Transmission lines and electromagnetic waves
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Lecture – 19
Electromagnetic Waves in a Conductive Medium

We will get started right. So, the last aspect that we saw was polarization, today we are going to just see how to start incorporating the effect of conduction current density specifically conductivity and the presence of free charges in the medium. The assumption thus far has been that we have medium free from the charges all right and now we are going to incorporate that part. The analogous part in the transmission line is introducing resistance and conductance in the equivalent circuit for the transmission line section.

However, the details are slightly different because here we have both conduction and displacement current densities. So, the interpretations and the number of cases are slightly different, but the analogy is exactly the same as what we saw in transition a, transmission lines ok.

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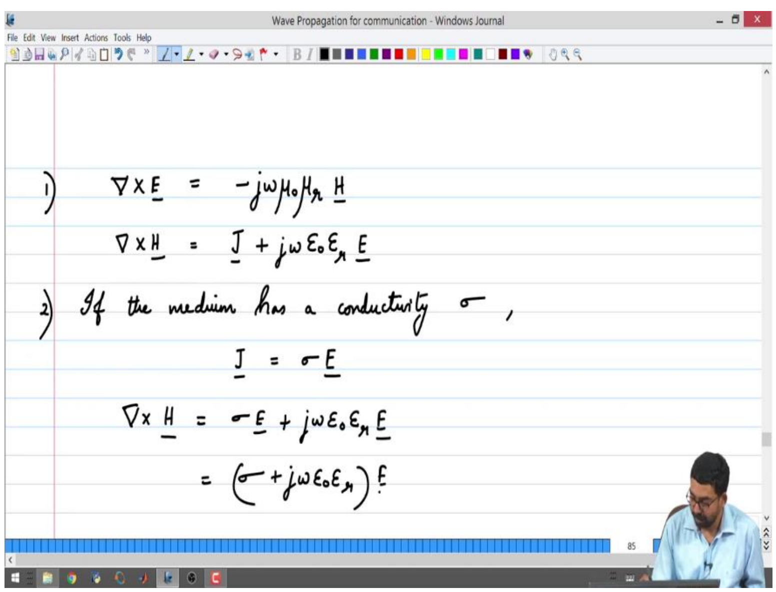

So, in order to start with this ok I am going to rewrite the two curl equations for now right ok. I am going to start with the $\nabla \times E$ and in the prior sessions we have written them in time domain ok.

Now, along with the introduction of losses, I want to introduce the effect of frequency because we know already that in the case of transmission line, we dealt with $j\omega l$ and $j\omega c$ unless you introduce the frequency, you will not be able to extract the correct value of inductive reactance and capacitive reactance. Similarly, here we want to have the effect of frequency specifically we want to analyze what a material medium will do in general at high frequencies how do they behave at low frequencies how do they behave.

So, we have to introduce the frequency ok. So,

$$\nabla \times E = -j\omega\mu_0\mu_r H$$

The assumption here is electric field and the magnetic field are time harmonic all right, they are periodic and the expression will be $E_0 e^{j\omega t}$ and $H_0 e^{j\omega t}$ ok. So, when you take a derivative of$H_0 e^{j\omega t}$ you will get $j\omega H_0 e^{j\omega t}$t.

So, we have just marked H_0 here again this part is exactly same as what we did with transmission lines there we would have had a voltage which was a sinusoid specifically we took a cosine and then we will take a real part of $e^{j\omega t}$. So, the same thing is being done here right. So, again $\nabla \times H$ previously we had neglected conduction current density now we are starting to incorporate the conduction current density also right. So, this would mean you have a conduction current density J right.

$$\nabla \times H = \sigma E + j\omega\epsilon_0\epsilon_r E$$

$$\nabla \times H = (\sigma + j\omega\epsilon_0\epsilon_r)E$$

So, this was $\frac{\partial D}{\partial t}$ all right and assuming that your medium is isotropic all right I mean isotropic then you will have ϵ_0 ϵ_r ϵ_r coming into the picture and then you are also assuming that its frequency independent for the ϵ_r value ok. The frequency dependence that we are interested in is using the ω and assuming that even if ϵ_r and ϵ_0 are going to be fixed.

They are going to be fixed right and if σ is going to be fixed thus ω play a big role in determining whether a material will behave like conductor or a dielectric this is the broad question and we are going to look only at the two curl equations right. So, we can begin this analysis by saying that, if the medium has a conductivity right and we represent conductivity with σ ok. We can write down Ohm's law for the conduction current density. You can say that the conduction current density which is in A/m right it's going to be

$$J = \sigma E$$

So, this is just Ohm's law unit of electric field volts per meter all right. So, correspondingly on the left hand side I have included a conduction current density. So, this means that the first equation does not change all right because we did not have any conduction current density related term over there, but the second equation $\nabla \times H$ we can expand J in terms of σ and E all right. So, we can substitute σ E instead of J in $\nabla \times H$ equation. So, we can start with $\nabla \times H$ ok you can write this down as

$$\nabla \times H = \sigma E + j\omega\epsilon_0\epsilon_r E$$

So, you can take the constants out you can say that

You can go one step further and a, we can try to rearrange the terms inside the bracket all right and try to see if we can divide this into some a, real part imaginary part extra all right. Already we have a σ term, which is not dependent on the frequencies what it looks like and then we have a $j\omega\epsilon_0\epsilon_r$ term suppose we rearrange all right.

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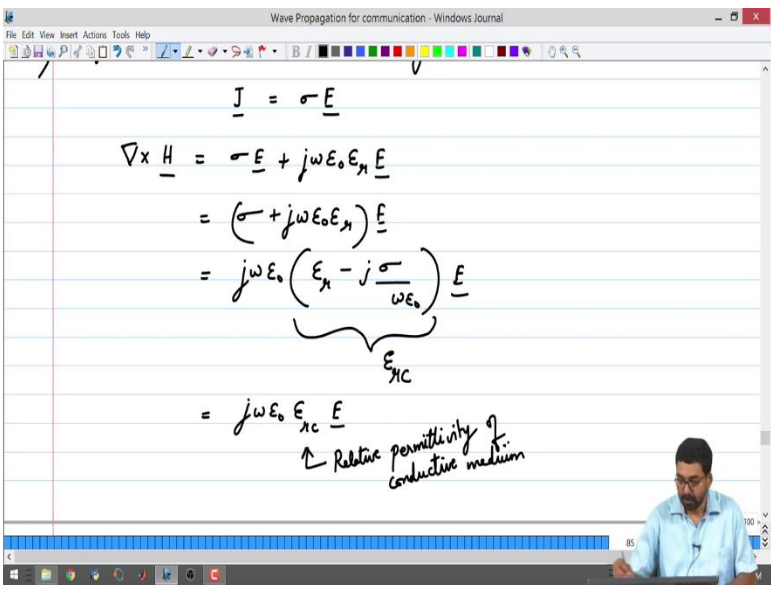

And we do some distribution of terms then can we make a different physical interpretation right.

So, what we can do is we can rewrite this as j omega ok ϵ_0 and then you can rewrite this as

$$\nabla \times H = j\omega\epsilon_0\left(\epsilon_r - j\frac{\sigma}{\omega\epsilon_0}\right)R$$

is the same thing all right where we have taken some common terms out and we have rewritten the same expression, the idea is very simple. If you look at the original $\nabla \times H$ equation we had j omega ϵ_0 ϵ_r E and you added a j.

Suppose I rewrite this down as j$\omega\epsilon_0$ some μ quantity times E, it will resemble the original equation that we wrote for vacuum also right. It will resemble, but the interpretation is going to be different.

So, what you are trying to say here is the term in the bracket right we can create a new constant or we can create a new variable name right and call this as ϵ_{rc}. c means complex all right. So, previously we had j$\omega\epsilon_0\epsilon_r E$, now we have created a new term, it has a real part and an imaginary part. And we call this as ϵ_{rc} meaning that we are saying that let us assume that the permittivity relative permittivity if it is to be a complex number, then we can account for the conduction current density also right. Then what happens is the way you write the equation becomes similar to the way you were writing prior to inclusion of j right.

So, you can write this as

$$\nabla \times H = j\omega\epsilon_0\epsilon_{rc}$$

It's just a simpler way to write the curl equation it resembles the original equation that we wrote except that we have suffixed c for the subscript of ϵ_r right. So, now, this term needs more attention right. So, this term needs more attention previously it was only j omega ϵ_0 ϵ_r all right now it is ϵ_{rc}That means, we have created some new term and we have to look into it in detail. That is it right.

So, we use this constant or variable right, this a, case its known as relative permittivity of a conductive medium, you can just say that relative permittivity of a conductive medium. Previously we just had relative permittivity all right now we are adding something to specify that, it could be a conductive medium if it does have conduction current density present ok.

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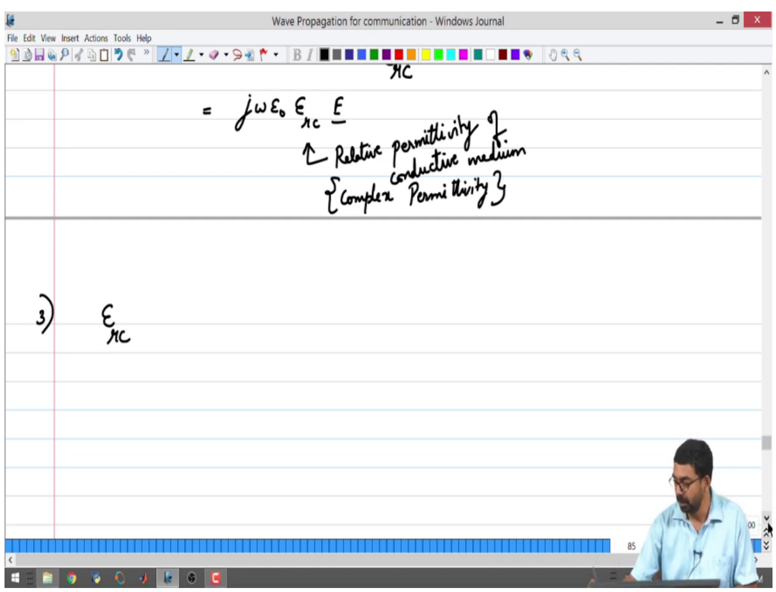

Once again the ϵ_{rc} is also known as a complex permittivity, in some text or complex dielectric constant all right. Complex permittivity is easier, complex dielectric constant is kind of weird because you are talking about a constant related to dielectric, but you are saying it is complex and you are including some conductivity, it may be confusing for a few people. So, you can just call it complex permittivity. It includes both the displacement and conduction quantities ok.

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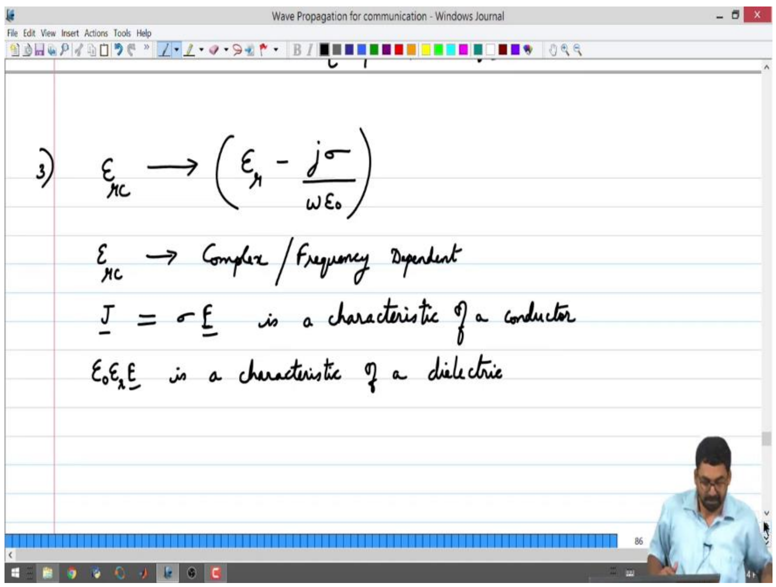

So, ϵ_{rc} the first thing that we note is let us write down this expression, it has the real part, it has an imaginary part ok. One of the things that we notice here is a, even if you assume ϵ_r to be fixed ok all right.

Even if you assume ϵ_r to be fixed that is the real part is fixed even if you start with some number say 1 and if you start with non zero σ and keep it fixed you can say it's also equal to 1 all right. ϵ_0 is the constant say $8.854 * 10^{-12}$ right.

But depending on the frequency, ϵ_{rc} can change right. The first thing that we notice here is even if you keep the relative permittivity conduction a, and you're a relative I mean absolute permittivity of the vacuum to be fixed, depending upon the frequency your complex permittivity can change all right. The only way you can remove any frequency dependence in such kind of media is by plugging σ equal to 0 correct. So, in a pure dielectric medium its possible to do that right over a wide range of frequencies, but at a conducting medium ϵ_{rc} depends on frequency.

So, this is the first thing that we have to deduce no matter what you do it's going to be complex and it is going to be say frequency dependent ok. One also can say that the term J = σE right is a, characteristic of a conductor ok its a characteristic of an electric conductor and $\epsilon_0\epsilon_r$E is a characteristic of a dielectric ok.

So, there is some term corresponding to characteristics of a conductor, there is some term corresponding to the characteristics of a dielectric in the most generic case where you write down the complex permittivity ok. Now then the question is is the material I am sorry usually I turn it off ok.

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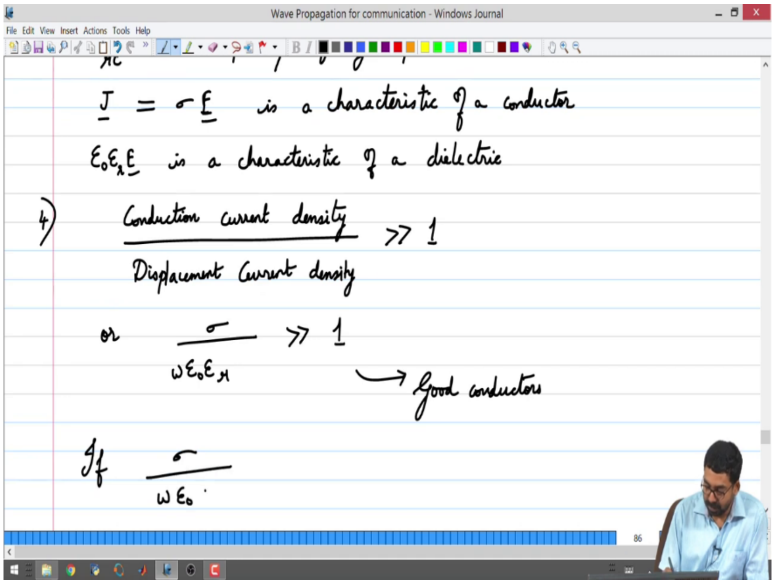

So, then the question becomes could we take the ratio of one quantity to another to determine whether the metal a material is behaving more like a conductor or a dielectric right. So, we can say that conduction current density divided by displacement current density supposes it is much greater than 1 ok. Suppose the conduction current density divided by the displacement current density is much greater than 1 or equivalently σ divided by omega ϵ_0 ϵ_r is much greater than 1 we call such materials as good conductors ok.

So,

$$\frac{\sigma}{\omega\epsilon_0\epsilon_r} \gg 1$$

we call it as a good conductor all right. Now there is a small detail that we have to notice over here. It's not a very big thing, but there is a small detail. σ greater than 1 does not automatically qualify for being a good conductor in these cases, you have to take $\sigma/\omega\epsilon_0\epsilon_r$ and that has to be much greater than 1 ok.

So, the ratio is more important all right. For example, one can always say that if you keep on increasing, the frequency of operation ok your denominator will keep on increasing invariably you will start with fixed value of σ and the ratio will be going towards 1 and then less than 1 extra.

So, it depends on the frequency and looking at this expression here, we can say that, there is a good chance that at very low frequencies, a material will be a very very good conductor, but at higher frequencies, the same material may actually behave like a dielectric ok. So, we can also write down if $\sigma/\omega\epsilon_0\epsilon_r$ << 1, you will call this as a good dielectric ok. So, the ratio is very much greater than 1 its a conductor, ratio is very much less than 1 you can call it a dielectric.

So, it means that if you start with fixed values of σ fixed value of ϵ_r and as you change the frequency, it is inevitable that at low frequencies you will observe that the material seems to be having a large amount of conduction current all right compared to the displacement current density. But at very high frequencies the material will be more or less dielectric this is the way it is all right.

Now we notice that the ratio goes between very much greater than 1 to very much less than 1 which means that has increased the frequency, the material has to pass through a frequency where both these quantities are nearly equal because you cannot go from one to the other without passing through that equality ok.

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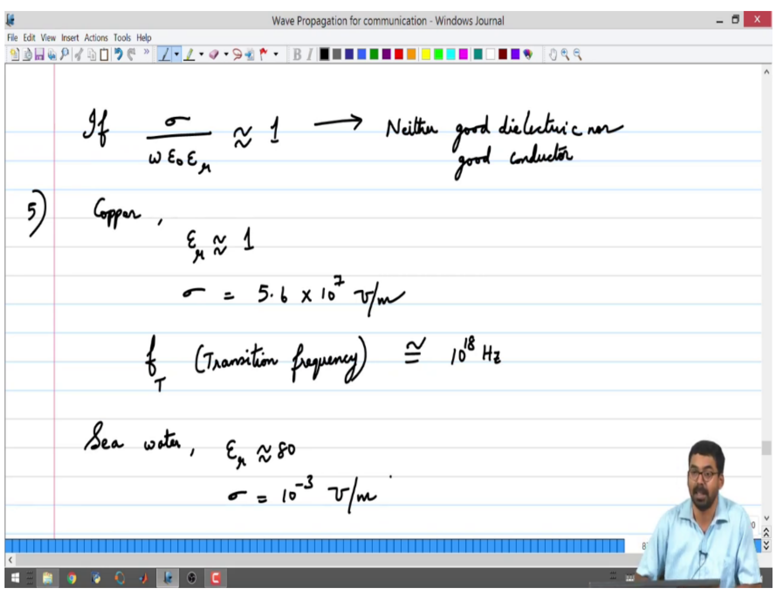

So, then you can have a case were $\sigma/\omega\epsilon_0\epsilon_r$, see in the order of 1 right ok. So, there will exist a frequency for a material where you cannot figure out whether it is a dielectric or whether it is a conductor ok.

So, these materials you can just say as its neither good dielectric nor good conductor ok. So, this means that if σ is approximately equal to $\omega\epsilon_0\epsilon_r$, you can expect that material to be neither a good dielectric nor a good conductor in other words you cannot say absolutely if the material is a dielectric or a conductor, this is true for any material all right and this happens at specific frequencies for specific materials.

For example, if you take copper it may have a value of σ all right assuming that σ does not change with frequency at all and assuming that ϵ_r of copper is 1 for all frequencies. You can calculate at what frequency omega σ will become equal to omega ϵ_0 ϵ_rThere will exist a frequency where both these components are equal and after that copper will start behaving more like a dielectric ok.

So, the trend here is at low frequencies many materials could be very good conductors, at high frequencies the same materials could exhibit dielectric-like behaviors ok. Now this is an important aspect because we use metals for interconnects on boards ok. Suppose you start increasing the frequency of operation of these chips and suppose you start increasing the frequency of operation of your entire system all right you need to be aware that the best conductors will not behave like the best conductors anymore ok.

So, it is a very important aspect while designing high frequency circuits or high frequency communication systems extra right. Just to give you an example all right all right. For copper I just looked up a value on the internet, some standard value people use this ϵ_r is approximately equal to 1 all right hard to believe, but copper seems to be having relative permittivity of just 1 similar to that, aware all right.

But where it differs from vacuum is the very high value of conductivity ok. So, I have written the distributed parameter $5.6 * 10^7\ \mho/m$ ok it's a very high value compared to ϵ_r right. So, we can say

$$\frac{\sigma}{\omega\epsilon_0\epsilon_r} = 1$$

$$\omega = \frac{\sigma}{\epsilon_0\epsilon_r}$$

So, we can say that the omega at which this transition happens right you can divide it by two π and you will get the frequency in hertz right. So, that frequency is known as transition frequency ok its known as transition frequency in this case just taking these values alright. It appears that the frequency in this case where I have taken these particular values is $10^{18}\ Hz$ ok seems abnormally high, that is simply because I have taken some values which I have assumed do not change with frequency also ok.

But in the simplest case it does look like at very high frequencies, copper should start exhibiting equal dielectric and conductive behavior and it's very high all right. For sea water all right. I saw that the relative permittivity from some source its about 80 all right and σ is $10^{-3}\ \mho/m$. Again these are not validated sources of this information but these are some sources that I have picked up just to demonstrate that there is a transition frequency.

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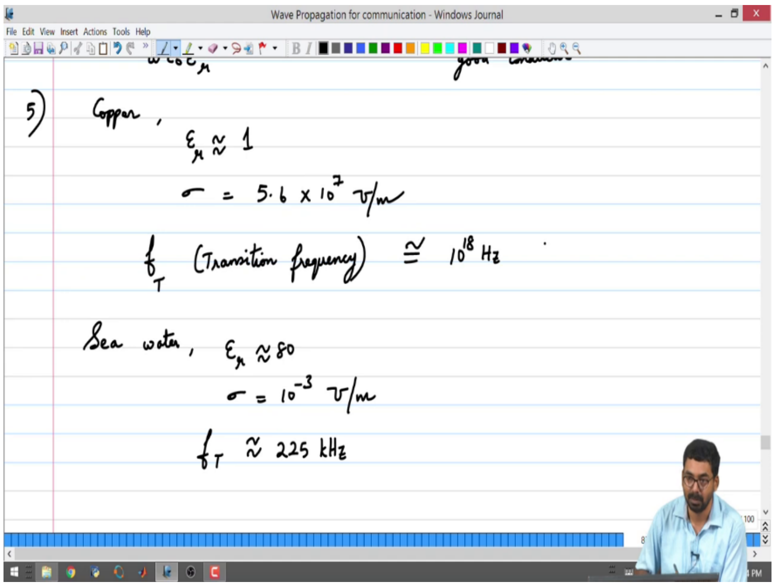

And in this case if you do the substitution for

$$\frac{\sigma}{\omega\epsilon_0\epsilon_r} = 1$$

and try to find out the transition frequency from there, it appears that the transition frequency is approximately 225 kiloHertz.

So, I just wanted to show the high range of frequency and relatively very low range of frequency by you know appropriate choices of ϵ_r and σ That is all right. This transition frequency in semiconductors they also call it as dielectric relaxation frequency, at this place you will not be able to figure out whether the material is behaving like a conductor or a dielectric. In conventional optics it's also common to call this plasma frequency because you cannot figure out the property as to whether it is a dielectric or a conductor ok.

So, these are just different terms, we will use the term transition frequency ok.

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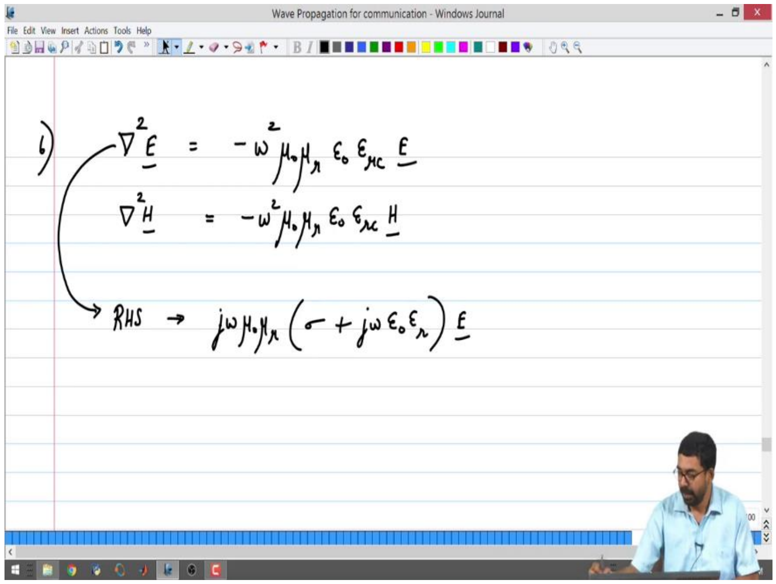

Now, we can go back and have a look at our modifications to the wave equation

$$\nabla^2 E = -\omega^2 \mu_0 \mu_r \epsilon_0 \epsilon_{rc} E$$

This is how we would have written it in the frequency domain, we had written in the time domain before the frequency domain assuming that you are having a single frequency source of electric and magnetic fields, this is how you would have written.

The only change now we need to make is change ϵ_r to ϵ_{rc} to indicate that yes you are including some conduction current also. In the same way for magnetic field

$$\nabla^2 H = -\omega^2 \mu_0 \mu_r \epsilon_0 \epsilon_{rc} H$$

In this case to indicate that you may be taking care of the conduction current density in the suffix. So, that rearrangement for the permittivity and conductivity is useful because we can revisit all our equations and add some subscripts all right and then we will be good to go right.

Now, let us take this first equation and start looking at the right hand side. So, once again you can go ahead and substitute for what is happening to ϵ_{rc} ok you can always substitute for ϵ_{rc}. So, there can be a rearrangement possible on the right hand side. So, I will just rearrange the right hand side $j\omega\mu_0\mu_r(\sigma + j\omega\epsilon_0\epsilon_r)E$

It is just to say that you can always separate terms, you can always rearrange it right, you can take some common terms out, but this I believe is hard to remember for the students, it's easier to just remember this right. So, if you remember this and later on if you want to substitute for ϵ_{rc} you could always do this and rearrange it, it's not a problem ok.

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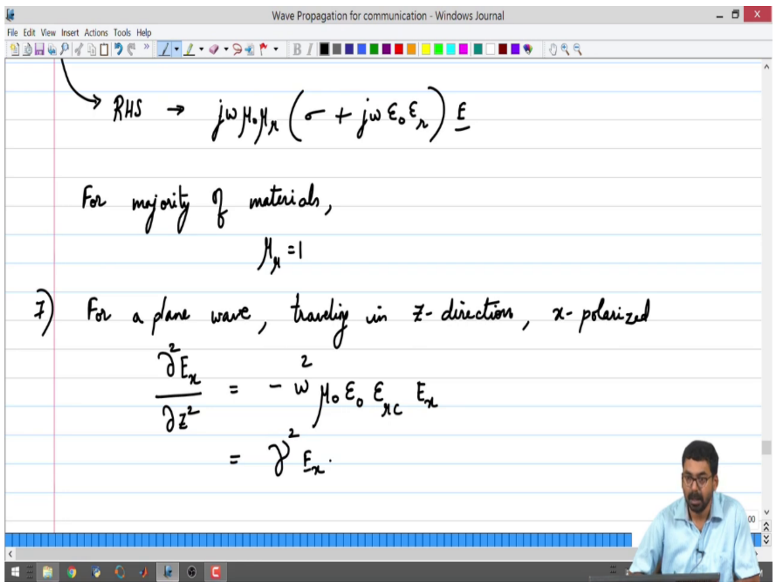

Now, we can also look at the equation again and say that a,for the majority of materials, for the majority of the materials it is safe to assume that a,the materials do not have any relative permeability greater than air, many of the materials are non magnetic all right. So, majority of the materials its safe to assume that μ_r is equal to 1. Of course, when you are doing serious research you can plug in the exact values and proceed, but it's not a, incorrect to assume that for a lot of the materials that you will encounter μ_r is actually equal to 1 that just removes one over term from the right hand side of the wave equation right.

So, which means that, I can now start to look at the solutions to the wave equation we already know that there will be a forward and a backward wave extra. We already know that there will be a propagation constant previously we had only phase constant now because you have included the conductivity part. There will be a propagation constant and we already know from transmission lines that the propagation constant will have a real and an imaginary part, real part will correspond to attenuation, imaginary part will correspond to the phase constant ok.

So, we are going to do the exact same thing as we did in the case of transmission lines ok. So, in this case there are some details that we will add because in the past we have come across these details we have to be specific to say that for a plane wave ok travelling in the z direction. So, now, we are becoming more specific in the way we a,you know talk about the plane wave, we are saying that it is travelling in the z direction and we also know from the prior class that we have to talk about polarization all right. So, we say that x polarized.

So, in the beginning or in the prior classes we have seen that the majority of the time we took Ex and then we took Hy and then the propagation direction was z. So, we are specifying that you know using a verbal statement saying that, its travelling in the z direction and it is x polarized. So, the wave equation can then be written as

$$\frac{\partial^2 E_x}{\partial z^2} = -\omega^2 \mu_0 \mu_r \epsilon_0 \epsilon_{rc} E_x$$

$$\frac{\partial^2 E_x}{\partial z^2} = \gamma^2 E_x$$

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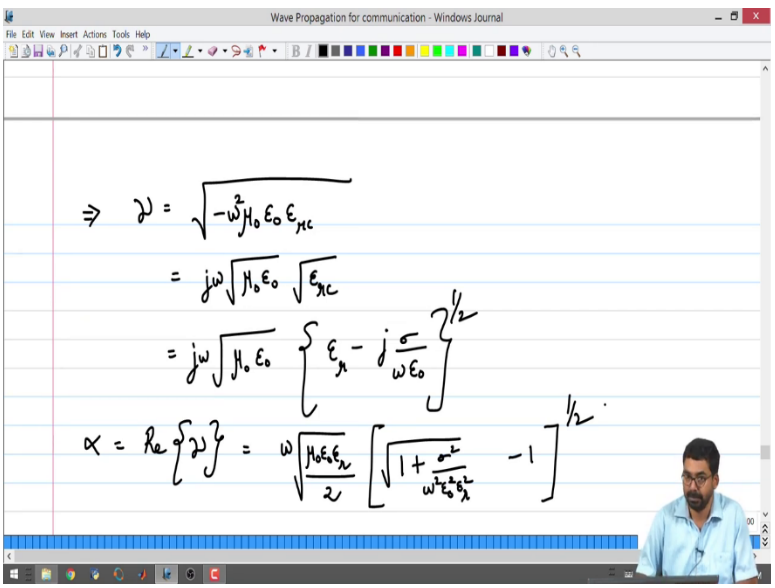

So, which means what we have done is we have created a new variable gamma is going to be

$$\gamma = j\omega\sqrt{\mu_0 \epsilon_0}\sqrt{\epsilon_{rc}}$$

This will be our propagation constant in the case of plane waves in a medium having conduction current ok ok. And now ok we also know that, by casual look we will make a deduction that we have j omega something which means that it is purely imaginary. But that is not true that is the illusion that a, this substitution for I mean ϵ_{rc} is giving us one should always remember in the back of the mind that this is again a complex number. So, even though you have written $j\omega$ Something here is a complex number. So, you will end up having a real and an imaginary part ok.

So, you will end up having a real and imaginary part and the task to separate them is not very simple ok. So, I will write down right anyway I will substitute for the ϵ_{rc},

$$\gamma = j\omega\sqrt{\mu_0\epsilon_0}\left\{\epsilon_r - j\frac{\sigma}{\omega\epsilon_0}\right\}^{\frac{1}{2}}$$

If we put it like this, then it becomes clearer that there is a real and there is an imaginary part ok. So, one has to be very careful and it's obvious that there is a real and an imaginary part, but separating them is not very easy ok.

So, what I did is I just plugged it in wolfram α on the internet and I checked what is the real and the imaginary part ok. I am not going to go inside how to separate the real and the imaginary parts over here, but I can make use of a symbolic math tool like x maxima or wolfram α and quickly see what is the real and the imaginary part ok. And I saw that α which is the real part of the propagation constant, this case turns out to be

$$\alpha = Re\{\gamma\} = \omega\sqrt{\frac{\mu_o\epsilon_0\epsilon_r}{2}}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon_0^2\epsilon_r^2}} - 1\right]^{\frac{1}{2}}$$

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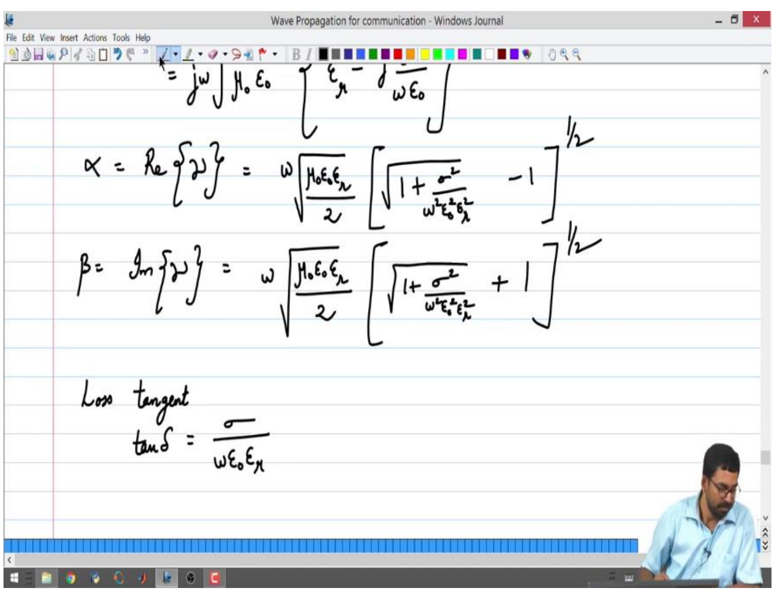

So, it was justified that I did use a tool to calculate ok. β It is an imaginary part of gamma. I am writing down these expressions just for the sake of completeness. I do not believe that you have to remember this for your exams and quizzes. If we come to a stage where you need to make formula sheets extra you definitely could do that all right, but even better will me to ask questions that do not involve this kind of expansion. So, I will see to it that that happens ok.

So, α is the real part of gamma say attenuation constant, β is the imaginary part of a, gamma which is the phase constant all right and they are very complicated expressions for us to make any direct a, evaluation all right. The only thing that one can say is α and β depend on frequency alright and I can say that there is omega multiplied by something and then I have

$$\beta = Im\{\gamma\} = \omega\sqrt{\frac{\mu_o\epsilon_0\epsilon_r}{2}}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon_0^2\epsilon_r^2}}+1\right]^{\frac{1}{2}}$$

Generally I can notice that since there is an ω term coming over here, I can make a general remark saying that as omega increases α could increase as omega increases β could increase these are the only things that broadly you can make out from here, but the prior argument was much easier to follow than this one ok. So, a, there is only one more term that people often use ok which is known as loss tangent ok we use the term loss tangent ok. So, this is usually given by

$$tan\delta = \frac{\sigma}{\omega\epsilon_0\epsilon_r}$$

So, if you were to consider this in the form of a, σ being represented in one axis, jω I mean $\omega\epsilon_0$ ϵ_r being in another axis you could mark and angle between the real a, axis alright. And between whatever value of $\frac{\sigma}{\omega\epsilon_0\epsilon_r}$ with respect to the origin right and it will resemble the triangle and you are taking the tan of that angle.

So, people call this loss tangent alright. So, they also mark it with a, term tan delta ok and you have $\frac{\sigma}{\omega\epsilon_0\epsilon_r}$ ok. And higher the value for this higher is going to be your attenuation that is all ok that we already know because if σ is going to be higher right because σ is present in the numerator of α . Obviously, you are going to be having more attenuation right which means that the electromagnetic waves that we are talking about ok.

So,

$$tan\delta = \frac{\sigma}{\omega\epsilon_0\epsilon_r}$$

is the loss tangent and usually higher this value ok one can immediately notice that you are having this term coming into the picture over here so, obviously, higher is going to be your attenuation alright it's another way of looking at it and a, there is also some other things that can be used to simplify things right.

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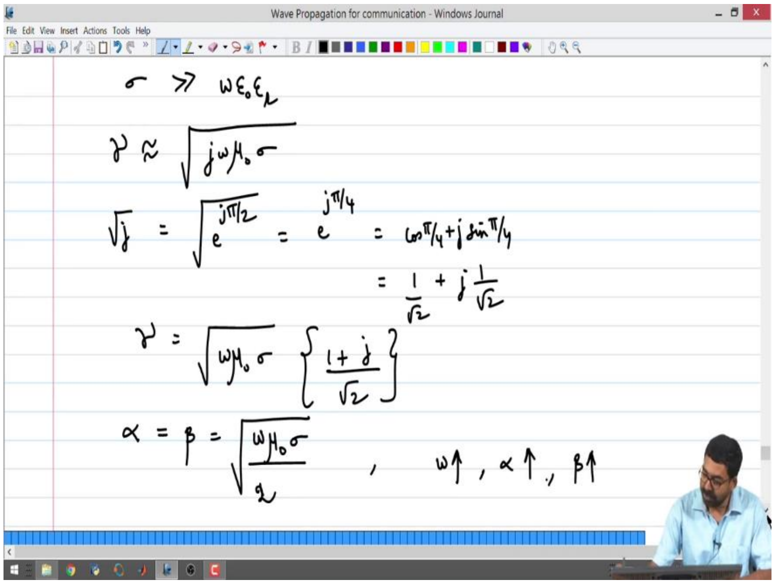

Let us take a material for example, where σ is much greater than omega ϵ_0 epsilon r.

The reason why I am writing all these things is because in the real world you can make some approximations by saying that one is much larger than the other and scratch out terms, so that it becomes easier to simplify that is why we are doing this in a number of places. So, if σ is very much greater than $\omega\epsilon_0\ \epsilon_r$ all right you can write down the propagation constant all right to be simply

$$\gamma \approx \sqrt{j\omega\mu_0\sigma}$$

Once again, this will have a real and an imaginary part ok and square root of j is usually confusing for some students right. So, if you want to write down the square root of j the best way to do this is to make use of Euler's theorem all right. Write it as you know

$$\sqrt{j} = \sqrt{e^{\frac{j\pi}{2}}} = e^{\frac{j\pi}{2}} = Cos\left(\frac{\pi}{4}\right) + jSin(\frac{\pi}{4})$$

$$\gamma = \sqrt{\omega\mu_0\sigma}\,\{\frac{1+j}{\sqrt{2}}\}$$

$$\alpha = \beta = \sqrt{\frac{\omega\mu_0\sigma}{2}}$$

So, in this case one can notice that its very very easy to separate α and β, you will have a j term alright and you will have a non j term that is all. The other observation that we make over here is when σ >> $\omega\epsilon_0\epsilon_r$, it's rather a peculiar case where α is equal to β and it's just

$$\alpha = \beta = \sqrt{\frac{\omega \mu_0 \sigma}{2}}$$

So, if you make some simplifications, in this case we said first μ_r is equal to 1, then we said that if σ >> $\omega\epsilon_0\epsilon_r$, then you can start cutting terms and then the expression for α and β becomes much simpler and then you can make some reasonable deductions you can take one parameter at a time increase and see the effect on α .

Now, it is clear that if I increase the value of ω, α will increase if I increase the value of ω, β will increase ok. So, at higher frequencies you will experience higher attenuation in conductive media ok at higher frequencies the phase constant is also higher, that means, the wave will go from 0 to 2π much faster than what it did in vacuum ok.

So, these things become clearer if you start making some assumptions scratch out terms and then you know looking at a simpler expression ok. So, in general a, one can write down that for non magnetic media where conduction current density is much higher than displacement current density or the material is almost a good conductor ok. Omega if it is high it will have high attenuation and it will have high phase constant.

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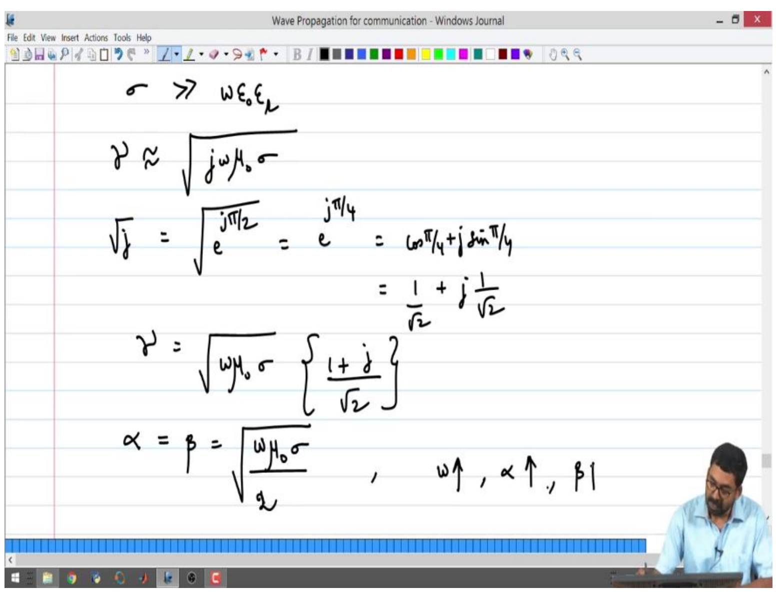

The other way to write this down is if the conductivity is very high which is the case with this structures I mean with the with this materials all right σ is very high compared to $\omega\epsilon_0\epsilon_r$ in this case. You can also write down that your α will be high β will be high. These are probably easier to remember than the a, large expression that you had for α and β ok.

So, I hope that now you have some general idea about the behavior of materials itself, it is safe to assume that any material at low frequencies will be a good conductor and the same material at very high frequencies will be close to a good dielectric ok. So, this is the natural trend at low frequencies whatever conducts at very high frequencies it will become you know dielectric all right. So, there is not much you can do about it.

Secondly, at low frequencies α is low that means you can get a long range of propagation of electromagnetic waves all right. So if you send a plane wave at a very low frequency it will travel a much longer distance in the medium than a plane wave at very high frequencies right. So, as you start increasing the frequency of the electromagnetic wave, you will notice that in the same medium it will start propagating lesser and lesser and lesser distance ok. So, it means that these things pose some technological challenges ok. The evolution of communication has been from low frequency and it is going towards higher and higher frequency of operations.

So, when they had radio waves right when they had long range when they had medium range when they had short wave communication all these things alright you would have had some propagation distance, they were able to communicate over long distances, but they needed maybe large areas extra. But then later on we have switched to much higher frequencies, but the challenge is as the frequency increases the range of your system for communication will naturally decrease because of attenuation present.

So, even in the air you will have some content of say water, water vapor extra this will also have some tiny amounts of salts and other things and which will increase the conductivity you know and then as higher frequencies happen, you will end up with α becoming higher ok. So, this is a practical scenario where one can assume that at low frequencies you can expect a high propagation lens ok and you can expect the material to be highly conductive. Losses are going to be very low, at high frequencies the materials will heat up very fast ok your losses are going to be high propagation lenses are going to be really low and this is inevitable.

Now, there is one more thing that we need to look at in the case of plane waves which is the power. In the case of electric circuits you would have learnt that a, if you applied a voltage and the system draws a current of I, then you will multiply the voltage with the current and then you will say that the power supplied is in watts The power delivered is in so, many watts extra. In this case simply because we are dealing with vector quantities ok we do not deal with the scalar voltage and a current we are dealing with vector quantities all right the calculation of power is slightly different from that of transmission lines ok.

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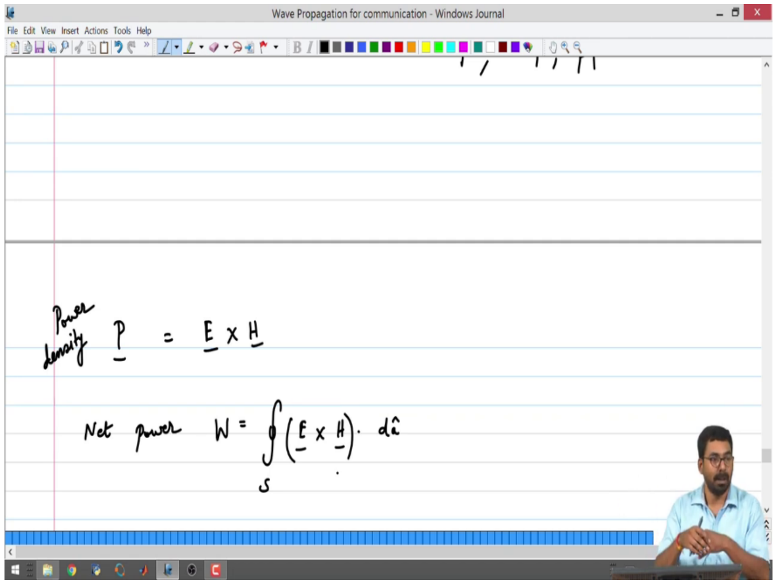

So, the power density ok ok is defined as

$$P = E \times H$$

. Remember that the power density is a vector alright its a vector it does have direction. It tells you where the source is and where the sync is ok. So, it is a vector and it is given as a cross product between your electric and magnetic fields. So, the unit for the electric field again is V/m, the magnetic field is A/m. So, you will have VA/m^2 all right. If you assume that there is no phase difference between your electric and magnetic fields all right instead of writing down volt-ampere you could use watts and say that its watts per meter square is ok.

So, dimensionally it is fine and it means that if your electromagnetic wave is arriving at a particular medium, you will have to take the size of the beam that is hitting the medium. Calculate the power density and integrate it over the entire area because you are having only $Watts/m^2$. To get watts you will have to integrate this power density over the area in which your beam is hitting, then only you will be able to get the net power all right that is crossing a surface ok.

So, the net outward power alright which is P sometimes it can become confusing. So, I will just use W it's going to be your surface integral ok

$$W = \oint (E \times H). d\hat{a}$$

It is slightly different from how you will calculate in electric circuits or even in transmission lines because you did not have vector quantities ok.

So, it is E cross H which also resembles what we had seen earlier classes for the discussion of the polarization, we had used the right hand rule where we we would use the electric field to be along the index finger, magnetic field along the middle finger and then you had a direction of propagation to be your thumb. $E \times H$ just represents the thumb all right. So, the power is going from this side to the other side which is along the direction of propagation all right.

So, it's safe to assume that the source is somewhere here and the sync is somewhere on this side ok. So, you can use the same rule all right to figure out where the power is actually going and there are some more details that we will have to slowly uncover. For example, we have to start looking at how the power depends on space and how the power depends on time.

Because we are having time harmonic electric field, time harmonic magnetic field it's quite possible that there is a phase difference between electric and magnetic field there is no phase difference between electric magnetic fields all right. And instantaneous power versus average power, just like in electric circuits you would have had instantaneous power and average power, you would have had real power, reactive power and apparent power all those things. So, similarly we will have to see what we can figure out from here right. So, that will be the goal for the future lecture right. So, I will stop here.