Transmission lines and electromagnetic waves
Prof. Ananth Krishnan
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture – 20
Plane Waves

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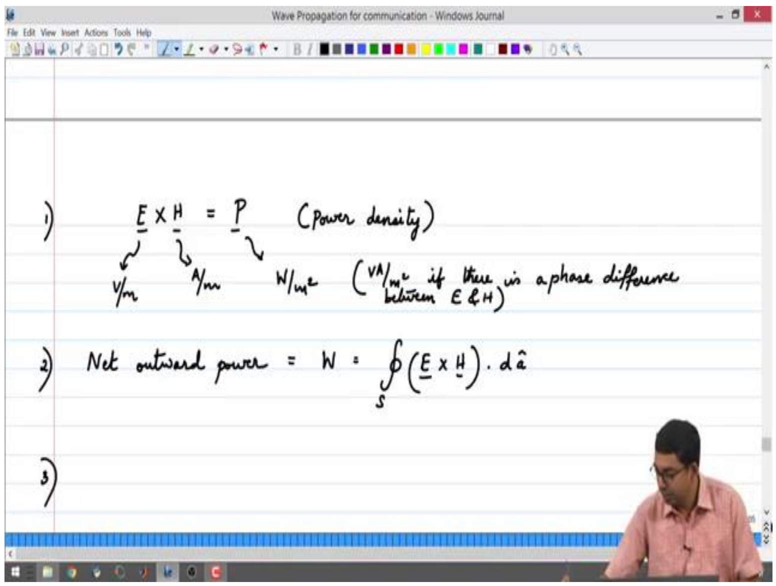

I think we will get started. So, we were starting to look at power ok in electromagnetic fields and which briefly opened up the calculation of power. I think we will be expanding on that in this lecture. So, towards the end of the previous lecture.

We just wrote down that

$$E \times H = P$$

or the unit of electric field is in V/m and the unit of magnetic field is in A/m and the unit of the right hand side quantity all right is VA/m^2 ok, but if the phase difference between the electric and the magnetic field is 0 we you could always write this down as $Watts/m^2$.

So, instead of calling it power it's more appropriate to call this as power density ok it gives you a notion about how concentrated the power is with respect to a given area right. So, its power density ok since we already saw that the unit of electric field is in volt per meter and magnetic field is in A/m the right hand side has to be equal to VA/m^2 all right and only when the phase difference between electric and magnetic fields is 0 this becomes watts per square meter all right.

So, in a generic sense the right hand side represents a quantity similar to apparent power in electric circuits where you had the unit to be volt amperes ok. So, in the case of electric power you would have talked about apparent power, active power and reactive power all right. So, this right hand side equivalently is a generally apparent power in electric circuits ok ok.

Now, given an area all right one can calculate the net outward power all right. The net outward power is usually given by this variable W ok. So, it's the closed integral of power density over the given area.

$$W = \oint (E \times H).d\hat{a}$$

All right that is the net outward power suppose you were to calculate the intensity of an electromagnetic beam that is hitting a surface you will take the power density at every place integrated over the area ok that is the net power that is delivered ok ok.

Now let us start to look at this in a little bit more detail because we already know that the electric field that we are considering now is a time harmonic quantity for this portion of the lecture and the magnetic field is also time harmonic. So, what is the effect of time on the power that is passing through a unit area is first that we need to understand ok.

So, for this let us go ahead and make the expression for electric and the magnetic fields clear with respect to the spatial and temporal variable then write down the power and then let us start looking at the power. What kinds of power are there other than apparent real and reactive power all right and which power should we analyze which power should we not analyze which power gives you meaningful conclusions which of them does not give you meaningful conclusions all right all ways extra right.

So, these are the things that we need to see.

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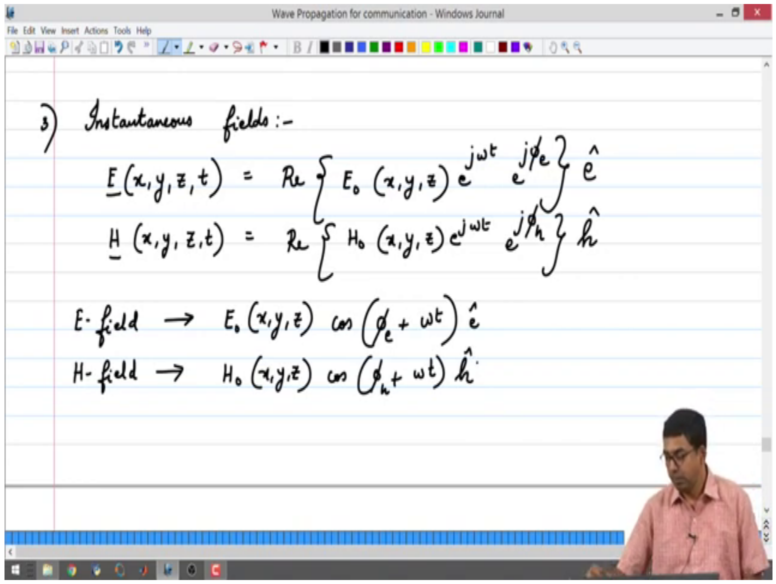

So, for this we start to look at instantaneous electric and magnetic fields ok ok by instantaneous I mean that I will be using $e^{j\omega t}$ for describing both the electric and the magnetic field it has the effect of time all right. So, I am using $e^{j\omega t}$ to describe electric and magnetic fields we had seen before right the beginning when we had started with the ac all right a electromagnetic field.

So, we have included the periodicity with respect to time to be an exponential and then we will take the real path of this exponential right. So, when we talk about instantaneous this means that you are going to write down the electric field which is a function of position x y z and time. So, three space variables and one time variable alright and we are simply writing this down as real part of E_0 which is a function of x y z special variables alright and I am also including the effect of time where it is periodic with respect to time ok ok.

$$E - field \rightarrow\ E_0(x,y,z)Cos(\phi_e + \omega t)\hat{e}$$

This is what we mean by instantaneous field and in this case I have taken a very generic scenario where the electric field is a function of x y and z ok. Similarly the magnetic field can also be written as

$$H - field \rightarrow\ H_0(x,y,z)Cos(\phi_h + \omega t)\hat{h}$$

But we can make this even more generic right now we have included all the space variables and the time variable

We can also make it a little bit more generic. A part comes from including the phase difference between E and H ok. So, it is possible for you to have a phase difference between E and H all right. So, we can now take a case where we would say that right. So, I will say that the electric field with respect to a reference has a phase ϕ_e ok and the unit vector is given by $\hat{e}$ ok. Similarly for the case of the magnetic field the most generic case with respect to a reference ok it has a phase ϕ_h and the unit vector describing the magnetic field is given by $\hat{h}$ $\hat{h}$ yeah.

This becomes a very generic expression for electric and magnetic fields ok. So, it includes the position it includes the time it also includes some phase with respect to a reference for electric and magnetic fields ok. It's quite possible that you choose the reference in such a way that ϕ_e is equal to 0 and then you know say with respect to the reference same reference what is the value of ϕ_h in that way you will be able to eliminate one of these two variables all right.

But in the most generic case with respect to a reference it's completely ok for an electric field to have a phase with respect to the same reference magnetic field as some other phase, that means that electric and magnetic fields could have a phase difference between them in the most generic case ok. So, now since we are talking about the real part of these exponentials ok we could always say that the electric field right is going to look like some value E_0 at some position x y z ok.

Since we are taking the real part we can directly go ahead and say that it looks like a cosine ok. It look like a $Cos(\phi_e + \omega t)$ is the net phase or its the total phase ok and it points in the direction $\hat{e}$ ok one could always orient the coordinate axis in such a way that $\hat{e}$ is pointing to one of the Cartesian coordinates, say x, y or z depending upon the problem.

Similarly, the magnetic field can also be written as

$$H_0(x, y, z)Cos(\phi_h + \omega t)\hat{h}$$

Now since we are dealing with time also ok. So, we have included the effect of time also and we know that this function is periodic with respect to time ok if you were to calculate the power the density directly by using $E \times H = P$ our understanding is that that is also going to depend on time ok.

So, let us first start with the the simple calculation for power density all right and since this is instantaneous electric field and instantaneous magnetic field that we are using the power that we are calculating will be known as instantaneous power all right or instantaneous power density where we used to call it ok.

So, ok ok and we have already written down the expression for the electric and the magnetic field. So, we just substitute the cosines that we have just written we have written

$$E_0(x, y, z)Cos(\phi_e + \omega t)\hat{e}$$

all right.

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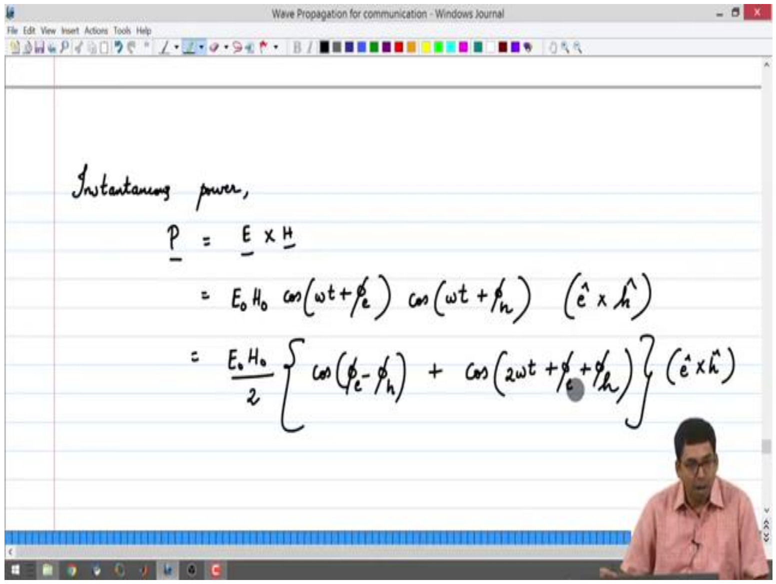

Similarly, we have written an expression for the magnetic field with the real part all right. So, we just substitute that and we get

$$P = E \times H$$

$$= E_0 H_0\, Cos(\omega t + \phi_e)\, Cos(\omega t + \phi_h)(\hat{e} + \hat{h})$$

So, you have to take the unit vector for electric and magnetic fields and cross them. So, that you are right hand rule is going to be preserved right ok ok

Now, that we have written an expression which is the product of two cosines can always use some trigonometric identity and convert this to a sum of cosines ok. So, we could write this down as

$$= \frac{E_0 H_0}{2}\{\, Cos(\phi_e - \phi_h) + \; Cos(2\omega t + \phi_e + \phi_h)\}(\hat{e} + \hat{h})$$

So, all we have done is we have taken the electric and the magnetic field expressions and nearly substituted them in the power expression that we had started with all right and this quantity if you look at it on the right hand side all right it depends on time ok. So, this quantity is known as instantaneous power or instantaneous power density.

Now, the question that one can ask is, what is the relationship between the instantaneous power and the time, all right. It's very clear that you have a $Cos(2\omega t)$ plus something all right. Means that its a periodic power instantaneous power density also all right which means that we have some possibilities over here we know that the cosine can be positive cosine can be negative or the cosine can be 0. It could also be in such a way that it exactly balances this quantity all right when it is negative and the entire term becomes 0 at some instant of time the entire term is positive at some other instants of time and the entire thing is negative at some other instants of time.

So, instantaneous power with respect to time you will see that the vector is pointing in very different ways all right. So, you may either have a positive quantity or zero or negative quantity ok, which means that to derive to make some meaningful conclusions it's not easy to do it with instantaneous power ok. Because instantaneous power at some instant of time can be 0 you cannot draw a conclusion that there is no power that is passing through the given area right.

Same way you can have instantaneous power to be positive which is fine for you to draw some conclusions normally, but in some other instance of time you will have negative all right. So, this is not an easy quantity to interpret to make physical this coming inferences ok. So, it is not the best quantity for making physical inferences because it can be positive, negative or 0.

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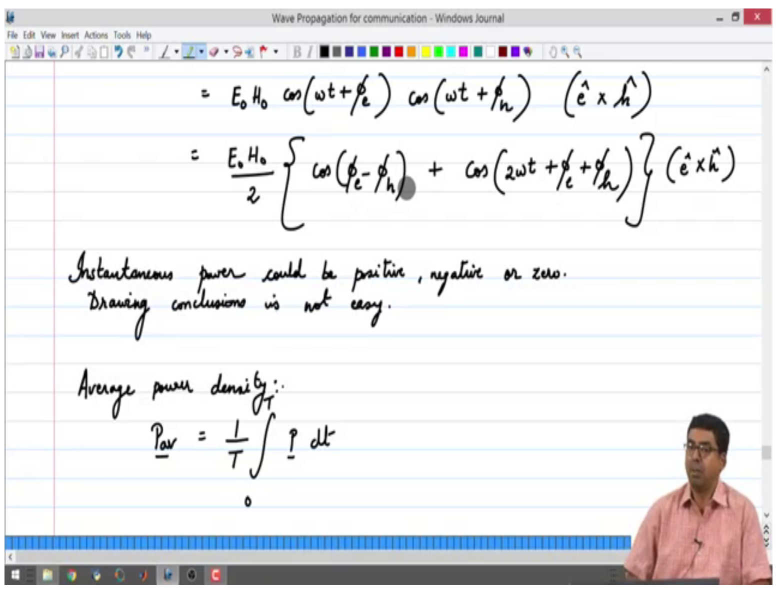

So, to just write this down all clearly right, ok instantaneous power could be positive or negative or 0 which means that you know drawing conclusions is you know not easy ok ok then what other option do we have.

Now one of the things that we look at is the right hand side is depending upon time at some instant of time you are having positive power, at some other instant of time you could have negative power. And another instant of time you could have zero power extra, which means that we need to look at a quantity that is not dependent on time to make some meaningful analysis all right and that quantity is known as average power density which is similar to your electric circuits ok. So, you will be having average power in your electric circuits. So, similarly you are having average power density over here ok.

So, the way the average power is defined is over a time period ok, I want to integrate your instantaneous power ok.

$$P_{av} = \frac{1}{T}\int_0^T \quad P\, dt$$

Now this could be the time period of E, this could be to the time period of H, both of them are going to be at the same frequency. So, it is perfectly fine to take the time period of E or H all right which means that you are going to be looking at substituting the value of instantaneous power in this expression.

And one of the things that we can immediately notice is the the instantaneous power broadly can be split into 2 quantities $\frac{E_0 H_0}{2}\{\, Cos(\phi_e - \phi_h)\}(\hat{e} + \hat{h})$ That is one term. The other term is $\frac{E_0 H_0}{2}\{\, Cos(2\omega t + \phi_e + \phi_h)\}(\hat{e} + \hat{h})$. So, so one term I mean one term and the second term is dependent upon time and it is dependent upon time in a periodic manner and the function being precisely a cosine all right.

So, the first term, yes , we can understand that it seems to be independent of time. So, if I integrate it I will just have to multiply it with the time period in this case and then I have a 1/T coming out. So, the time and time gets cancelled. So, the first term remains as such in the case of average power density in the case of the second term when you integrate the cosine over an entire time period you will end up getting a 0 all right.

So, what that means is you are removing that quantity from the picture completely and then what you are left with is something that is not dependent on the current value of time and that is average power density and that should give you a valid quantity to make some conclusions on ok. So, I will write down the expression.

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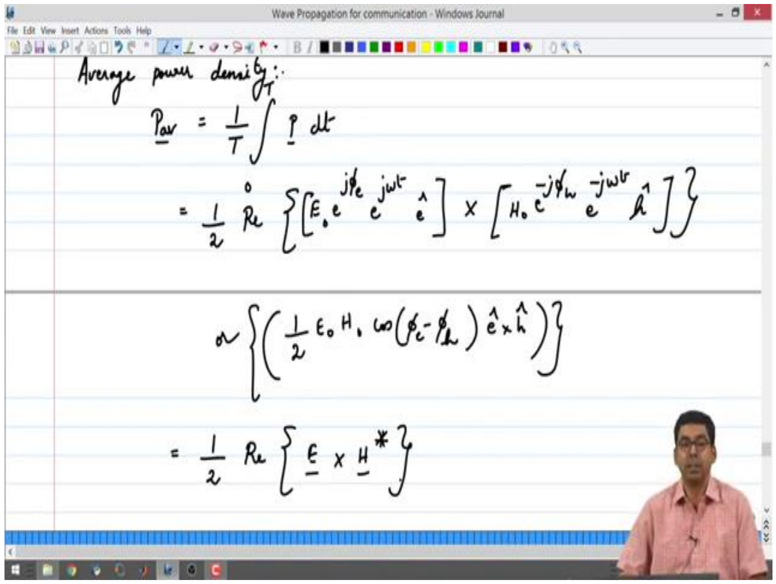

So, this is

$$P_{av} = \frac{1}{T}\int_0^T \quad P\ dt$$

$$= \frac{1}{2} Re\{[E_o e^{j\phi_e} e^{j\omega t}\hat{e}] \times [H_o e^{j\phi_h} e^{j\omega t}\hat{h}]\}$$

Or I mean you could also write this down as the expression that we just now got you can just say that this is

$$= \{\left(\frac{1}{2} E_0 H_0\ Cos(\phi_e - \phi_h)\hat{e} + \hat{h}\right)\}$$

Now if you look at the first expression that I have written all right there is a ah you know small problem with that expression ok there is a small problem right this is expression this is what we already got. So, we cannot doubt that all right.

Now, one of the things I notice here is that if I take this cross product I will never get ϕ_e - ϕ_h all right. So, which tells you that there is something else going on. So, here I have the expression for the average power to be

$$= \{\left(\frac{1}{2} E_0 H_0\ Cos(\phi_e - \phi_h)\hat{e} + \hat{h}\right)\}$$

So, I just cannot simply write this as half real part of $E \times H$ because that will never give me ϕ_e - ϕ_h it only gives me ϕ_e+ ϕ_h which is not consistent with the derivation I just made all right which means that we need to make some adjustment to one of these quantities all right.

The phase is not being calculated correctly. So, the only way to get ϕ_e - ϕ_h is to tweak the phase of your magnetic field in the expression all right. So, the correct way to write this down is by actually saying it is -jϕ_h and $-j\omega t\hat{h}$ ok. Otherwise you will not end up with this expression. Remember that we are trying to write this expression all right using the exponential that is all all right and when we are doing that we have to take into account the phase accurately all right. So, the only way you will be able to get it is this we just means that we could always write down the average power to look like

$$P_{av} = \frac{1}{2} Re\{E \times H^*\}$$

So, $E \times H^*$ is the correct way to write the actual power. So, the complement there comes because you have a phase difference between E and H ϕ_e - ϕ_h to account for that phase difference the way to calculate power is to take $E \times H^*$. Many of you may remember in low frequency circuit you would have had VI^* in ac circuits power calculation you would have had half VI^* to be the average power in ac circuits all right.

This is exactly a similar case alright where to account for the phase difference between E and H you have to take the complement of h ok. Now since this quantity is independent of time ok. It gives you some meaningful quantity to make actual conclusions ok it gives you whether there is an average power flow or not whether there is an average power flow into the surface or out of the surface or not all right. So, this is a quantity that you can use to make some meaningful conclusions and what kind of meaningful conclusions can one make whether it is a source or a detector.

So, things like that or whether it is passing through a surface or not all right. So, you may have inward power being equal to the outward power on the other side extra. So, the average power is the quantity that you need to look into ok. The other thing that we are looking at is the power is dependent upon $E \times H^*$ now whenever we have something like this $E \times H^*$ all right and if we write down only the real part you will get cos of ϕ_e - ϕ_h all right.

This means that if you have a configuration where the electric and the magnetic field unit vectors are perpendicular to each other following your right hand rule. So, direction of propagation is say your thumb all right electric and magnetic fields are given by your index and middle fingers ok it forms a perfect right handed configuration for your plane wave to travel.

But we still are not talking about the phase when we are talking about the right hand rule we are just talking about the unit vectors we are not talking about the phase when we look at ϕ_e - ϕ_h it tells you something about the phase ok. So, it means that you can have a perfect right handed configuration of electric magnetic fields and your pointing vectors are perfectly defined according to the direction of your fingers.

But it's quite possible that the electric and the magnetic fields are 90 degrees out of phase. It's possible it still forms a right handed configuration and it still seems to be a valid solution for your pointing vector all right if that happens you do not get any power ok. If your electric field and magnetic field are in quadrature phase ok quadrature phase means one fourth of your wavelength alright. So, it is $\pi/2$ ok.

Then you may not get any power transfer at all it is very similar to the case where you have passive components in electric circuits which are l and c ok in resistor we will say that the current and the voltage are in phase and the power will simply be V*I. And if you want to be very specific you can say $VI\ Cos\phi$ where $Cos\phi$ is your power factor all right or ϕ is the phase difference between your current and voltage because ϕ is equal to 0 degrees you will substitute cosϕ as 1 and then you will get it to be VI watts.

In the case of pure inductor we know that

$$V = \frac{Ldi}{dt}$$

All right which means that you will be having voltage and current out of phase by 90 degrees. More specifically you will say that the current lags the voltage by 90 degrees in phase and then when you take $VI\ Cos\phi$ you will get 0, that means, that your inductor cannot consume any power there would not be a power transfer to a pure inductor.

Same way in a capacitor you will be having

which means that your voltage and current are going to be out of phase by 90 degrees or current will lead the voltage by 90 degrees again you cannot transfer power to capacitor or power I mean power cannot be dissipated by the capacitor all right. So, the equivalent can happen here also. All right, you can have a media in which the electric and the magnetic fields will not have any phase difference between each other.

You can have some other media where the electric and the magnetic fields have some phase difference between each other. The phase difference can be positive phase difference can be negative phase difference can be 0 depending upon the choice of the medium and in the most general case half real $E \times H^*$ should give you about should give you an idea about all that ok. So, most of the cases are very similar to your electric circuits and that is something to bear in mind.

And one of the conclusions that one can draw from this is not all electromagnetic fields carry power it is completely possible to describe an electromagnetic field with an electric magnetic field component perfectly aligned to your right hand coordinate system all right, but still have a relative phase difference between the electric and the magnetic field to be 90 degrees and then electric electromagnetic wave does not carry any power at all ok.

So, not all electromagnetic fields can carry power. So, this is one thing to remember all right. So, for you to effectively deliver power from one side to the other the phase difference between E and H is very important ok. The other conclusion that you can also draw from these is that the electric and the magnetic fields do not cross each other right. That is the subtle difference that we will see when we do a computational simulation of the electric and the magnetic fields in the em field right.

Now, that some idea about the power should be there and some analogy between low frequency circuits and plane wave electromagnetic should be there all right. We can go to some more generic cases all right. A more generic case is where we are talking about a plane wave travelling in arbitrary directions. So, here we are already talking about plane wave being a function of x y z time and we have mark the direction of the plane wave to be say $E \times H$ for the pointing vector all right.

We have already made it generic enough, but we can also make it even more generic by looking at how we describe electromagnetic fields travelling in arbitrary directions in the form of plane waves ok.

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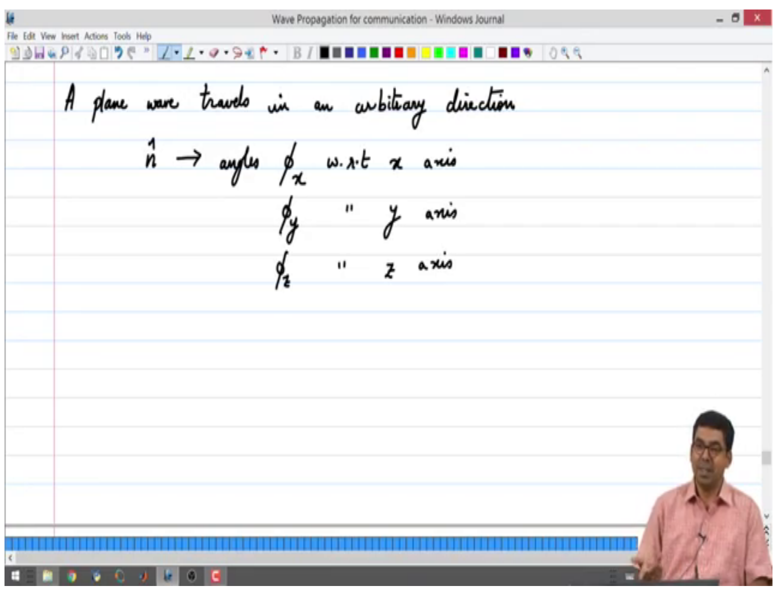

Now, I will just describe what I am trying to do all right. Let us say that ok. Many times in problems you will be given something like it is travelling in a direction all right it hits some medium try to calculate reflection and transmission coefficients and then you may be asked to figure out what is the power transferred extra.

In those cases first you should be able to write down the expression for the plane wave travelling in an arbitrary direction then only you can do cross product dot product extra accurately otherwise you would not be able to do it. So, we need to know how to describe a plane wave travelling in an arbitrary direction with respect to a fixed coordinate axis ok. So, let us say that this arbitrary direction that we are talking about all right has a unit vector $\hat{n}$ ok and this $\hat{n}$ is making an angle ϕ_x with respect to x axis ϕ_y with respect to y axis.

Please remember that previously when we were talking about plane waves we made it very convenient in such a way that the direction of propagation was assumed to be z. You just assume the e field to be x oriented and the magnetic field to be y oriented all right. So, it was very easy if you had very few you know unit vectors in different directions to manipulate, but it's also possible that the coordinate axis is fixed all right and a plane wave is allowed to travel. You should be able to describe that also.

In practice it's a good approach to do either one you can always orient your thumb to the direction pointing I mean to the direction of the travel and say that that is going to be my z axis in that case you are defining a coordinate axis for the problem the opposite can also be fine if you are comfortable with having fixed coordinate axis and then describing what a plane wave is doing that is also fine all right depends upon what the student finds more comfortable all right.

Most of the time I find it comfortable to orient my thumb to the z axis all right that way it becomes easy for me to describe the fields, but this is also a valid thing right ok.

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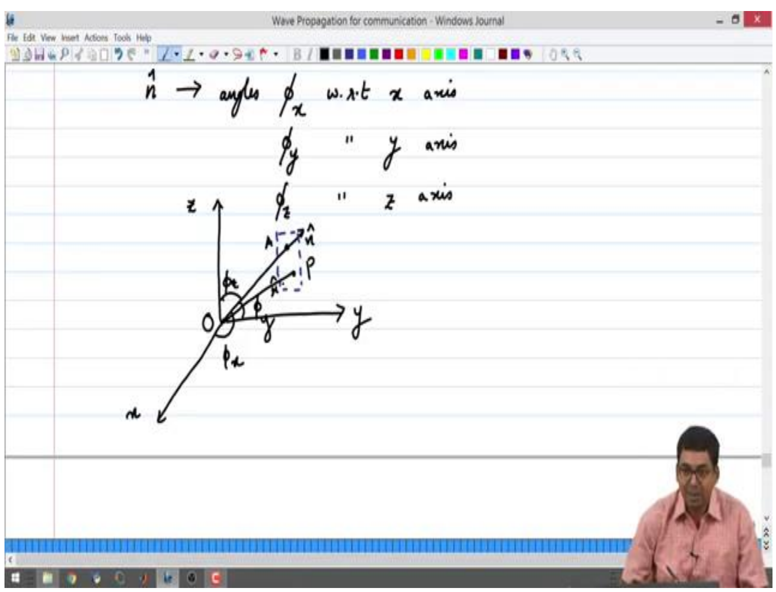

So, let us draw a diagram ok and let us mark our coordinate axis alright say I am having xy and z ok and it is going to be tough for me to draw in three d ok. So, you have to make some imagination right.

So, I am having a direction of travel and say something like this ok I am calling the unit vector to be a hat. So, the angle it makes with respect to the x axis I will call that as ϕ_x the angle it makes with respect to the y axis is ϕ_y right with respect to the z axis say ϕ_z ok and I will mark the origin with some letter o ok. Now we can also be a little bit more clear about what is happening here we can always draw a plane of constant phase ok.

This is a plane wave travelling in the direction in the plane perpendicular to it it's a constant phase plane all the electric field is having the constant phase this has the same phase only when it moves to another place at another instant of time the phase has changed ok. So, this aspect where we are drawing something like this all right. So, a plane of constant phase is ok and let us say that this point is ok.

So, now we have a unit vector in the direction of propagation to be $\hat{n}$ ok coordinate axis defined according to the right hand system x y z should form a perfect right hand triode all right angle between the x y z are defined and we have drawn a plane which is describing a constant phase plane at the point A. Let us see here that is a setup that we are having now we can also have another point on this plane ok and that plane would have the same phase as the wave at point a all right that is why its a its a plane of constant phase ok.

So, you can always have another point and call it P ok even though you have marked a different point the wave that is travelling in this plane at that plane and all the points has the same phase ok remember that when we are orienting our direction of propagation $\hat{n}$ to be in z direction we wrote down the equation to be $Cos(\omega t - \beta z)$ ok. So, $Cos(\omega t - \beta z)$ means that in x and y directions the plane is infinitely large where ever you take a point on that plane the description of the field is $Cos(\omega t - \beta z)$ right.

So, you could draw a large plane and if you take the point P its its lying on the same phase ok, but the way we have draw a coordinate system over here it's perfectly possible for me to draw a vector from point O or the origin going to this point P, but it will not look like $\hat{n}$ ok it is the way we have defined the coordinate system you could always draw a different vector ok

We have defined an origin to be a point and we are taking a plane of constant phase its already understood that I can draw a vector going straight to the centre of these plane all right and hitting the plane at 90 degrees I could also draw another vector going from the origin say to the edge of the plane, but its not hitting the plane at 90 degrees all right, but given the coordinate system if I have to describe that point on the plane that is what I have to use all right. So, it's perfectly possible for us to have points on this plane all right and you could connect it to the origin using different vectors all right.

So, this makes it a little bit TDS to describe a plane wave all right in this kind of a scenario where it is travelling in an arbitrary direction it is a plane wave, that means, it has a large plane where the phase is constant, but you have a coordinate axis with the fixed origin and you have to describe at every point on this plane the phase has to be constant all right. So, it becomes a little bit more challenging to think about it, but once you look at this problem and orient everything to a coordinate axis then it becomes very simple all right.

But in an arbitrary direction it's always a little bit more complicated ok. So, let us give some mathematical descriptions to the quantities that we have written ok $\hat{n}$ is the unit vector we know how to write that ok.

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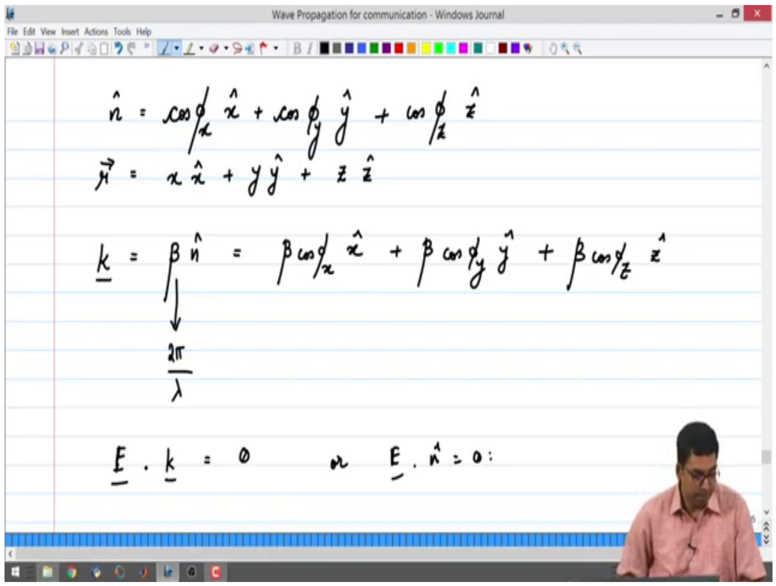

So, we can write this as

$$\hat{n} = Cos\phi_x\,\hat{x} + Cos\phi_y\,\hat{y} + Cos\phi_z\,\hat{z}$$

So, this is the unit vector ok n which describes the directions of propagation

So, the next phase front will be a space where you know what to say. It's a version which has travelled further or ahead extra. So, if you assume the direction is like this then the next plane that you draw will be on the unit vector passing exactly perpendicular to this plane again all right. So, we have drawn

$$\hat{n} = Cos\phi_x\,\hat{x} + Cos\phi_y\,\hat{y} + Cos\phi_z\,\hat{z}$$

This is the expression for the unit vector

This is how you will be writing the expression for the unit vector the position which we have just discussed ok that is the point P is described by vector r ok and whenever you have a vector that you have describe all right in this kind of a scenario all you would just do is you will just write this down as x y z coordinates of that particular point. So, you can always say that the r vector which describes the position which you want to consider ok it's simply going to be given by

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

This is the only way we can describe right. Now the objective of this exercise is to write down the electric field expression is to write down the magnetic field expression once you have that you can find out the power extra ok now before going into more details n describes the direction of travel of your wave that should be very clear now all right, but we already have talked about E and k all right k describes the direction of your pointing vector or direction of power transfer ok.

So, which means that k should be some multiple of n or k could be equal to n also, but the direction of k should be the direction of n all right. So, that should be clear. So, let us make that clear in a little bit more sensible manner the relationship between k and n is just multiplying with the constant β right and $\beta = \frac{2\pi}{\lambda}$ where λ is the wavelength in the medium that you are considering. Remember that we have talked about the coordinates.

We have not described the medium. The medium could be air, the medium could be water, the medium could be slightly conductive or it could be a poor dielectric extra in any of these media. In the previous classes we have seen that β could be different we also saw that as omega increases alpha could be different β could be different all right extra. So, β multiplied by $\hat{n}$ which means that the expression for k vector ok is

$$k = \beta\hat{n} = \beta Cos\ \phi_x\hat{x} + \beta Cos\ \phi_y\hat{y} + \beta Cos\ \phi_z\hat{z}$$

Now, we also have to remember that when we are doing this the immediate thing that should come to our mind is we are talking about a plane wave $E.k = 0$, that means, the dot product between the electric field and the poynting vector is 0, that means, the projection of electric field on the pointing vector or the projection of the poynting vector on the electric field vector is 0 ok.

They are orthogonal to each other if you take a projection of this on this you will get nothing right ok. You can also write this down as previously we had all right this means that we are talking about the transverse electromagnetic wave ok and we are also talking specifically about plane waves in this case ok.

Now, to just make you feel comfortable with the where we have written the k we can always think of the problem in the past that we have done all right we always said that the let us assume that the direction of travel is z this was our most a you know go to Cartesian coordinate system direction of travel is z suppose direction of travel is z all right one can ask what is k right.

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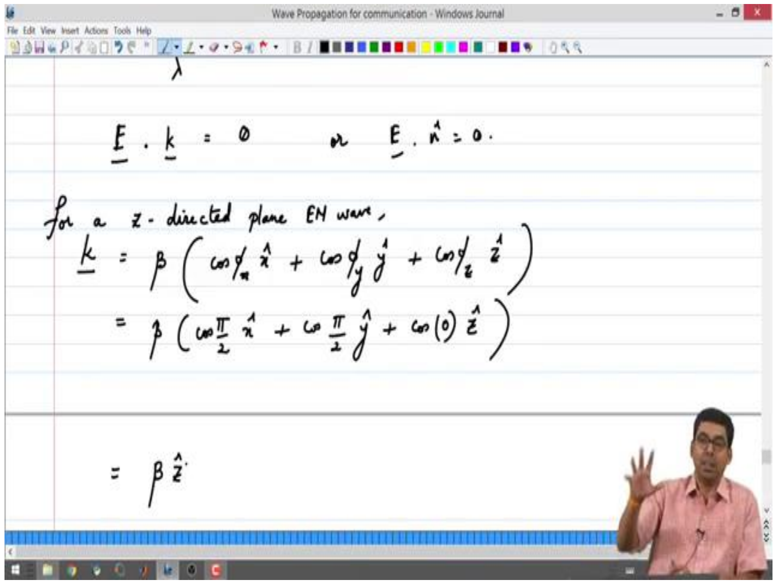

So, let us look at k for a ok how do we write this ok. So, let us look at the expression

$$k = \beta(Cos\ \phi_x \hat{x} + Cos\ \phi_y \hat{y} + Cos\ \phi_z \hat{z})$$

$$= \beta(Cos \frac{\pi}{2} \hat{x} + Cos\ \frac{\pi}{2} \hat{y} + Cos(0) \hat{z})$$

$$= \beta \hat{z}$$

A z directed wave all right the way you will draw the unit vector $\hat{n}$ is along the z axis that is it. So, in the direction I mean in the diagram that we had before this $\hat{n}$ will be along the z axis and it will be going up all right.

It will starting here it will be going there ϕ_x will become 90 degrees ϕ_y will become 90 degrees because it's the angle measured from the x axis to the vector all right. So, ϕ_x will become 90 degrees ϕ_y will also become 90 degrees which means that you will be writing this as

$$k = \beta(Cos \frac{\pi}{2} \hat{x} + Cos\ \frac{\pi}{2} \hat{y} + Cos(0) \hat{z})$$

Now, since it is along the z direction ϕ_z is going to be 0 degrees all right. So, you can just scratch that ϕ_z over there and just write as 0 degrees or 0 radiance right. So, this means that I will end up with an expression that just says $k = \beta \hat{z}$ Ok, the first step is to be able to write down the k vector for something which is given to you. For an electromagnetic wave is travelling in some direction, how do you write down the k vector? This is the first step ok.

So, in a generic case you should be able to take a coordinate system draw the direction of travel mark the unit vector multiply that unit vector with β in that medium then measure the angles from the x y and z axis write down cos ϕ_x cos ϕ_y cos ϕ_z write down k vectors this is the first step ok. And then there are only a couple more steps. I will describe that in the next class ok and once you have done that you know how to write a plane wave when you are given a coordinate system and an arbitrary direction of travel.

Once you know that things are very simple you can always orient it in the way you want and you can start writing down things in many practical problems if you orient the coordinate axis to suit one part of your analysis. The other part may not be exactly in a convenient location, for example if you have a transmitter that is located at 000 all right and you say that its giving something in x direction I mean in z direction extra.

You may have some medium which reflects it let us say 45 degrees alright and it goes to another place you want to describe that and then you do a series of manipulations and then you have a detector that is finally placed in some other direction. You do not want to rotate the coordinate system for every component in your analysis. You want to keep it fixed and then see what happens. It will help you in those kinds of scenarios. But the first step is to identify k vector all right k vector is β times whatever your unit vector is to the direction which is normal to the surface ok you cannot take point p in our diagram.

You have to take points all right for defining your k. This is the first step. The remaining steps are very simple. We will go over it in the next class and in the next class you should be comfortable writing down plane waves in an arbitrary direction. Once we have done that, then the scope for dealing with interfaces comes into the picture. We can deal with arbitrary directed waves hitting some reflecting medium refraction. All these things can come into the picture. So, that is the idea right.

So, I will stop here and we will meet in the next class.