1}, \quad \beta_2 = \omega \sqrt{\mu_2 \epsilon_2}$$

$$\Rightarrow \eta_2 \cos \theta_B - \eta_1 \sqrt{1 - \frac{\beta_1^2}{\beta_2^2} \sin^2 \theta_B} = 0$$

$$\& \eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}}, \quad \eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}}$$

Now, if I have written it with

$$\beta_1 = \omega \sqrt{\mu_1 \epsilon_1}, \quad \beta_1 = \omega \sqrt{\mu_1 \epsilon_1}$$

$$\eta_2 \cos \theta_B - \eta_1 \sqrt{1 - \frac{\beta_1^2}{\beta_2^2} \sin^2 \theta_B} = 0$$

$$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}}, \quad \eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}}$$

Then you could substitute that into your expression for finding out the Brewster's angle; it only gets more and more complicated when you make a lot of these substitutions ok. But let us go back and look at why we are doing all these things ok, the reason why I started with  $\beta$  and not with  $n$  is because simply we remember and to be a square root of epsilon  $\mu_r$ . In majority of the cases it's only indicative of a relative permittivity and relative to permit permeability and permittivity all right.

So, sometimes people tend to confuse us to what is going on beta, however, has an expression which is saying that is  $\omega \sqrt{\mu_1 \epsilon_1}$  does it have the effect of the vacuum permittivity and your relative permittivity vacuum permeability and relative permeability. So, chances of making mistakes while doing some calculations are a little lower is what I observed right.

So, once you have something absolute like this then there are some other inferences that one can start drawing for example, if you were to take the original expression where we started with numerator of the reflection coefficient is equal to 0 that's where we started this, and we were to substitute all right. So, you will be having

$$\eta_2 \cos \theta_B - \eta_1 \sqrt{1 - \frac{\beta_1^2}{\beta_2^2} \sin^2 \theta_B} = 0$$

This is the last expression that I have for  $\cos \theta_t$  is equal to 0, this is what that same expression would look like right. So, if you were to remember the previous expression it's more than enough for you to determine what is the Brewster's angle, but we are just trying to see if instead of writing with terms of a  $\eta$  and  $\theta_i$  and a  $\theta_t$  is it possible to draw some more inferences that are all right.

If you do the substitutions and if you wish to solve for  $\theta_B$  you will get a very large expression ok. I will not do the solving part alright I just plugged it into wolfram online and I got a solution for  $\theta_B$  which seems to match with the expression that people have been mentioning in the textbooks, I will write down the expression I got from wolfram right.

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So, I got

$$\theta_{B\perp} = \left\{ \frac{\mu_2}{\mu_1} \sqrt{\frac{\mu_2 \epsilon_1 - \mu_1 \epsilon_2}{\mu_2 \epsilon_2 - \mu_1 \epsilon_1}} \right\}$$

This is another expression that you can write for your Brewster's angle, the simplest expression is what we had before if you know that you are all set right.

So, given a problem configuration if you are asked to calculate you know how to calculate all right, this is just to see in which cases this Brewster's angle can exist in which cases it cannot exist extra ok. Now that we have written this right, we can do the same thing for the other polarization.

So, for gamma parallel to be equal to 0 we can take the numerator of the expression for the reflection coefficient and equate it to 0 right so we can write  $\eta_1 \cos \theta_i - \eta_2 \cos \theta_t = 0$  and then you make the same assumptions let us say that an experiment you do not have a control over everything, but only the incident angle and let that incident angle which is a special incident angle where the reflection coefficient becomes 0 be known as  $\theta_B$ .

Then you will rewrite as  $\eta_1 \cos \theta_B - \eta_2 \cos \theta_t = 0$ , this implies that

$$\theta_B = \cos^{-1} \left( \frac{\eta_2}{\eta_1} \cos \theta_t \right)$$

This is the simplest expression that you can use to get  $\theta_B$  for both the polarizations, one of the things that we notice here is that because the numerators for the reflection coefficients were different for both the polarizations the angle at which the reflection coefficient goes to 0 is also slightly different.

So, the angles at which the reflection coefficient becomes 0 for different polarizations is different all right. So, this is also known as Brewster's angle. The only thing is that Brewster's angle is also polarization dependent.

If you have perpendicular polarization you have to use one expression if you have parallel polarization you have to use some other expression which means, that when we are talking about Brewster's angle we also have to indicate the polarization for which the Brewster's angle is calculated all right.

So, likewise you can say that this is perpendicular this is parallel ok so, Brewster's angle for perpendicular polarization, Brewster's angle for parallel polarization this is how it is usually expressed ok. Now we can go through the Snell's law make substitutions for  $\eta_2 \eta_1$ , make substitutions for  $\theta_t$  in terms of  $\theta_B$  all right and you could make substitutions in terms of  $\beta_1 \beta_2$  then make it to a  $\epsilon_1 \epsilon_2$  extra it is the same drill over here right

Except that the angle here, turns out to be slightly different because our starting expression itself was different alright so, to indicate that we are dealing with parallel and perpendicular cases so, I am just writing a suffix of perpendicular and parallel, turns out that

$$\theta_{B_{\perp}} = \left\{ \sqrt{\frac{\epsilon_2}{\epsilon_1}} \sqrt{\frac{\mu_1 \epsilon_2 - \mu_2 \epsilon_1}{\mu_2 \epsilon_2 - \mu_1 \epsilon_1}} \right\}$$

It is a very big expression. I do not expect you to remember all this right. If you are able to do the first case which is this  $\cos^{-1}\left(\frac{n_2}{n_1} \cos\theta_t\right)$  extra it is more than enough.

So, there is a slight difference between these two expressions right, and a one can pay some attention to these slight differences all right one of the things that a I have touched upon in the past lectures is  $\mu_1 \mu_2$  being non a you know not the same as vacuum permeability and permittivity for I mean permeability.

So,  $\mu_1$  and  $\mu_2$  are permeability all right you can write them as  $\mu_0 \mu_r$  ok, however, in the majority of the materials there are no magnetic properties magnetic materials are rarer than the other dielectric materials. So, it is quite possible all right that for majority of the materials that you will encounter  $\mu_1$  and  $\mu_2$  is equal to  $\mu_0$  all right.

In general case this is the expression for your Brewster's angle for perpendicular and parallel polarizations in the most expanded form suppose you are not given anything except  $\mu_1 \mu_2$  you will be able to do this all right all right.

Notice that this expression has got nothing you just need  $\mu_1 \mu_2$  you will be able to find out the expression if you were to go back incident suppose you are transmitted angle is a given to you alright then you will be able to find out the Brewster's angle in the other case, but if nothing is given you will have to do some arithmetic's, but it is possible to write down the incident angle purely in terms of material parameters and arrive at what is the Brewster's angle ok.

Now this is the final expression, but one could also go one step further and make it a little bit simpler for many people, in the sense that the majority of the materials. I am just writing the majority of the materials; not all materials have to undergo this.

So, this is the expression that we have got as final and it's correct all right, but in case you want to make some other inferences which are more common than some other cases right. I can say majority of the materials have  $\mu_1 = \mu_2 = \mu_0$  so, that is you do not have any magnetism related properties in these materials.

So, vacuum permeability is the same as the material permeability or  $\mu_r$  is equal to 1 it's very common for most of the dielectrics, most of the metals also you will find that  $\mu_r$  is equal to 1 ok ok.

What then ends up happening is if  $\mu_1$  is equal to  $\mu_2$  is equal to  $\mu_0$  you can always go back to the final expression that you have substitute for  $\mu_1 \mu_2$  as  $\mu_0$  ok, the first thing you will notice is on the left hand side over here this ratio will become

$$\theta_{B\perp} = \left\{ \sqrt{\frac{\mu_0 \epsilon_1 - \mu_0 \epsilon_2}{\mu_0 \epsilon_2 - \mu_0 \epsilon_1}} \right\}$$



Now you can take  $\mu_0$  common from the numerator and denominator and you will just get a ratio which will look like ok, square root of  $\mu_2$  by  $\mu_1$  is just 1 all right and then I make a substitute for  $\mu_2$  and  $\mu_1$  to to be  $\mu_0$  so I can take  $\mu_0$  outside right.

$$\theta_{B_{\perp}} = \left\{ \sqrt{\frac{\epsilon_1 - \epsilon_2}{\epsilon_2 - \epsilon_1}} \right.$$

Once again I can take a minus sign all right the ratio has to be equal to minus 1 so, I get tan inverse of minus square root of  $\mu_0$  right ok, tan inverse of square root of minus  $\mu_0$  is what I would get ok. Generally tan inverse a of a square root of minus  $\mu_0$  means that you do not have a angle that you can you know attached to  $\theta_B$  perpendicular.

So, tan inverse of square root of minus some quantity means that tan inverse of j something right. So, there is no real, angle real angle  $\theta_B$  which will give you this relationship all right.

So, people argue that in the case of perpendicular polarization for non magnetic materials, you may not have a Brewster's angle ok. So, in this case. So, there is so many conditions in the case of perpendicular polarizations, if you have nonmagnetic material on the left hand on the right you may not get a  $\theta_B$  perpendicular because you will end up having tan inverse of square root of minus  $\mu_0$  all right.

Student: Minus 1.

Minus 1 all right whatever ok, so it is a.is it

Student:  $\mu_0$  is common.

Mu naught is common in the numerator and the denominator is it not?

Student: Yeah.

Ok so, it's  $\sqrt{-1}$  so, it becomes tan inverse of a j right. So, it becomes tough to find out what this  $\theta_B$  perpendicular is. So, in case of planar angles it is not going to be possible. There are some other arguments you can say that you can say that the angle is out of plane and all that.

So, if you have the entire diagram in the plane you can always argue that if you shoot the beam out of the plane you will get something else right, but we are not getting into solid angles ok.

So, it is not possible to find out a real angel  $\theta_B$  perpendicular so, people tend to say that for perpendicular polarization Brewster's angle does not exist ok, but it is under the assumption that you have nonmagnetic materials right. Now how about we apply the same condition to the right hand side ok.

So, for  $\mu_1 = \mu_2 = \mu_0$

$$\theta_{B_{\perp}} = \left\{ \sqrt{\frac{\epsilon_1}{\epsilon_2}} \right\}$$

So, if you consider majority of the materials to be non magnetic ok, then for a particular polarization you may not be able to find out a Brewster's angle, but for a other polarization you will be able to find out a Brewster's angle which, means, that for a given material configuration that is nonmagnetic.

The interface will have reflection coefficient being equal to 0 for some component of the electric field that is parallel, that is in parallel polarization, but if your incident wave is going to have some other polarization it may have some reflection coefficient coming in.

So the polarization plays a major role at the interface ok, in determining your reflection coefficient. This aspect is very different from transmission lines, the in the case of transmission lines you strictly add  $(\eta_2 - \eta_1) / (\eta_2 + \eta_1)$  or equivalently we wrote it as  $(z_l - z_0) / (z_l + z_0)$ .

If you are material properties dictated  $z_l$   $z_0$  extra then you are pretty much fixed whatever voltage and current waves you are launching will adder to this  $(z_l - z_0) / (z_l + z_0)$ . But in this case merely the material configuration is not enough, you also need to have the polarization being aligned in some direction all right. So, it has to be parallel polarization in the case of non magnetic materials to have reflection coefficient equal to 0 ok. So, this means that impedance matching in the case of electromagnetic waves should also consider polarization ok.

It is perfectly possible to have no reflection of 1 polarization, but a lot of reflection of the other polarization, it is perfectly possible all right. So, that is what it is going towards, so impedance matching in the case of electromagnetic waves having oblique incidence on interfaces is not a very simple problem ok.

Now, this has a lot of practical applications. Many of you when you buy eyeglasses ask for some anti reflection coatings extra. Essentially what they are trying to do is make sure that you know you get a lot of light inside, but you could also look at it as another problem. It say that there exists some angle Brewster's angle for the light that is coming to your glasses, you do not want any light to be reflected from the glasses directly because it needs to reach your eye ok.

So, you can say that I do not want reflection, so I want reflection from coating there, you could also determine. What could that it could say that  $\epsilon_1$  and  $\mu_1$  correspond to air,  $\epsilon_2$  and  $\mu_2$  are corresponding to some material that you are going to be coating. You could calculate a Brewster's angle. You can say that for this angle a lot of light is going to be coming in, that is only for 1 polarization or other polarization. It is not going to be working very easily.

So, you will need a more complicated you know coating for your glasses all right. So, impedance matching in the case of E m waves is not a very easy thing to do all right unless you have very

good control of the polarization extra. It is a disadvantage in that case, but it is also an advantage in some other cases for example, if you are driving all right or if you are outdoors, it seems that the polarization of glare ok, coming from natural sunlight it's very different from the polarization of use full information that is also coming from these material.

So, you will have glare cutting glasses ok or polarizing glasses extra. So, it is also possible that if you have reflection coefficient 0 for the useful information to come and reflection coefficient not equal to 0 for the glare part it is actually coming in glasses right. So, it could be used in useful ways or it can become a disadvantage when you are actually trying to solve problems. So, the only thing that I can say at this point is impedance matching in the cases of electromagnetic waves is non trivial ok, using just manipulating angles you need to do something else all right.

So, I think we will stop here one of the reasons why, I mentioned these is because a in the case of transmission line impedance matching was trivial all you needed to do was find out  $z$  I minus  $z_0$  by  $z$  I plus  $z_0$  and then make sure that you go to the center of the smith charts somehow ok here, however, if you go to the center of the smith chart for one polarization you may not be at the centre of the smith chart for another polarization. So, it becomes very important to draw smith charts for different polarizations.

And then decompose every incident wave into a parallel and a perpendicular component strictly a refill need not be parallel or perpendicular to the plane it can be anywhere in between those cases you will decompose it into two and then you will have reflection coefficient for one component being some value, the other component being some other value and then you have to take the summed up effect.

So, effectively what we are saying is polarization is a very important property. In the next class what we are going to do is account for polarization, when it travels from one medium to another medium ok

So, here we have made an assumption that the E field is perfectly perpendicular, the other cases it's perfectly you know in plane it's components what if it is in between ok? What happens and what happens to say linear polarization circular polarization on the interfaces? What can you expect? So, these are something's that we are going to see ok.

So, for now I will stop.