Transmission lines and electromagnetic waves

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Lecture – 21
Plane Waves at Normal Incidence

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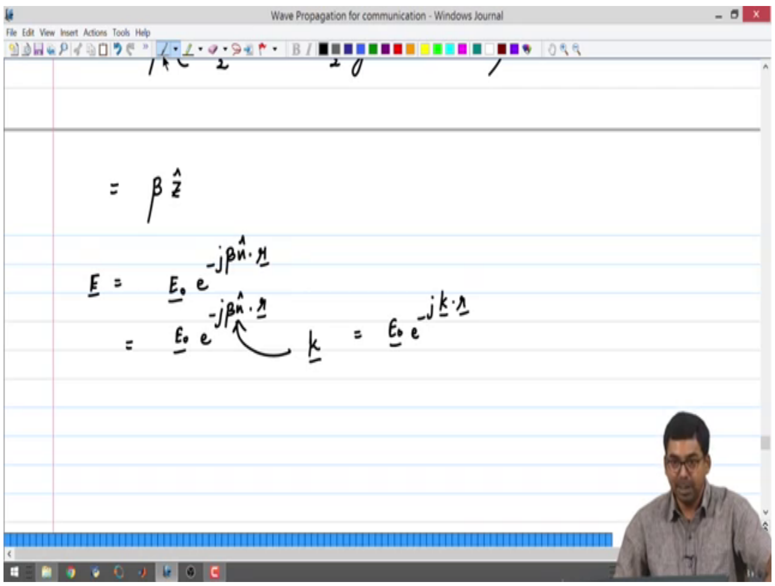

Now, we will begin ok. The place where we stop was writing down the expression for k

$$k = \beta \hat{z}$$

And a the expression that we could write for the electric field is

$$E = E_o e^{-j\beta \hat{n}.r}$$

$$= E_o e^{-j\beta \hat{n}.r}$$

$$k = \beta \hat{n}$$

$$E = E_o e^{-jk.r}$$

So, we had to define a problem statement like this where you have a point in space all right.

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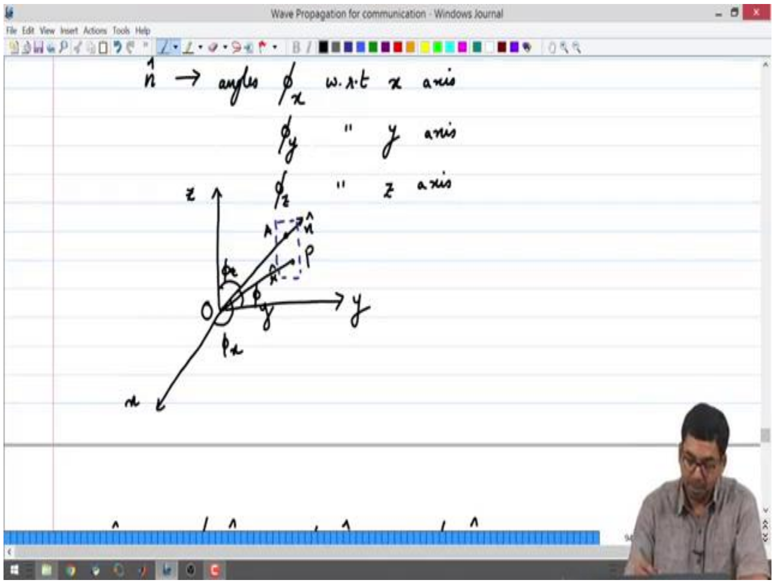

And you have a plane wave front, you are having the direction of the normal, a vector connecting the point on that plane to the origin all right. You have to take the dot product between r and n all right. So, you are taking a projection of one vector on the normal and trying to figure out the equation for the electric field ok.

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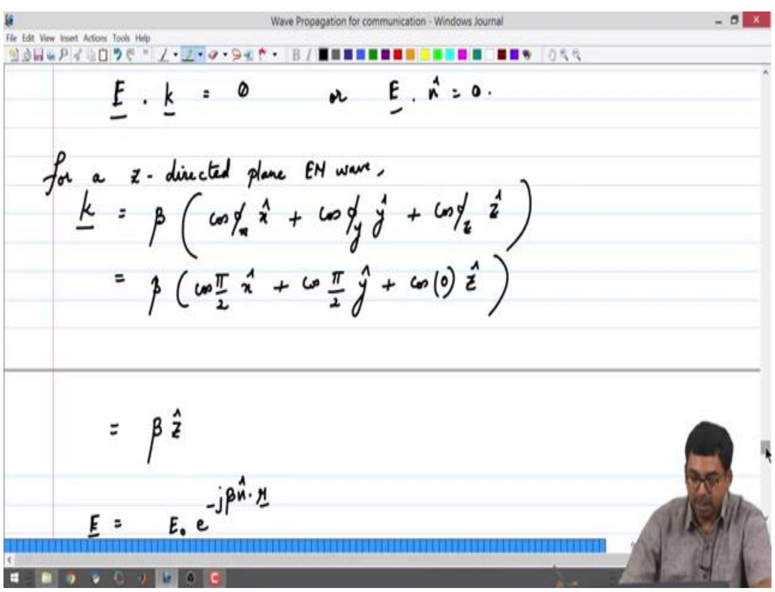

So, last time we had seen how to write down the k vector all right. You will have

$$k = \beta(Cos\ \phi_x \hat{x} + Cos\ \phi_y \hat{y} + Cos\ \phi_z \hat{z})$$

$$= \beta(Cos\frac{\pi}{2}\hat{x} + Cos\ \frac{\pi}{2}\hat{y} + Cos(0)\hat{z})$$

$$= \beta\hat{z}$$

So, end up with $\beta\hat{z}$ ok, which means that one could start writing down the expression for the electric field over here.

So, you could say that this

$$E = E_o e^{-j\beta\hat{n}.r}$$

So, here we can say that the beta n hat that we have found is

$$k = \beta\hat{n}$$

is the normal to the surface that we have already found that is the k. So, you could always write this down as $k.\vec{r}$ all right. Now,

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

corresponding to xyz coordinates of that particular part. This is how some you will write down the expression for a plane wave in an arbitrary direction, given a coordinate system.

So, you will try to find out k which is a point all right which is a vector joining the origin and the point on the plane that you are going to be talking about that will have $x\hat{x} + y\hat{y} + z\hat{z}$. And then you are going to be dealing with a finding out k, which is going to be simply $\beta\hat{n}$. So, you have to find the normal to that plane and then you have to multiply it with the constant all right.

And so, when you take this expression, this is giving you the expression for a plane wave electric field right travelling along a particular direction given an arbitrary coordinate axis ok. And in order to verify what you can do is you can substitute $k = \beta\hat{z}$, for example, you can take a plane you know which is perpendicular to the z direction all right, that is how your face front is going to be for a z directed wave. The face fronts are going to be like forming slices in the z axis.

So, you can take a point for example,

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

was the point corresponding to r. You can simply take one of the points to be $z\hat{z}$ ok, right at the center x equal to 0, y equal to 0 and z and you will end up getting

$$E = E_0 e^{-j\beta z}$$

ok. So, that is how you write down an expression and to check whether the expression is valid or not, you can always substitute $k = \beta\hat{z}$ for a z directed wave. $\vec{r} = z\hat{z}$ and if we take the dot product of the two, you will get $e^{-j\beta z}$ which is the original expression that we had written for the plane waves before.

Now that we know this all right, the problems can be expanded to a variety of things. We can start to look at what will happen at an interface, what will happen if the wave travels at a particular angle to an interface all right. What happens when it is normal to an interface extra. So, that is that is why we are writing down these expressions. We want to understand what a plane wave will undergo if it hits an interface at normal incidence or at some other angles of incidence, we want to be able to write down the expressions ok.

But just before we proceed I will also write down a couple of things that will need some thoughts ok.

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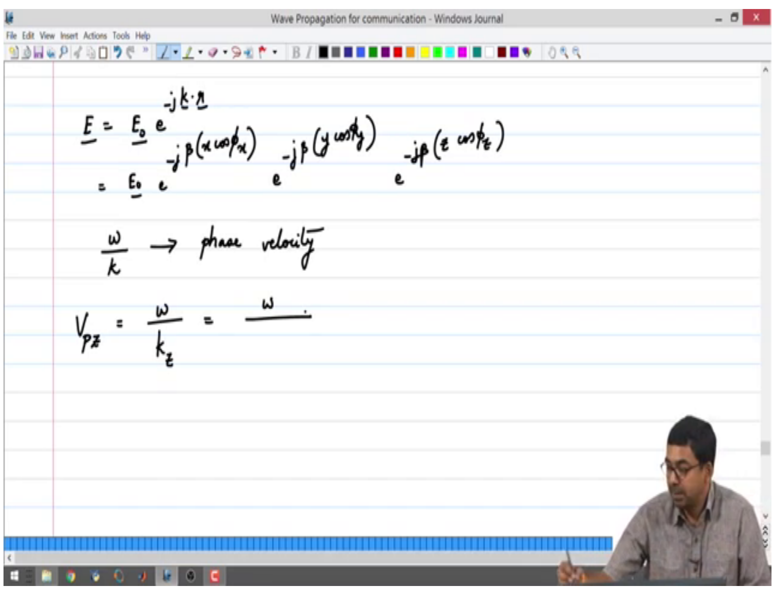

So, we have the electric field expression to look like

$$E = E_0 e^{-jk.r}$$

$$= E_0 e^{-j\beta(x cos\phi_x)} e^{-j\beta(y cos\phi_y)} e^{-j\beta(z cos\phi_z)}$$

So, you can write this down as beta ok and you can take the dot product with r,

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

Now another thing to remember in this cases is similar to your transmission lines, ω/k is phase velocity all right and suppose I want to find out, so, it is represented by say

$$v_{pz} = \frac{\omega}{k_z}$$

Remember that ω is in rad/s all right. So, it is not a vector quantity or anything, but the denominator is k. Now we have just now written k to be an expanded vector,

$$k = \beta\hat{n} = \beta(Cos\ \phi_x\hat{x} + Cos\ \phi_y\hat{y} + Cos\ \phi_z\hat{z})$$

it had an x y and z. So, k is a vector over there. Suppose I want to take the z component of the velocity,

$$v_{pz} = \frac{\omega}{k_z}$$

I already know from the previous class all right,

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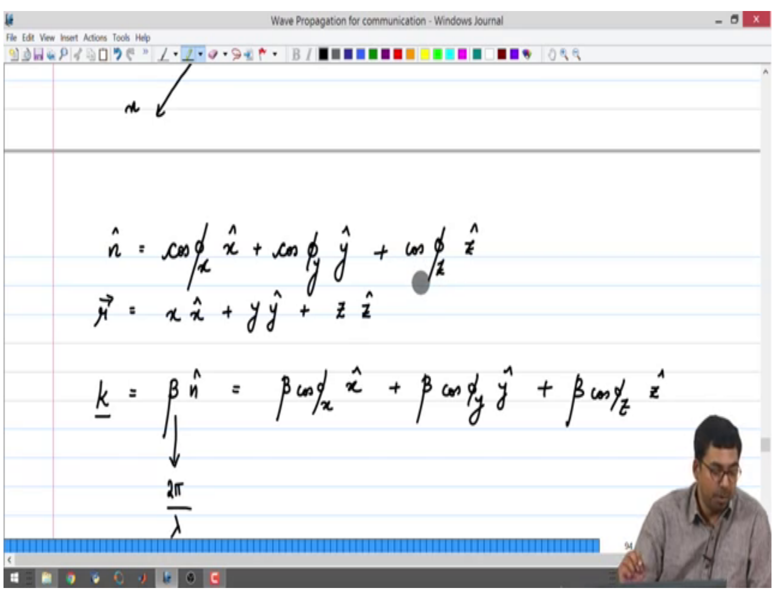

kz is going to be alright this part over here all right all right. So, I am going to be having $Cos\ \phi_z$ all right.So, I am going to take only the only part which is not the vector part, I am only going to take the coefficient of the vector right.

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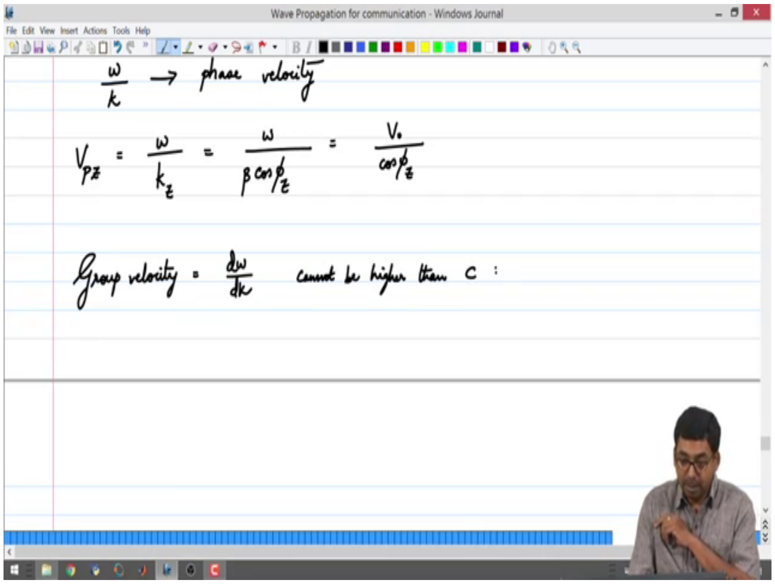

So, I am going to be having

$$v_p = \frac{\omega}{k_z} = \frac{\omega}{\beta Cos\phi_z} = \frac{v_z}{Cos\phi_z}$$

just taking the unit's, ω is in radians per second, beta is in radians per meter

So, I am going to be having some velocity divided by $Cos\phi_z$ all right. Now, suppose we think of this medium to be vacuum or air all right, the value of ω is going to be something all right some radians per second. So, you can take a specific wavelength, we can calculate the frequency corresponding to that wavelength and then you can calculate the ω for that particular wavelength or frequency ok. Similarly you can calculate the beta for that particular wavelength or frequency as $\frac{2\pi}{\lambda}$ all right.

Then you can substitute ω/β will correspond to the velocity in free space which will be $3 * 10^8\ m/s$ second ok and it is divided by $Cos\phi_z$. Now, one will have to be clear about what we are looking at, divided by $Cos\phi_z$ means that too many questions will start arising. $Cos\phi_z$ will go between plus and minus 1, it passes through 0, all right which means that the phase velocity here that what we are talking about can be equal to the bulk velocity that we are talking all right in the medium can be infinite can be positive can be negative [laughter] all right.

And there is no you know limits to what the phase velocity can do, which if you go back to my original you know discussion that we had about phase and group velocity in one of the lectures said that we should not be deriving lot of inferences from phase velocity ok because, it simply does not have any limit's. It could depending upon the value of ϕ which is the angle with respect to the z axis in this case right. It could be positive negative 0 extra I mean it could be infinity for all we know right. So, the phase velocity could be any value if you take it like this all right. So, it does not convey any meaningful information all right.

So, conclusions based on phase velocity should be very carefully made because it may appear like you will get propagation in one direction faster than the speed of light in vacuum for specific value of $Cos\phi_z$ ok. Immediately questions will start arising as to whether you know you are doing something wrong with the theory of relativity and all these things. So, one has to be careful. Phase velocity being higher than velocity in free space is not at all uncommon ok. Simply by the expression that we have written, the phase velocity could be higher than the velocity in free space but it does not mean anything ok.

So, this is something that one has to keep in mind. We have not yet seen the other type of velocity that is more important, we will be going there slowly. The other meaningful one that people always talk about is what is known as group velocity all right. It's dω/dk in the strict access small change of a k giving rise to a small change of ω and you take the ratio of that and that actually gives you the group velocity all right.

So, group velocity is a more meaningful term where you can draw inferences ok and we can write down so far from what I know is at least in most of the claims, this has not been the case. There have been a few claims about a couple of terms, superluminal propagation all right, but the publications have drawn heavy criticism saying that the experiments are not very clear or they need additional proof. So, it is not clear whether they are talking about group velocity or phase velocity extra. So, even the most experienced researchers could make these mistakes because experimentally it is very tough to separate things all right. So, in Google you can have a look at the term superluminal propagation all right.

You will have a lot of hit's and you will have also run into some papers on which there were some tough debates ok. It is good to know this because you will then understand you know a publication journal is not free from criticism even after it is published. So, all right. So, most of the time in very good journals after it is published is when people start reading. So, you will get more criticism only after that, but it is a good article nevertheless because the rebuttals tell you about group velocity and phase velocity in a broad sense and while some claims need to be re looped all right. So, we can have a look at superluminal propagation on Google.

So, now that we know how to write down the expression for an electromagnetic wave and also we know the pitfalls that is if you did get a solution for a phase velocity is higher than c, it is nothing to panic could actually be the solution to a problem. Now that we know this much, I think we can proceed to the next part, which is dealing with interfaces ok. So, there are multiple things that were the deal with respect to interfaces. So, interfaces means that you have one medium on one side another medium on another side and interface in between these 2 media. In transmission line the equivalent would be you had one transmission line through L & C parameters on one side and then you had another transmission line that is joint to it.

So, it had different L&C, it had so, it had different characteristic impedance, it had different absorption coefficient extra, but they are connected to each other and the wave travels from one transmission line to another transmission line. So, in this case you are going to be having wave going from one medium to another medium. We will start with the case which is similar to the transmission line all right, where we are talking about propagation in such a way that the wave hit's the interface normally. This would be the case equivalent to transmission line.

Suppose you connected 2 transmission lines the direction of travel is you know in such a way that it hit's the interface perpendicularly. So, this is a case where it would be very very similar to transmission line reflection coefficients extra. But since we have written down the expression for electric and electric fields in arbitrary directions, we can see there is a difference between the electromagnetic waves and the voltage and the current waves in the transmission lines that we will see a little later.

Also we know that another difference is polarization, we have not considered polarization in transmission lines, but here we have seen circular elliptical and linear polarization. We have to see what happens to the polarization when the wave goes through an interface; these are the broad objectives ok.

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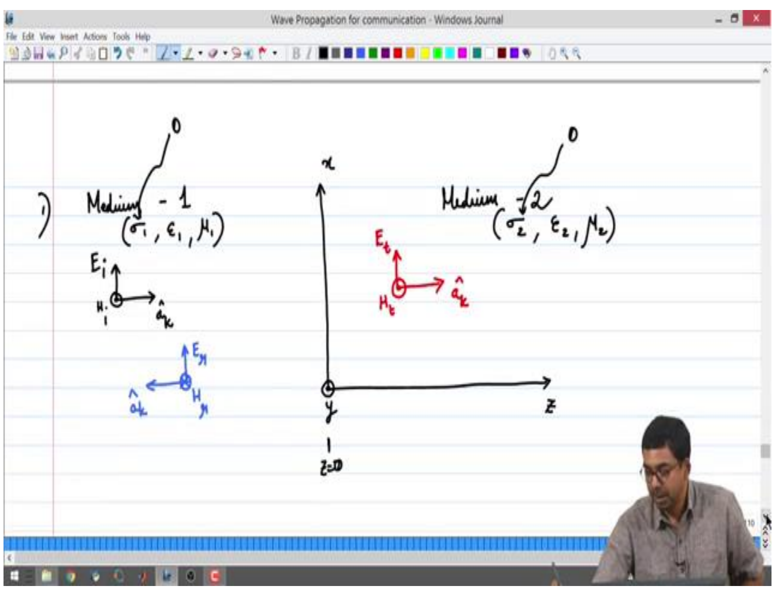

So, I will start with a set up. The setup here is I am going to take an axis all right all right. I am taking an axis and the direction of travel is along z ok. So, inside I can take the right hand, y is out of plane ok and I will mark medium 1 on the left side all right, medium 2 on the right side ok and most generic case medium 1 can have a value of conductivity, permittivity, permeability and here it could have σ_2, ϵ_2, μ_2, all the values could be different or one of them could be different with the most generic case everything is different.

Now, we also need to look very carefully at these interfaces. We are starting with dielectric dielectric interfaces. Later on we will see interfaces between dielectric and metal extra. So, for now I am going to say that let us assume that the conductivity is 0 in both cases ok. So, I have ϵ_1 μ_1 on one side, ϵ_2 μ_2 on the other side right and I am going to be having a wave travelling going from left to the right, hitting the interface and something happens ok. So, in order to make that clear ok, I am going to be drawing some vector diagrams right. So, I am saying that the unit vector in the direction of k is going this way all right. So, I am having a wave travelling from left to right ok.

And now I have to make some assumptions. So, I am going to start with the case where the electric field is oriented this way ok. Electric field is oriented this way, direction of propagation is horizontal which means that using my right hand rule I can figure out how my magnetic field is going to be. In this case it is a right handed system. So, the magnetic field is going to be out of plane in this case ok.

Now, this wave is a plane wave alright and it's direction of travel is from left to right it goes, hit the interface ok it's the interface. Now, we know that since ϵ_1 μ_1 on one side is different from ϵ_2 and μ_2 on the other side there is going to be a change in characteristic impedance between the 2 media all right.

So, already we know from transmission lines that when we have differences in characteristic impedances between the 2 transmission line sections, you will be having some reflection ok. So, there is going to be some reflection and reflection is also going to be a plane wave alright and we have to make some assumptions ok in order to proceed further all right. Now what we are going to do is we are going to have a reflected plane wave also and we are going to assume that let the electric field not flip it's direction ok, k vector is flipped, obviously, it is travelling the reflected wave is travelling from right to the left.

So, if you were to mark you know the unit vector for k, in this way you cannot have the same configuration of electric and magnetic fields ok. So, if you assume that the electric field is unchanged all right then you will have to flip your magnetic field ok which is the only way it will form a right handed triad right. Now in order to distinguish between these 2, we can add some subscripts. I can say that Ei corresponding to E incident, Hi corresponding to H incident ok and Er corresponding to reflected, Hr corresponding to reflected ok.

Let us also make a few more things clear. The point on the left going to call it has z equal to 0. So, z equal to 0 forms the interface between the left and right hand side semi infinite media ok and the wave is travelling from left to right, portion of it gets reflected and assumption is the electric field does not flip. If the electric field does not flip then the magnetic field has to flip. This is one assumption. On the other hand, if you assume that the electric field has flipped then the magnetic field direction will remain unchanged ok.

So, that is also one way of looking at it ok. Now the other thing that we know from transmission lines is that you may have some reflection, but for the reflection coefficient to be equal to 1, we needed to have a short circuit or an open circuit. In the case of electromagnetic waves, you needed to have a perfect electric conductor or a perfect magnetic conductor to have a reflection coefficient of 1, which means that if you do not have a perfect electric conductor or a perfect magnetic conductor, you are not going to be having a reflection coefficient of one for the electric field. That means that some portion of it is going to be transmitted ok, some portion of it is going to be transmitted.

So, now, we are going to say that the transmitted wave has no change in the direction of the electric field or the polarization is fixed right. It travels from left to right once again ok, which means that the magnetic field is going to have similar configuration as the incident magnetic field and the subscript t transfer stands for transmitted. So, now, we are dealing with 3 waves. one which is incident a portion of it is transmitted and a portion of it is reflected back all right and the incident on the transmitted have similar orientation for the electric, magnetic field and the k vector. For the reflected case the k is flipped magnetic field is also flipped ok.

This is the way we are setting up the problem. And what we want to know is the ratio of Et to Ei and Er to Ei. We want to know what fraction of the electric field is reflected and what fraction of the electric field is transmitted, this is the objective.

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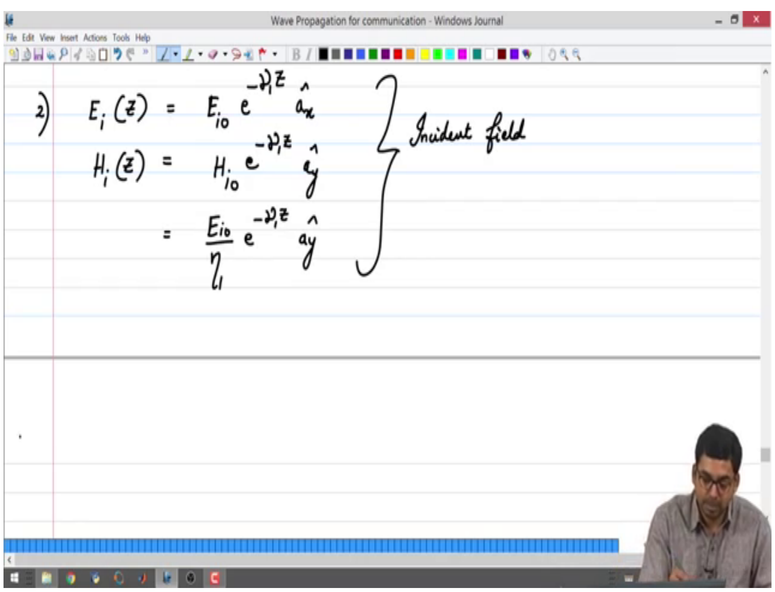

So, we can start with the expression for the electric field. Since our coordinate system is very, you know, wisely chosen, it is known as normal incidence. So, the interface is like this and your wave is hitting in this way. So, it is a normal incidence. Angle of incidence is equal to 0 ok, normal incidence means an angle of incidence is 0 because you have to draw a normal from the interface and measure the angle between the normal and your direction of propagation

So, normal incidence is all right. So, I am going to say what to write down some expressions for the incident electric field ok.

$$E_i(z) = E_{io}e^{-\gamma_1 z}\widehat{a_x}$$

Similarly, we can write down the expression for the magnetic field that is going to have a very similar form except that the unit vector for that magnetic field is in the different direction. That is all ok.

$$H_i(z) = H_{io}e^{-\gamma_1 z}\widehat{a_y}$$

By now we are very familiar with how to write these expressions. You can go one step further and say that you can apply Ohm's law for calculating the value of the magnetic field that you have written here.

So, you can always write this down as

$$H_i(z) = \frac{E_{io}}{\eta}e^{-\gamma_1 z}\widehat{a_y}$$

So, this is how one would write down incident fields.

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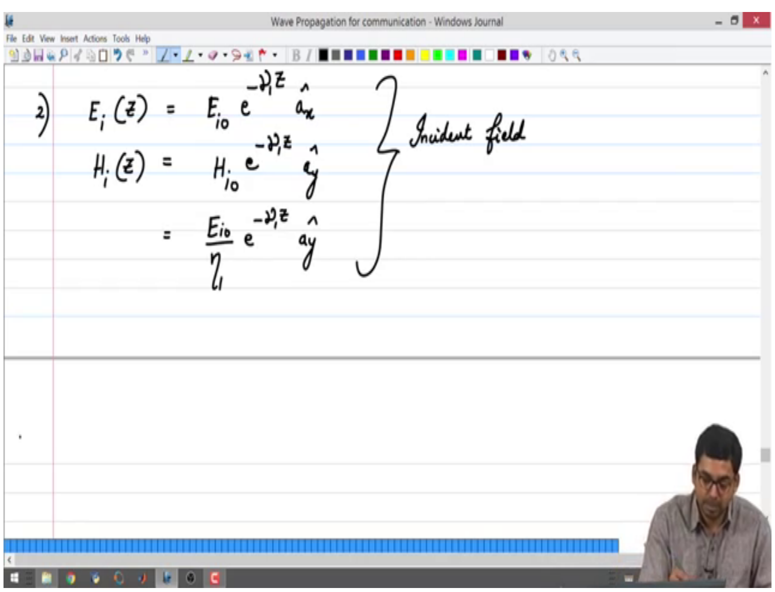

Now, let us write down the expressions for the reflected field ok. So, we are going to be having E_r right it's going along this direction all right. So, it is going to have some magnitude E_{r0}, once again it's travelling through the same medium all right. So, it experiences $\gamma_1 z$, notice that the sign is $e^{\gamma_1 z}$ because it is a backward wave all right, in the wave you would traditionally describe from the transmission lines and even from Maxwell's equations. It is a backward wave all right and orientation of the electric field is still the same. We have assumed it to not flip it's direction all right, so, it is a $\hat{x}$.

Magnetic field corresponding to the reflected wave is $H_{ro} e^{\gamma_1 z}$ ok, but now it has flipped it's direction. So, you just write this as ok minus a y hat ok or you could also go ahead and do this as

$$H_r(z) = \frac{E_{ro}}{\eta_1} e^{\gamma_1 z} \widehat{a_y}$$

This is how you would write down the expression for the reflected field.

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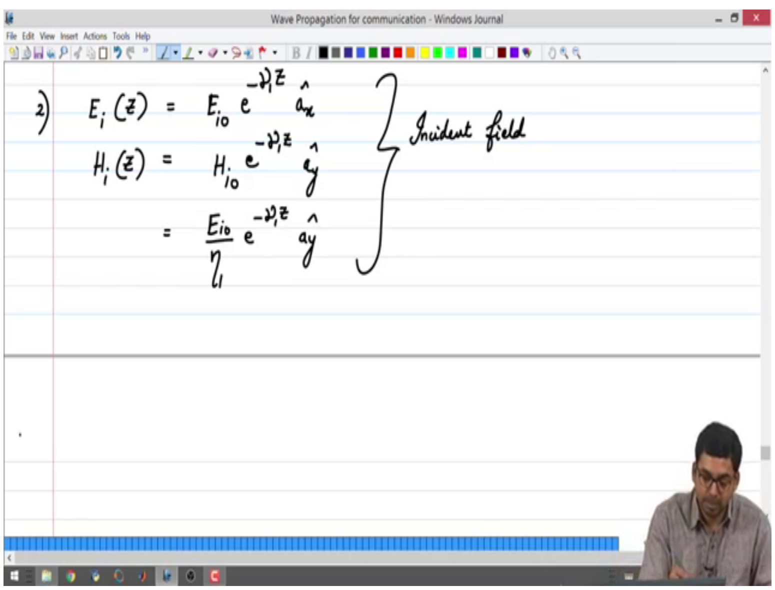

Now, we can also write down the electric and magnetic field expression for the transmitted part ok. So, that would be $E_t(z)$ Et plus some magnitude right E_{t0}. Now it is travelling in the same direction as the incident wave which means that a you will be having minus z coming into the picture a, but it is travelling through a different medium, the medium has a propagation constant say γ_2 all right. So, you will have to make that γ_2z all right. And we are assuming that the transmitted field has the same electric field orientation as your incident field all right. So, once again, so, it's $\hat{x}$ ok.

And we can write down the expression for the magnetic field that is transmitted. So, you will be having $H_{to}e^{-\gamma_2 z}$ and it's orientation is the y direction ok. So, it is clear that we are launching an electromagnetic wave from the left side and it is of a specific wavelength or frequency ok and it is going and striking this interface part of it is reflected. So, at any given point of time the total amount of electric field that you will have in medium number 1 is going to be the sum of the electric field that you have launched and the electric field that has that is coming back.

It is very similar to the standing wave case that we have seen in the transmission line right. So, you will have a sum of the two. In other words, if there is an absolute reflection from this interface then you will be having perfect standing waves all right in medium number 1 ok, but since not everything needs to be reflected. So, we can write down the total fields ok, so, when we talk about the term total field in medium 1 ok, you can write this down as

$$E_1 = E_i + E_r$$

So, it is a linear superposition of the incident electric field and the reflected electric field and you can do the same thing for the magnetic field.

$$H_1 = H_i + H_r$$

So, when you talk about total fields it is a very common terminology that is used. You are talking about the incident plus the reflected field in that medium all right. In medium number 2 for example, if I want to write down total fields it is just only one thing that is going. So, it is going to be the transmitted field. So, it is

$$E_2 = E_t$$

$$H_2 = H_t$$

Now, in certain formulations when they do simulations they use this term total field scattered field formulation ok. So, there they are talking about something very specific all right. We will go through that in subsequent classes, but the terminology is total field means $E_i + E_r$ and usually when they talk about scattered fields in this case we are considering a pure reflection. The scattered field will just be here.

So, in many cases when you place a photo detector in your computer simulation on one side, it will be actually responding to the superposition of the incident and the reflected field. So, people will want to separate the incident field and observe only the reflected field. So, they do some tricks with the equations ok. When they want to extract only the scattered or only the reflected part they usually run some tricks or some run some algorithms not exactly tricks, but algorithms to do this right.

And there are very specific ways of doing that. We will be seeing it later ok, but the term is very common. Total field means that you are considering incident pulses reflected ok.

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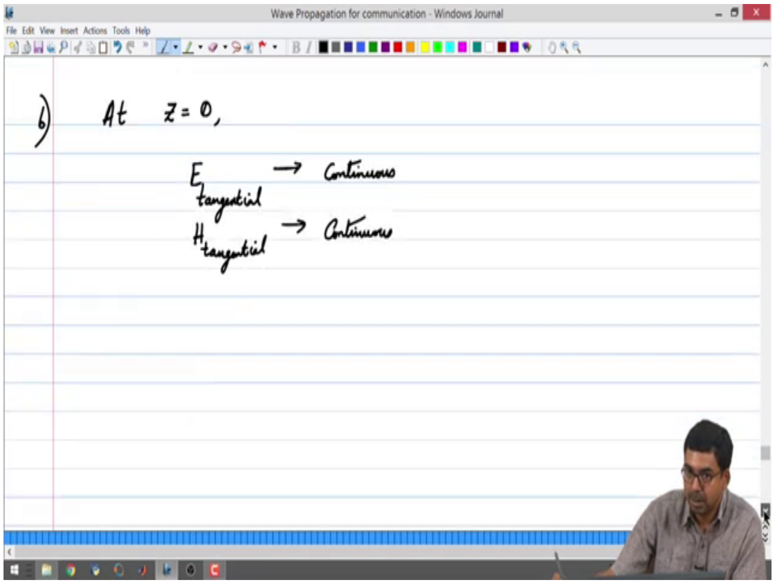

Now, let us look at this interface ok, at z equal to 0 ok. Now thus far we have been talking about the 2 Curl equations alright and we did not talk too much about the 2 Gauss's law equations then I said that it was implicit boundary conditions for the particular case that we are considering right.

So, we were saying that in the case of you know electromagnetic solvers where you are using the 2 curl equations right, the 2 Gauss's laws become implicit boundary conditions ok. So, one has to remember that and then when you are using it for interfaces the Gauss's law is very important for you to determine continuity of fields ok which fields are equal on medium 1 and medium 2 which fields are not equal in medium 1 and medium 2 is actually determined by Gauss's law. So, it is known as a boundary condition or an implicit boundary condition.

If you assume your charged density to be equal to 0 it becomes dielectric dielectric interface right. Now I do not want to elaborate because this should have been done in the undergraduate part already. The tangential part of the electric field or the tangential component of the electric field is continuous ok. This comes directly from Gauss's law. This is for a dielectric dielectric interface, I am writing H tangential continuous ok. Again dielectric dielectric interface both the fields are continuous in the tangential ok, tangential components are continuous here. Is there a problem?

Student: Continuous means no currents are there right.

Yeah. Continuous means in a strict sense is just equal. At that point whatever is coming here is equal to that is all it means no. There is no abrupt discontinuity at that point that is it. So, that does not change sign does not change value very easily, it is just equal.

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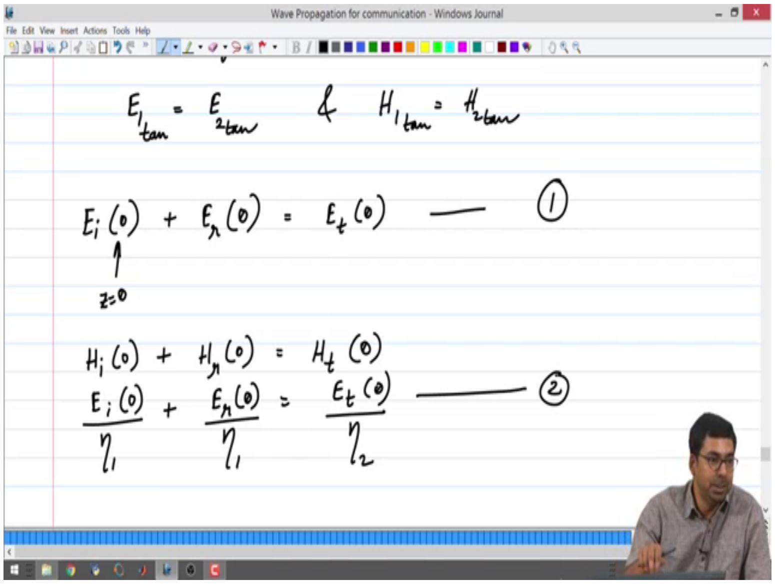

So, I mean if you want very easily you can write that

$$E_{1,tan} = E_{2,tan}$$

That is that is all that is all it means and you can write this for the magnetic field also. You can write that

$$H_{1,tan} = H_{2,tan}$$

So, mathematically that is all it means a continuity ok.

$$E_i(0) + E_r(0) = E_t(0) \ldots \ldots \ldots 1$$

Remember that in this case the electric field is already tangential, the way we have drawn the system ok. Your interface is like this and your electric field is oriented upward. So, if it is already tangential. So, your total fields in medium number 1 have to be equal on each side ok.

So, you have an incident field plus the reflected field in medium 1, both are pointing tangentially all right, both are pointing in the same direction and have to be equal to the field on the right side that is according to the Gauss's law. So, you can mark this as equation number 1 ok. And you can say

$$H_i(0) + H_r(0) = H_t(0)$$

$$\frac{E_i(0)}{\eta_1} + \frac{E_r(0)}{\eta_1} = \frac{E_t(0)}{\eta_2} \ldots \ldots \ldots 1$$

Then if you consider that Ei, Er, Et are your variables then you have 2 equations and one would say that you have 3 unknowns all right. I mean then you cannot find out all the unknowns, what is the thing that you can find. We were interested in finding Er divided Ei, Et divided by Ei. So, you could divide equation number 1 by Ei(0) on both sides all right.

Then what happens is you will have the ratio, two ratios will be then you will have 2 variables. Same way you can divide the left and right right hand inside of equation number 2 with by $E_i(0)$, you will end up with ratios you will have 2 variables.

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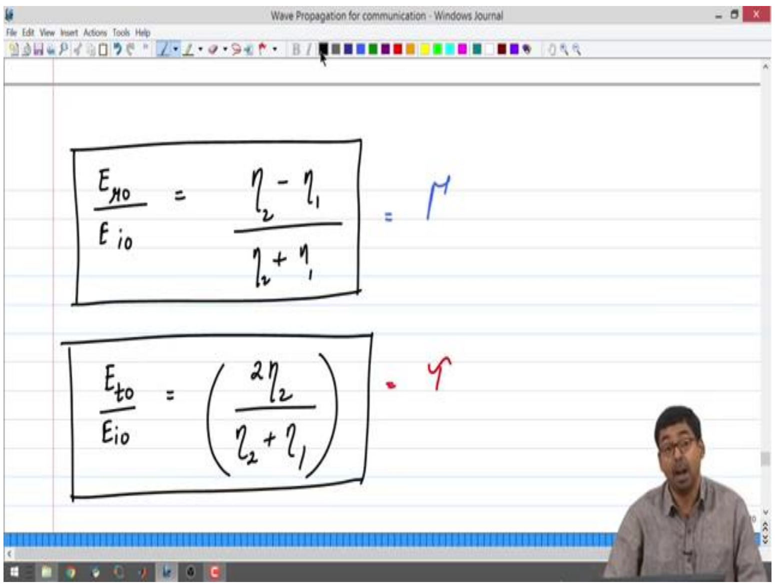

Then you will have 2 equations, 2 unknowns and then you can solve for the unknowns which would be Ei by E0 and Er by a a Er by Ei and Et by Ei sorry ok. So, 2 equations 2 unknowns straightforward to solve. So, what happens is it looks exactly like what you had in transmission lines ok, z l minus z naught by z l plus z naught is what we had in the transmission lines, it looks exactly like the reflection coefficient in the transmission line.

So, for the normal incidence on an interface all right when you are assuming that there is no flip in the electric field and all that it seems like it is exactly analogous to transmission lines. So, it is fine for you to assume that everything is a transmission line and then you know to join pieces together and you may arrive at the similar deductions right.

But, one of the things that we have not seen when we are dealing with transmission lines was actually we had not seen this at that time all right, we had seen only the reflection coefficient. Here we are also considering a transmission coefficient because these are semi infinite media and you will have propagation on the right side in that media also all right.

So, we can always take a Et by Ei, we did not do this for transmission lines, but we could always do it ok. So, here it turns out to be

$$\frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \Gamma$$

So, the first term or the first a boxed expression is called Γ, similar to your transmission lines is just the reflection coefficient. So, if you have a interface between 2 media

$$\frac{E_{t0}}{E_{i0}} = \frac{2\eta_2}{\eta_2 + \eta_1} = T$$

Straight forward transmission coefficient is represented by T ok.

In the case of transmission line T represented transit time, but here it just represents a transmission coefficient ok. There are also some other relationships that one can a find out between Γ and T ok.

So, since the denominators are equal $\eta_2 + \eta_1$, you could just take a sum and see what happens all right.

$$1 + \Gamma = T$$

So, there are a variety of things that you can do because the denominators are equal, you can always you know add subtract and see what happens, but you can also find out a straightforward expression between Γ and tau.

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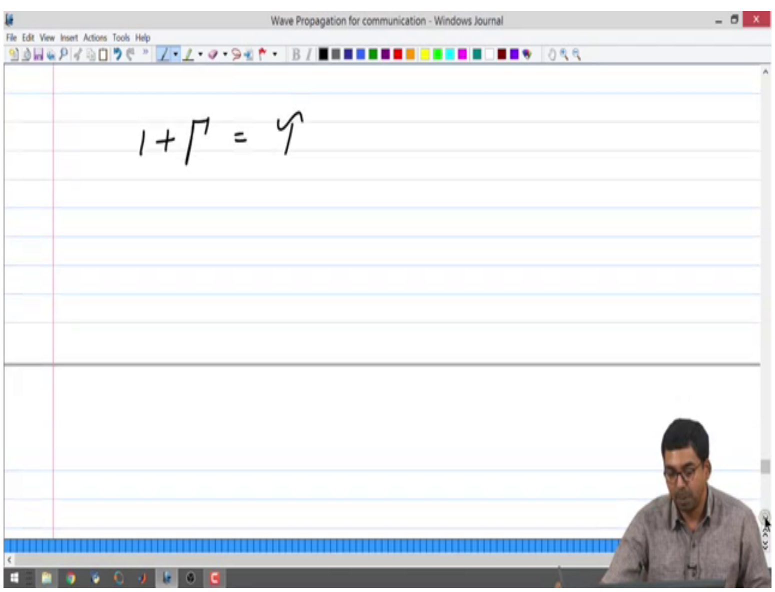

Now, this raises some important questions.$1 + \Gamma = T$ all right, 1 plus some quantity is your transmission transmitted electric field. Now remember that we are still talking about electric fields. Just like in the case of transmission lines, when we talked about reflection coefficient we were clear that we were talking about voltage reflection coefficient, we did not talk about current reflection coefficient a lot ok.

We just said the current reflection coefficient was negative of the voltage reflection coefficient. Here we are talking about electric field reflection coefficients ok. So, most of the time when we talk about reflection coefficient, it is about the electric field and for the electric field reflection coefficient in this configuration what we have considered?

$$1 + \Gamma = T$$

Now this needs a lot of thought, ok, needs a lot of thought because it means that your incident electric field could be say 1 V/m, your transmitted electric field can be higher than 1 V/m ok. So, it is all uncommon.

So, it is perfectly possible for you to have an electric field in the second medium higher than the incident electric field and it is not an absurd solution at all. one can always ask a question then how is the energy conserved in this kind of a system all right. The answer lies in the magnetic field. If the electric field goes up, the magnetic field will go down and adjust in such a way that there is no energy loss or no energy created in this system.

So, if you get a solution where your transmitted electric field is higher than an incident electric field, you need not panic. It could be a valid solution, if you need some you know verifications whether the magnetic field is decreasing by the same amount extra, but it is possible. So, you can have transmitted electric fields to be higher than the incident electric field, it is not uncommon. So, this is something that you have to keep in mind.

Typically, Γ is complex 1 plus Γ, obviously, T is also complex ok. We know that from our transmission line that Γ is going to be complex, it will end up having real and an imaginary part all right which means that 1 plus a complex number is again you are going to be ending up with a complex number ok. So, T is also going to be a complex number ok.

You can also do some validity tests with this and try to see what happens, but a more rigorous approach would be better, but just for the sake of completeness, I would just do a simple thing all right.

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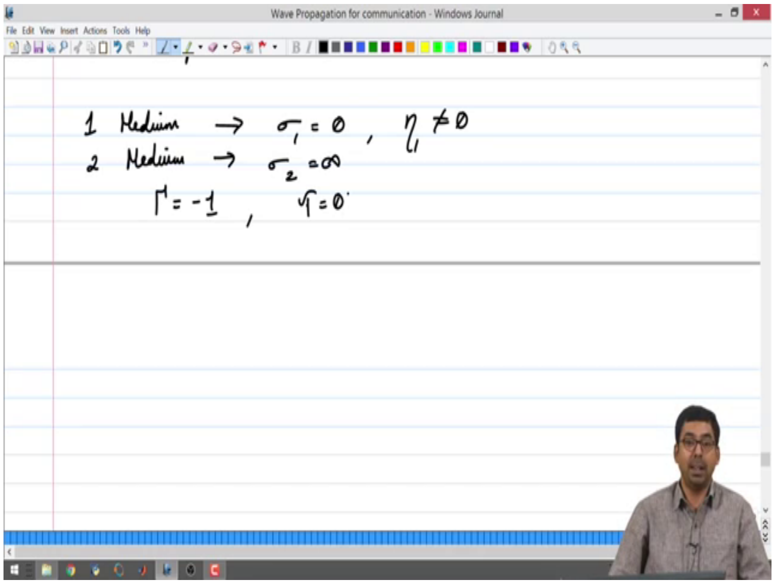

If there is a scenario where the first medium ok is having no conductivity, but obviously, characteristic impedance is not 0, right. And the suppose suppose we say that σ_2 is equal to infinity ok, directly you need not approach this problem. We can just want to see whether our equations are holding good for this case or not all right. Otherwise you could write down the same thing. You could have write the incident field, write the reflected field, write the transmitted field, try to find out the value of Γ and T using proper boundary conditions ok

Then you will find out that you know you are having a case where σ is equal to 0 and one side, σ is equal to infinity on the other side ok. We can guess right Γ = -1 all right corresponds to this case all right. σ at equal to infinity, that means, you have a perfect electric conductor and when we ran a simulation with our codes all right, we had written Maxwell's equation in one dimension and we had written a code.

Suppose we make one side to be a perfect electric conductor which means that you make the electric field equal to 0, whatever be the voltage you will not get an electric field over there ok. So, if you do that then you will get a reflected wave which is flipped all right. We saw that an impulse will travel, it will get flipped in direction and then it will come back extra. So, Γ = -1 all right. Consequently, $1 + \Gamma = T$.

We can make some inferences that actually T is 0. So, you know if you have a wave that is trying to penetrate into a perfect electric conductor that is not going to be possible ok. Any wave is not going to be able to penetrate into an infinitely conductive medium all right.

So, we say that T = 0, Γ = -1, $1 + \Gamma = T$ for this particular configuration.

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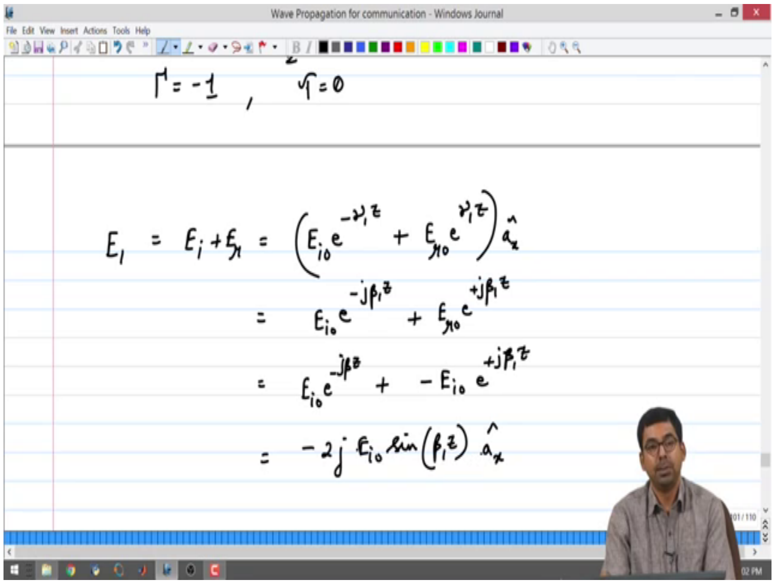

One could also write down the electric field, you know in medium number 1 to be equal to

$$E_1 = E_i + E_r = (E_{io}e^{-\gamma_1 z} + E_{ro}e^{\gamma_1 z})\widehat{a_x}$$

$$= E_{io}e^{-j\beta_1 z} + E_{ro}e^{+j\beta_1 z}$$

$$= E_{io}e^{-j\beta_1 z} + -E_{io}e^{+j\beta_1 z}$$

$$= -2jE_{io}Sin(\beta_1 z)\widehat{a_x}$$

What this is telling is that in medium number 1, suppose all of the electric field is reflected back you will be having a superposition of the incident and reflected electric field and the superposition is such that it is going to create a standing wave, all right.

It is going to create a standing wave in medium number 1 at some positions z in medium number 1, you will be having 0 electric field, at some other positions you will be having maximum electric fields, but they will vary only with respect to time with respect to the space in certain places you will be having 0 electric field. So, this is the same as what we had seen in transmission lines. If you in fact go back and see the expression, it will be identical for a short circuit condition right.

So, this is $-2jVSin(\beta z)$ is the expression that we had for the net total voltage present in the transmission line for a specific boundary condition short circuit boundary condition. So, this will look exactly the same all right. So, this means that you will be having a total wave in medium number 1 to look like having nodes and antinodes. So, in other words you will be having standing waves in medium 1 and this because the reflection coefficient is magnitude is equal to 1, it is a perfect standing wave means that the node is exactly 0 ok minimum is 0.

And if it is not equal then you will not have the forward and backward wave, cancelling each other completely. It will not be a 0. You will have some different voltage standing wave ratio in the case of transmission line, you will be having a different standing wave ratio in the case of electric fields ok. But I have done this all right without changing the boundary conditions and all that just to look whether there are sanity checks.

One has to be aware of all the boundary conditions possible and then use the correct boundary conditions to arrive at the same or similar expressions ok.

So, I think we will stop here. The next thing that we are going to do is simply see how this problem changes when we have the wave going at an angle with respect to the interface. Then we will go to the metal dielectric interfaces oblique and normal all right and then we will start to look at what happens to polarization extra right, what are the consequences. Here we have assumed the electric field to just do this all right. Suppose, if it is circularly polarized suppose if it is elliptical elliptically polarized extra what would happen ok. So, we will see about that.

In specific we are going to be dealing with what are known as Fresnel reflection coefficients for different polarizations and what are the consequences and what are the inferences from these 2 ok. We will stop here.