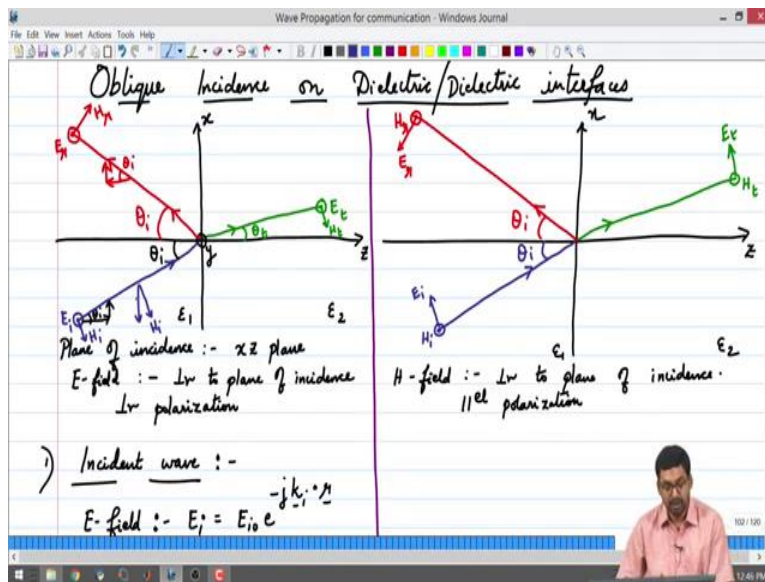


Transmission lines and electromagnetic waves
Prof. Ananth Krishnan
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture – 22
Plane Waves at Oblique Incidence- 1

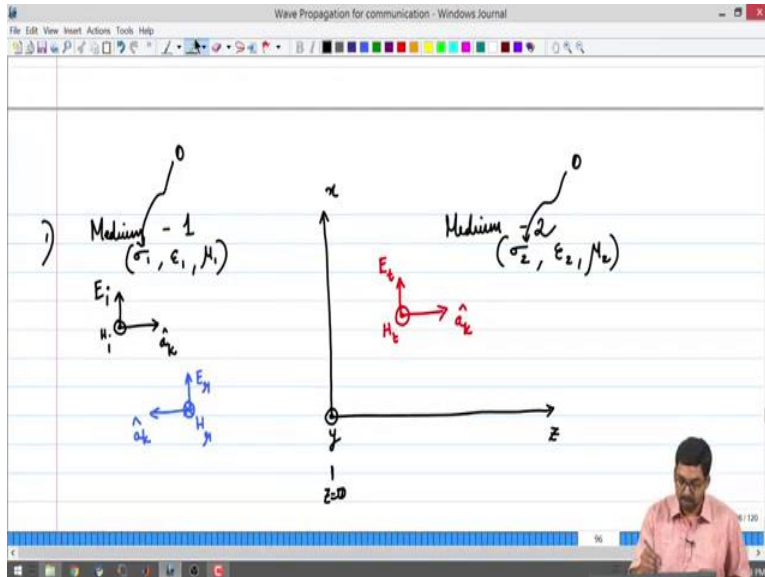
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We will get started right. So, I think we are going to start with ok. We are going to look at Oblique Incidence on dielectric dielectric interfaces for this class ok and I am going to divide the page into two columns for two different configurations ok.

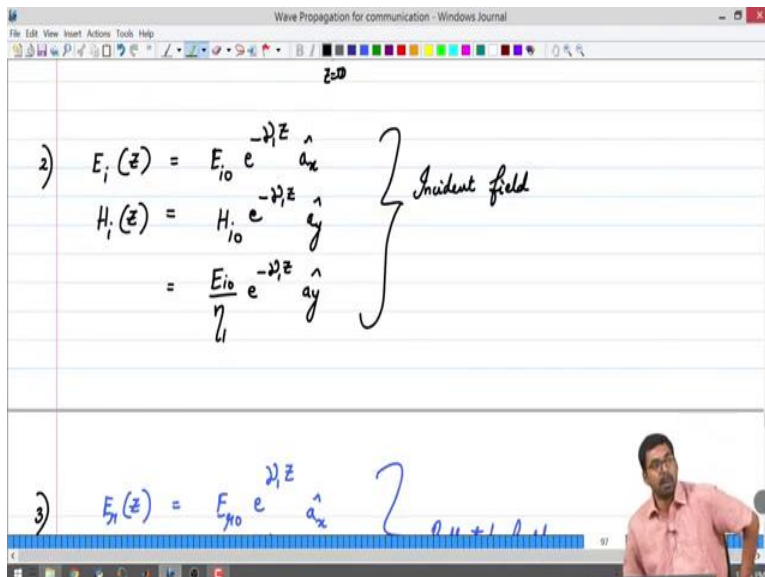
So, I will start with the first configuration ok. So, similar to the last lecture where we talked about normal incidence ok.

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So, we have an axis which is the x axis here, z axis here and to form a right handed triad we had a y axis marked in this way. We talked about a specific case where the polarization of the incident wave was having an electric field component in the plane of the diagram, and the magnetic field was perpendicular to the plane of the diagram.

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And the transmitted wave, the assumption was that there is no change in any polarization and then we calculated what is the value of a reflection and transmission coefficients. And we found that it looked exactly the same as what we got in transmission lines except that in transmission

lines we did not calculate transmission coefficient, we calculated only the reflection coefficient right.

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The screenshot shows a digital whiteboard with the following handwritten text:

b) At $z=0$,

$E_{\text{tangential}} \rightarrow \text{Continuous}$

$H_{\text{tangential}} \rightarrow \text{Continuous}$

$E_{1,\text{tan}} = E_{2,\text{tan}} \quad \& \quad H_{1,\text{tan}} = H_{2,\text{tan}}$

$E_i(0) + E_r(0) = E_t(0) \quad \text{---} \quad \textcircled{1}$

The whiteboard interface includes a menu bar (File, Edit, View, Insert, Actions, Tools, Help), a toolbar with various drawing tools, and a Windows taskbar at the bottom showing the time as 4:14 PM.

Nevertheless, the key thing that we saw in the previous lecture also was that the boundary conditions for the case of dielectric dielectric interfaces is that the tangential E field and the tangential H fields are continuous. Please remember that this is valid only for dielectric dielectric interfaces. A for dielectric metal interfaces we will have a separate lecture where we will derive the reflection and the transmission coefficients. But the boundary conditions are slightly different because it will involve surface currents ok.

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$$\frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \Gamma$$
$$\frac{E_{t0}}{E_{i0}} = \left(\frac{2\eta_2}{\eta_2 + \eta_1} \right) = T$$

So, we derived the Γ which looks like

$$\frac{E_{t0}}{E_{i0}} = \frac{2\eta_2}{\eta_2 + \eta_1} = T$$

You can assume this η_2 to be load in the case of transmission line and say that its Z_L or characteristic impedance and η_1 could be Z_0 characteristic impedance of the first transmission line and then you will get

$$\frac{Z_L - Z_0}{Z_L + Z_0} = \Gamma$$

This is what we had seen plus we also calculated another new quantity called T all right. The T was the transmission coefficient and it had the form

$$\frac{2\eta_2}{\eta_2 + \eta_1} = T$$

This is some quantity that we had not calculated before all right.

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$$\frac{E_{t0}}{E_{i0}} = \left(\frac{2\eta_2}{\eta_2 + \eta_1} \right)$$

$$1 + \Gamma = T$$

1 Medium $\rightarrow \sigma_1 = 0, \eta_1 \neq 0$
2 Medium $\rightarrow \sigma_2 = \infty$
 $\Gamma = -1, \eta = 0$

And there was a relationship between the transmission and reflection coefficients ok. And we found that $1 + \Gamma = T$ all right. This means that the transmission coefficient can be greater than 1 when we are talking about the value of the electric field all right. So, it means that your incident electric field can be 1 V/m, the transmitted electric field can always be higher than that, but it does not mean that the energy conservation is violated.

Whenever the electric field goes up in the second medium the magnetic field will proportionally go down to make sure that the conservation of power or conservation of energy is held correctly in the system, this is what we had seen broadly. And in today's lecture we will create a slightly more complicated system, where a wave is going to hit the interface at an angle with respect to the discontinuity of the plane of incidence. And this kind of setup is not there in transmission lines ok. So, this is a difference ok. So, I am going to mark the axis in the exact same way as we did before.

So, I am going to have the x axis marked vertically, here is my x axis ok. The z axis is what we were taking on the horizontal ok and to form the right handed triad you will need to have your y axis going out of the paper is the same coordinate system as what we had used in the last class ok.

Now, there are some small definitions that we will need to look at ok. First of all the term that we will call a plane of incidence is ok, it's a very important term plane of incidence ok. So, before defining this I will define a couple of things much more clearly ok. So, I am having a medium all right on the left hand side the dielectric constant is ϵ_1 on the right side the dielectric constant is ϵ_2 .

So, its a dielectric dielectric interface and I am also assuming that a μ_1 could be equal to μ_2 like in majority of the cases ok even if it is different does not matter, but there is no sigma coming

into the picture σ is 0 in both the sides ok. So, ϵ_1 and ϵ_2 , the interface between ϵ_1 and ϵ_2 is at z equal to 0.

So, this point here corresponds to z equal to 0 all right. So, z less than 0 corresponds to 1 dielectric with dielectric constant ϵ_1 , z greater than 0 corresponds to another dielectric with ϵ_2 . And we are assuming that these 2 media are semi-infinite which means that on the left hand side this medium is extending infinitely and on the right hand side this medium is also extending infinitely.

The reason for doing that is we want to avoid more interfaces coming into the picture suppose you had another material coming in then you would have one more interface and there would be a reflected wave from there and all that. So, right now we are taking a very simple case where there are 2 semi-infinite dielectric media on the left and the right ok.

So, the plane of incidence in this case is going to be the xz plane. So, whenever people talk about planes of incidence, you have to draw this diagram with the dielectric discontinuity ok and you also need to mark some incident reflected, transmitted waves extra.

So, I will go ahead and draw an incident ray ok. So, the incident ray goes and hits the interface at the origin ok, it can get reflected and part of it can get transmitted extra. So, that means, that we are going to be having some reflected ray going out like this and we are going to have a transmitted ray let us say going like this ok.

So, in this configuration the plane of incidence is xz ok and z has the discontinuity in the dielectric permittivity ok. So, now, having drawn these rays there are many configurations possible simply because our E and H fields are actually vectors all right, they are vector fields. So, directionality can come into the picture ok. Now, the arrows that are marked here correspond to the direction of the k vector or the direction of propagation more specifically its the direction of your pointing vector all right.

So, in order to satisfy these pointing vectors it is quite possible that you can have multiple configurations of E and H , we have already seen that in the case of plane waves. You can orient your thumb towards the direction of propagation, but you are free to rotate the other 2 fingers all right about the wrist and still you will be able to satisfy the condition of direction of propagation.

But the E field H field configurations can be known rotating about this transversal plane. So, we need to put some guidelines and we need to talk about more details ok. So, one of the configurations that I am going to draw on the left hand side ok is such that the electric field is out of plane, it's pointing out of the paper ok this is one configuration all right.

So, now I can orient my thumb and I can position my electric field to be going out of the paper and I get the direction of the magnetic field to be pointing downwards in the plane of the diagram. So, I can now mark the direction of the magnetic field as H . To be clearer I will use some suffix. I will say that this is my incident electric field, this is my incident magnetic field. Please make a note that the electric field is pointing out of the plane of incidence ok.

In other words, the electric field here is perpendicular to the plane of incidence ok. E field here has only y component ok right, its perpendicular to the plane of incidence and such a configuration we call this as perpendicular polarization ok. So, given a problem you should be able to identify the plane of incidence, then you should be able to draw an incident wave and clearly indicate what polarization you are considering. So, in this case the electric field is pointing out of this plane of incidence diagram that you have drawn.

So, it is perpendicular, so you can call it perpendicular polarization ok. Now, this forms a right handed triad and the wave is going and hitting the interface, since we are talking about oblique incidence we can start marking some angles ok. Now, we have to mark what is known as the angle of incidence ok. The angle of incidence is the angle made by the wave with respect to the normal drawn to the interface.

So, the normal here to the interface is the z axis itself all right. So, we have to mark some angle and we will say that this angle is θ incident or θ_i ok. Now, because ϵ_1 and ϵ_2 are different, you are going to be having intrinsic impedance mismatch at this boundary.

So, that means, that your wave is going to get reflected and part of it is going to be transmitted. So, let us start marking the red part which is the reflected ray. Now, we have to make some assumptions all right. Now, we will say that in this left hand side we are going to be sure that the electric field is always going to remain perpendicular to the plane of incidence right.

So, we are making an assumption that there is no polarization change all right with respect to the directionality of the E field. So, there is no suppose we say that the E field is going to be pointing out of the plane of incidence right, the magnetic field will adjust itself to compensate for the direction of propagation right.

So, if that is the case all right we can go ahead and say that my reflected electric field is going to have the same characteristics of the incident electric field in terms of direction all right, but now the k vector is going in another direction. So, in order to satisfy that we are saying that the magnetic field is going to be aligning itself to form a right handed triad.

So, since the direction is reversed over here the magnetic field has to flip its direction compared to the incident. So, we can go ahead and mark a reflected magnetic field looking like that right ok. Now, there is another quantity that we need to still mark the angle of reflection all right, I think many of us already know that the angle of incidence is going to be equal to the angle of reflection. So, you could mark it directly as θ_i ok, θ_i is equal to θ_r . So, we just use 1 variable to denote the angle of reflection ok.

Now, proceeding to the transmitted part ok. A, the transmitted part again we are making sure that the E field always is in a is perpendicular to the plane of incidence. So, I am going to have E transmitted pointing out of the paper magnetic field is simply going to follow a similar form as your incidence fields, so that means, it's going to look like this to form the right handed triad ok.

Now, there is one more quantity we need to mark here which is θ_t and θ_t can be different than θ_i ok. So, we do know that there is something known as Snell's law where

$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

So, it is quite possible that θ_t is different from θ_i . So, if ϵ_2 is different from ϵ_1 θ_t will be different than θ_i and to just indicate that I have marked the green colour with a smaller angle could also be a different angle right depending upon the ratio of permittivity extra, we will get to that later right.

So, this is one configuration and in this configuration we refer to this system as oblique incidents on a dielectric dielectric interface, but specifically we are talking about perpendicular polarization, that means, the E field is always for the transmitted reflected incident it is always perpendicular to the plane. Under this scenario we wish to calculate what is the value of the reflection and the transmission coefficient ok.

So, that is the objective of the lecture for this configuration. Now that we have drawn this configuration, let me draw the other configuration on the side. So, that we can work out one part and then go to the other part and try to mimic the same thing again right. So, I will go ahead and draw the axis again ok ok.

So, I am having the x axis and I am going to be marking an incident ray ok. In this case what we are doing is the magnetic field ok is going to be out of plane ok, out of plane of incidence ok. So, the plane of incidence is xz. So, I am going to make a mark here and I am going to call this as H_i , the direction of the arrow that is going to the interface is the direction of k. So, you can use your right hand all right and try to figure out what direction your E field will be. So, the E field in this case is pointing up all right. So, I am going to be marking an arrow.

Let us say that this is my E_i ok, the angle is there an issue error all right there is an error you immediately point it out ok before it gets big. So, its θ_i is the angle of incidence again all right. Now, again we can have a reflected and transmitted wave. So, I will also mark ϵ_1 , ϵ_2 ok, I am having the ray going out like this and the assumption we are making is the magnetic field does not flip its direction at all.

So, what happens is you mark the magnetic field in the exact same way. So, this is going to be your H_r , if this is your H_r the direction of the thumb will point to the direction of propagation and your magnetic field is going to look outwards. So, that means that your electric field is going to flip its direction.

So, now it is going to point something like this, ok ok. Once again we mark the angle of reflection to be the same as the angle of incidence and then we mark a transmitted ray also yeah. Once again the assumption is that the magnetic field remains out of plane and is not flipped extra.

So, we just mark this to be H_t transmitted and in order to form the right handed triad with the k vector pointing in this direction the E field will look ok. So, just to be clear over here. So,

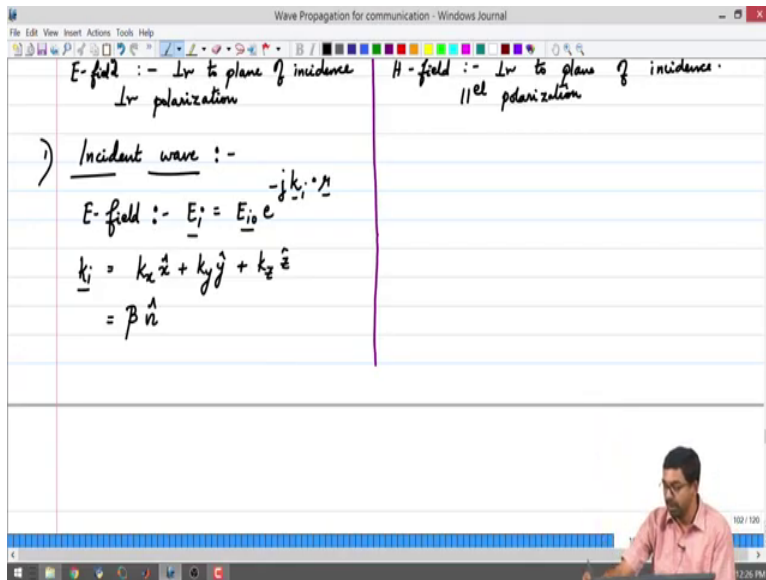
sometimes it can become difficult to remember this, E field perpendicular is perpendicular incident, E field in the plane is parallel ok because if you say H field perpendicular you may be remembering perpendicular for both the cases.

So, it's easier to remember in terms of the electric field because we define polarization as you know a dependence on the electric field and how with respect to time the electric field direction is changing. So, this particular case is known as parallel polarization ok. There are also other notations that are used widely for example, s and p polarizations, TE and TM polarizations extra.

But I think I will just stick to perpendicular and parallel because it's easy for me to remember, s and p is from some other language I do not remember what s and p usually means right, TE and TM, one can say because the transverse part of your electric I mean transverse electric field or transverse magnetic field extra that is also one way of remembering. So, you can pick one of them, but I am going to stick to perpendicular and parallel for this set, yeah ok.

Now, once having created these two systems then we have to start doing a few things. The first thing that we want to write down is expressions for incident reflected and transmitted waves in both these configurations ok. So, we start with the case on the left hand side.

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So, I am having ok, I am having an incident wave and I am going to write down some expressions specifically, I want to write down the expression for the electric and the magnetic fields ok. So, I will write down what we had seen before and then build on it ok ok.

So, I am denoting the electric field incident to be E_i ok, it does have some magnitude and a direction all right and it varies with respect to the space right as $-j(k_i \cdot r)$. We had seen that for

a plane wave that is traveling in an arbitrary direction the way to write down the expression for the electric field is

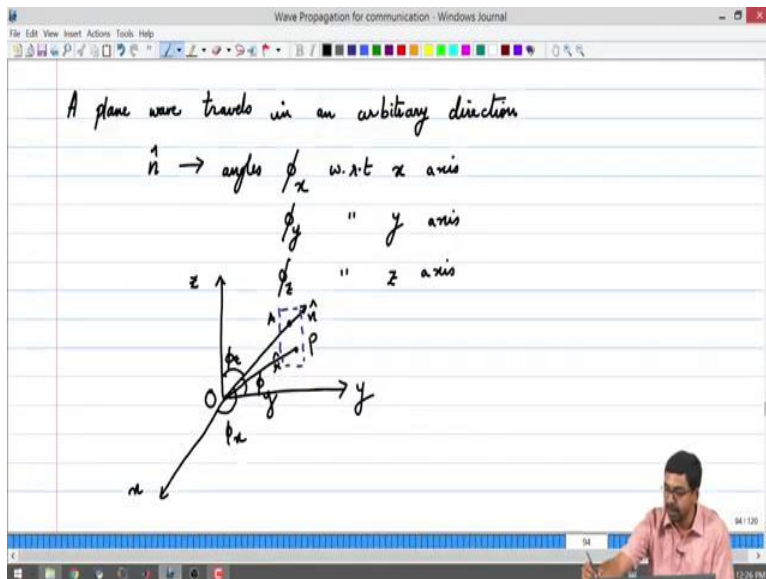
$$E = E_{i0} e^{-jk \cdot r}$$

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

So, we will do all of these now systematically, but you need to remember that this is the generic expression for the electric field. Now, our objective is to calculate correctly the value of k . Plug it into this expression and then start to look at what we can do with it ok. So, from one of the previous lectures we will remember that k_i in an arbitrary direction will have components in the x y and z axis ok.

So, in order to find the dot product between k and r you need to know k_x , k_y , k_z , then only you can find out the dot product right. We also saw that the k_i is actually $\beta\hat{n}$ ok. In the earlier lectures we had said that it's multiplied by the combination of the phase constant pointing in the direction of the unit vector along k right. So, if you go back a couple of lectures ok.

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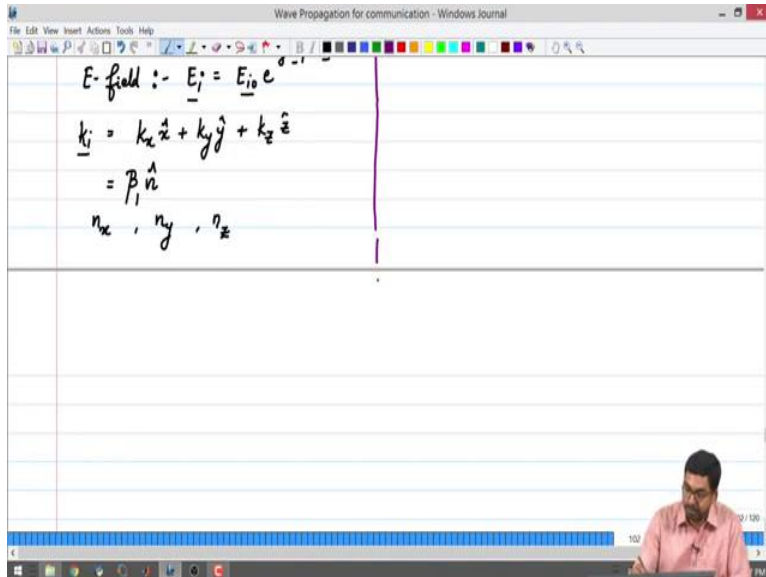
$$\hat{n} = \cos \phi_x \hat{x} + \cos \phi_y \hat{y} + \cos \phi_z \hat{z}$$
$$\vec{n} = x \hat{x} + y \hat{y} + z \hat{z}$$
$$\underline{k} = \beta \hat{n} = \beta \cos \phi_x \hat{x} + \beta \cos \phi_y \hat{y} + \beta \cos \phi_z \hat{z}$$
$$\frac{2\pi}{\lambda}$$

So, we had written the expression k to be equal to

$$k = \beta \hat{n} = \beta (\cos \phi_x \hat{x} + \cos \phi_y \hat{y} + \cos \phi_z \hat{z})$$

β times \hat{n} , where \hat{n} is the unit vector in the direction of travel. So, β is the phase constant it's simply $\frac{2\pi}{\lambda}$. So, now, we know two we can estimate $\frac{2\pi}{\lambda}$ provided λ is given to us in the problem all right it's a constant that can be evaluated. You need to get some input in the problem. Which means, that now I have to find out what my \hat{n} or the unit vector is going to be all right in order to find out the expression for the plane wave system that I am doing right.

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So, I can always say that β all right for the medium on the left hand side is β_1 or $\beta_1 \hat{n}$ all right to be very clear because permittivity is ϵ_1 and we already know that the phase constant is going to be different in different media all right. The velocity is also going to be different in different media these are 2 things that we have seen from simulations in the past ok. Which means that now the problem is really how I correctly ascertain what my n_x , n_y , n_z components are going to be. These are the components of the unit vector ok ok.

So, now, I go back to my diagram and the direction of travel is given by the arrow on this blue colour ray over here, this is the direction of the unit vector \hat{n} all right and its magnitude is going to be equal to 1 because it's a unit vector in that direction. So, I can take that unit vector, I can decompose it into having an x component and a z component sorry because we have drawn a weird axis all right, x component usually we mark it along the horizontal, but we have drawn x to be in the perpendicular direction.

So, all we have to do is the unit vector has to be resolved into a x component and a z component. Now, I can see clearly that I can decompose this vector ok by drawing a vector in this direction and a vector in this direction, that means, I am having positive z axis and positive x axis all right.

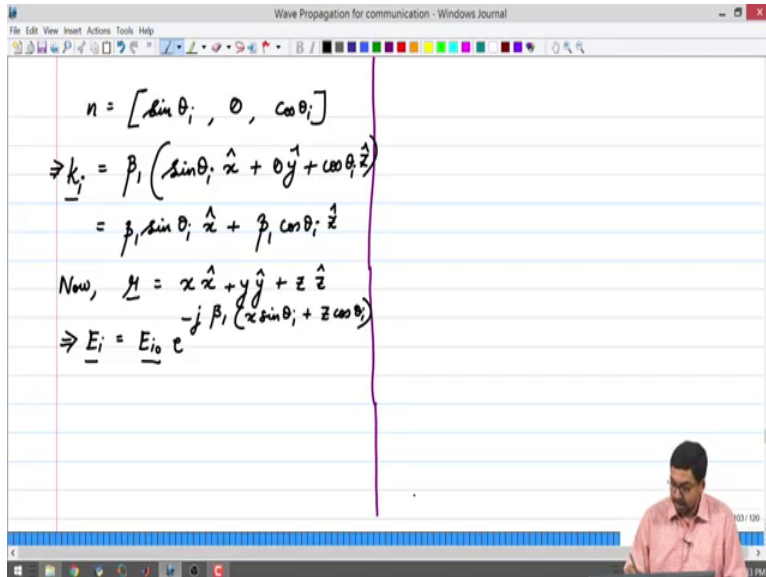
So, if I go along the positive z and along the positive x if I draw 2 vectors I can decompose my unit vector in that direction right. So, now, I have to find this vector horizontal and this perpendicular. So, I can use some identities for angles. So, this angle that we have marked is θ_i . So, in this triangle using the alternate interior angles are supposed to be equal. So, I can mark the angle on the left hand side here to be also θ_i ok. Now, I can write down all right \sin and $\cos \theta_i$, n_x which is the vertical component divided by n is equal to $\sin \theta_i$ all right. So, we will just write down the components of n ok.

$$n = [\sin\theta_i, 0, \cos\theta_i]$$

$$k = \beta_1(\sin\theta_i\hat{x} + \cos\theta_i\hat{z})$$

$$= \beta_1\sin\theta_i\hat{x} + \beta_1\cos\theta_i\hat{z}$$

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Now, what we can do is we can denote the position vector to be

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

We need to find the dot product of k_i and r ok.

So, now I can directly write down the electric field expression is equal to

$$E_i = E_{i0} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

So, given a configuration now you should be able to systematically do this and write down the expression for a plane wave electric field going in an arbitrary direction towards the interface. That part should be clear by now as to how you have to do it right. Now, in order to do this a little bit further now, that we have the expression for the incident wave, why don't we try to write down the expression for the transmitted and reflected wave. So, I will start with the reflected ok.

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2) Reflected Wave :-
$$\underline{E}_r = \underline{E}_{r0} e^{-j\underline{k}_r \cdot \underline{r}}$$
$$\underline{k}_r = \beta \hat{n}$$
$$n = [\sin\theta_i, 0, -\cos\theta_i]$$
$$\underline{k}_r = \beta (\sin\theta_i \hat{x} - \cos\theta_i \hat{z})$$

Now, your reflected wave can be written as say

$$E_r = E_{r0} e^{-jk_r \cdot r}$$

$$k_r = \beta \hat{n}$$

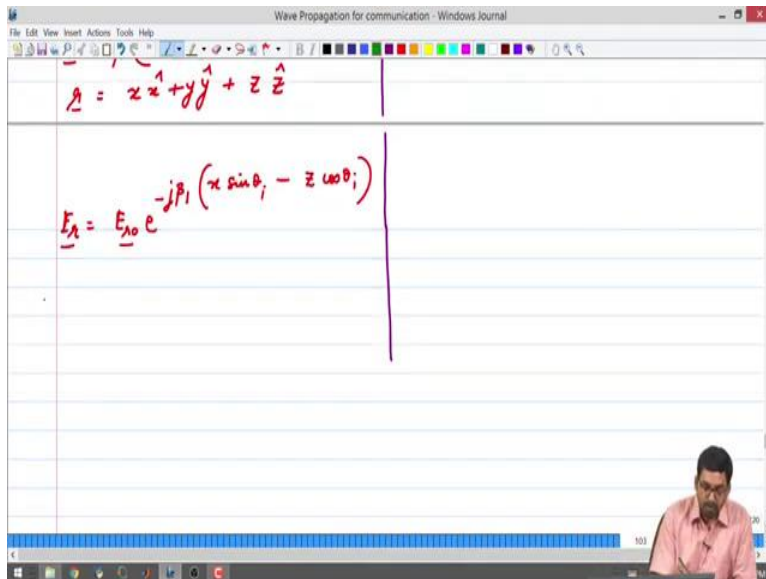
$$n = [\sin\theta_i, 0, -\cos\theta_i]$$

$$k_r = \beta (\sin\theta_i \hat{x} - \cos\theta_i \hat{z})$$

Once you have found this the next step is very trivial now all you need to do is write down the form for the position vector.

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

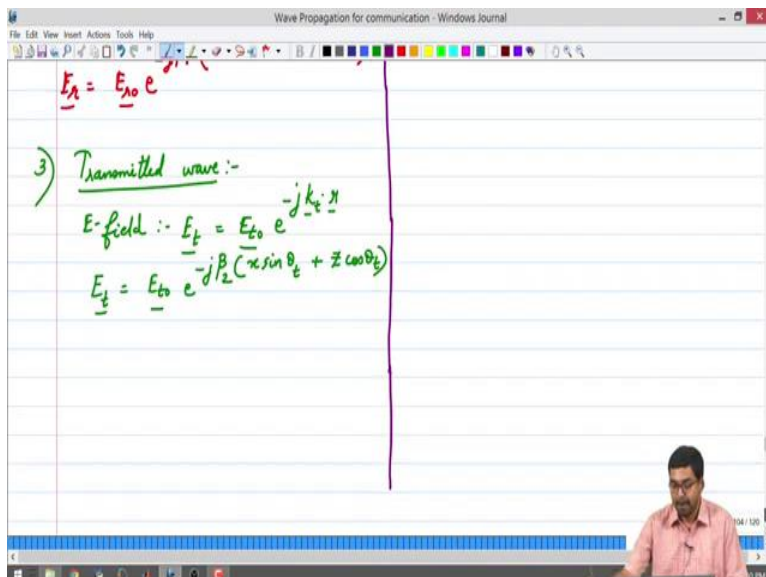
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Now I can write down the expression for my reflected electric field right.

$$E_r = E_{r0} e^{-j\beta_1(x \sin \theta_i - z \cos \theta_i)}$$

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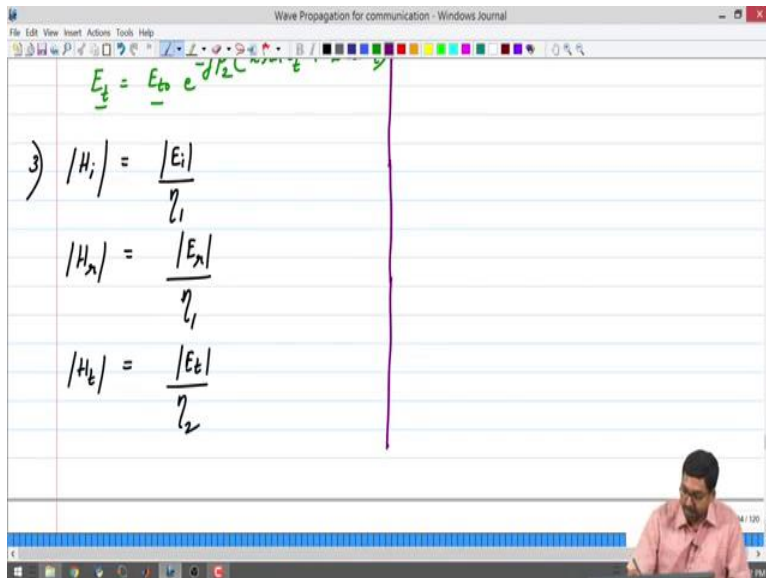
Now that we are experts with this part we can write down what is going to be the form for the electric field for the transmitted. Transmitted field let us say that I call it E_t , it's going to look like

$$E_t = E_{t0} e^{-jk_r r}$$

$$E_t = E_{t0} e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

Now, we have written the electric field expressions for the incident wave reflected and the transmitted wave all right.

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What we can do is we can make our life a little bit simpler by just saying that look at the incident magnetic field is going to follow Ohm's law. Magnitude of it it's going to be

$$|H_i| = \frac{|E_i|}{\eta_1}$$

So, I can simply say that this is going to be divided by eta that gives me the magnitude directly all right I know that its oriented along y direction ok wait which configuration are we seeing oh it's in the xz plane sorry ok H_r is simply

$$|H_r| = \frac{|E_r|}{\eta_1}$$

And just to be specific I will just mark this as η_1 for these 2 cases because they are in the same medium ok ok.

If you wanted to write down the detailed magnetic field expressions the procedure is going to be exactly the same as what you did for the electric fields ok. You can resolve the components of the math only thing is you will have to be careful with the components which component is going along the x direction which component is along the z direction extra and you will have to be a little careful with your a k vector that is it right.

But the procedure is exactly the same, there is no big difference here right. Now, once we have written these two expressions for example, we have written electric field expressions for the incident reflected transmitted wave in detail. Magnetic fields we have just said that using Ohm's law magnetic field is going to be equal to electric field divided by characteristic impedance that is what we have done so far right.

Now, what are we interested in? We are interested in finding out the reflection coefficient and the transmission coefficients in this configuration right. So, all we need to do now is we need to look at the boundary or the interface.

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the interface is at $z=0$
 Apply boundary conditions

$$E_i + E_r = E_t$$

$$H_i \cos \theta_i - H_r \cos \theta_r = H_t \cos \theta_t$$

At $z=0$,

$$E_{i0} + E_{r0} = E_{t0} \quad \text{--- (1)}$$

$$\frac{E_{i0} \cos \theta_i}{\eta_1} - \frac{E_{r0} \cos \theta_r}{\eta_1} = \frac{E_{t0} \cos \theta_t}{\eta_2} \quad \text{--- (2)}$$

So, here the interface is at conveniently z equal to 0 ok. So, the position coordinate with respect to z I mean z equal to 0 marks the discontinuity between medium number 1 and medium number 2. So, at z equal to 0 is the exact interface all right. So, at the interface we need to apply boundary conditions and the boundary conditions for dielectric dielectric interfaces is tangential E fields are continuous, tangential H fields are also continuous that is it all right.

So, now what we are going to do is we are going to apply these boundary conditions ok. So, all we can say is that the way we have drawn the configuration here is perpendicular to the plane of incidence which means that all the electric field is tangential to the interface ok.

So, the interface is like this and your electric field is like this, so it's all tangential. So, you can simply write down that

$$E_i + E_r = E_t$$

This is simply because E_i and E_r are assumed to not be flipped in directions. So, E_i is pointing 1 way E_r is also pointing in the same way. So, we have added E_i with E_r and it has to be equal to whatever is being transmitted E_t .

So, this is actually a give should give you some idea that transmission coefficient could be greater than incident, I mean transmission coefficient could be greater than 1 and the transmitted electric field on the right hand side could be greater than incident electric field right. So, it's not a very you know unnatural consequence, we still have to remember that the magnetic field will adjust itself if this happens. So, this is one boundary condition ok and for the magnetic field we have to take the tangential components and then we have to see that it is continuous. So, we have to go back to the diagram ok.

Now, our H_i is looking like this for the sake of some clarity. What I will do is I will mark another H_i here right and I need to take the tangential component. So, it is pointing like this ok. So, it is pointing like this: I need to find this component and then I have to mark it for all the 3 rays at the boundary and I have to apply the tangential field to the right.

So, I can use trigonometry to find out the angles ok, I can use trigonometry to find out the angles I will not go into too much detail, but I will just write down, it will look like $H_i \cos \theta_i$. Now, remember that for the magnetic field the direction was flipped for the reflected case all right.

So, you have to be careful you are having 1 vector going like in 1 direction another vector in the other direction. So, you have to make

$$H_i \cos \theta_i - H_r \cos \theta_i = H_t \cos \theta_t$$

and this is that z equal to 0. So, you can also be more specific saying that at z equal to 0, this means that whatever you wrote within your exponential, exponential minus $j \cdot k \cdot r$ all right it is at z equal to 0 ok.

So, this just means that you are having

$$E_{i0} + E_{r0} = E_{t0} \dots \dots \dots 1$$

$$\frac{E_{i0}}{\eta_1} \cos \theta_i - \frac{E_{r0}}{\eta_1} \cos \theta_i = \frac{E_{t0}}{\eta_2} \cos \theta_t \dots \dots 2$$

So, now we have some equations. We have equation number 1, equation number 2, now we have to say what is known, what is unknown extra and what we wish to solve for. Let us say that

η_1 and η_2 are known that you are given some value of permittivity permeability for medium number 1 permittivity and permeability of medium number 2. You will be able to find out the characteristic impedance all right for medium number 1 and medium number 2.

So, η_1 and η_2 are known ok, E_{i0} , E_{r0} , E_{t0} extra alright let us say that we want to find out the ratio of E_{r0} to E_{i0} , E_{t0} to E_{i0} . So, there are actually E_{i0} if you consider it as unknown, E_{r0} as unknown E_{t0} as unknown and there are 3 unknowns and we have only 2 equations right. So, we cannot solve, so we take the ratio of E_{r0} to E_{i0} , E_{t0} to E_{i0} all right.

Student: Cos θ second equation.

Oops Cos θ_i , Cos θ_t both the cases. So, this will be Cos θ_t on the right side ok. So, again θ_i is given θ_t is something that you can calculate from a Snell's law because you have been given ϵ_1 , ϵ_2 , μ_1 and μ_2 right then you can calculate

$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

then you should be able to calculate θ_t these are known quantities. We wish to find out what is E_{r0} by E_{i0} and E_{t0} by E_{i0} right.

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The screenshot shows a Windows Journal window with the following handwritten equations:

$$E_{i0} + E_{r0} = E_{t0} \quad \text{--- (1)}$$

$$\frac{E_{i0} \cos \theta_i - E_{r0} \cos \theta_i}{\eta_1} = \frac{E_{t0} \cos \theta_t}{\eta_2} \quad \text{--- (2)}$$

$$\Gamma_{\perp} = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 \cos \theta_i - \eta_1 \cos \theta_t}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t}$$

$$T_{\perp} = \frac{E_{t0}}{E_{i0}} = \frac{2 \eta_2 \cos \theta_i}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t}$$

So, in other words you can solve these 2 equations simultaneous equations and you can write down Γ which is nothing, but E_{r0} divided by E_{i0} ok. The solution looks like

$$\Gamma_{\perp} = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 \cos \theta_i - \eta_1 \cos \theta_t}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t}$$

It is a big formula and it is a very confusing formula. You will not be expected to remember this for your quizzes extra alright if needed. All these formulas can be given or you could make a formula sheet. We will work it out, but you do not need to memorize it. To just make an additional point that this Γ and T that we had previously written before did not have the polarization marked all right. From now on we have to be very clear when we are talking about Γ we have to say which Γ is it parallel polarization or perpendicular polarization extra has to become clear.

So, we are talking about Γ suffix a perpendicular symbol to clearly indicate that this we have calculated only for a specific E field H field configuration ok. Now, once you have calculated this you can also go ahead and try to calculate what the transmission coefficient T will look like

$$T_{\perp} = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

You can already see that the denominators are identical all right and it will look like

$$1 + \Gamma_{\perp} = T_{\perp}$$

So, there are no issues with that. It looks similar as the case with the normal instance where $1 + \Gamma = T$ For similar I mean polarization configuration ok. So, given a problem like this you should be able to estimate the Γ T and also say for example, if an incident wave electric field is given you should be able to figure out what the reflected electric field and the transmitted electric field are going to be like ok.

Specifically, you should be able to tell what is the value of the electric field that is transmitted reflected given a configuration with an incident electric field ok ok. Now, we can always go back and start filling the right hand side for each of these right and we will do it in the next class and then we will start comparing the left and the right hand sides all right and we will also see some conditions. If you look at the Γ perpendicular condition over here, the numerator has a negative sign already it should trigger some you know thought that is it possible for the numerator to be equal to 0 ok.

What would that be all right, is it possible for the numerator to be positive, is it possible for the numerator to be negative all right, when these things would happen what material configuration do you need, what is the relationship between ϵ_1 and ϵ_2 that will give you these things alright.

So, we have to see when Γ will be 0, Γ will be negative, Γ will be positive all right and the same thing we have to do for the other polarization case also. And then we will try to see what condition corresponds to what and we will give unique names to each of these conditions all right. Some of these conditions could be you can say total deflection or you can say Brewster's angles something like that all right.

So, we are going to assign different cases for these different conditions. So, right now we have just done in for perpendicular polarization. In the next class in a little swift manner we will write this for the parallel polarization now that you are aware of how to do this ok. And then we will try to draw inferences from what can happen when a plane wave strikes the interface assuming that the E field or the H field do not flip right only one of them flips and the other one remains a constant ok. And then we will try to make some inferences from them, yeah I will stop.