Transmission lines and electromagnetic waves

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Lecture – 16
Octave Simulation of an Electromagnetic Wave Equation

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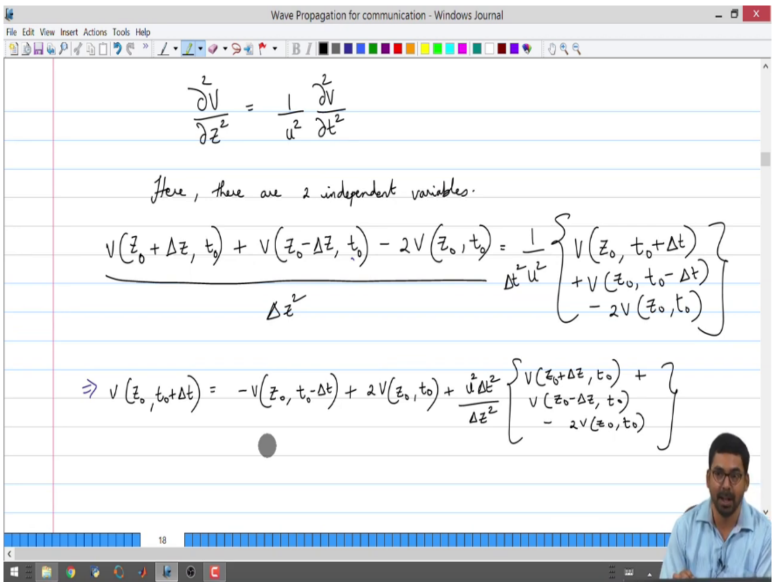

We will get started. So, the goal is actually to revisit the programs that we have written all right, and try to see the similarities a little bit more closely all right, and also expand the way we are thinking in one or two ways that is all, ok. So, I have pulled up the finite difference equivalent ok, from the previous notes for $\frac{\partial^2 V}{\partial z^2}$, you will need the octave in the second, ok (Refer Time: 00:45).

$$\frac{\partial^2 V}{\partial z^2} = \frac{1}{u^2}\frac{\partial^2 V}{\partial t^2}$$

What we had written in the prior class for the transmission line was, we had written a central difference approximation for the second order derivative alright on the left hand side, and a central difference second order approximation for the right hand side.

We identified that the unknown quantity is the one that has a timestamp of $t_0 + \Delta t$. So, we pulled that to one side and all the other remaining quantities were to the right side and we wrote a program for that. We are going to revisit that program, ok. Now we know all we need to do is change V to Ex in our case all right and we are going to be having the wave equation for the electric field, we will start with that all right and then we will talk about the remaining programs, ok

But we will make, since we are now in the advanced stage, we will make some small changes ok (Refer Time: 01:44), just to keep you know things are a little bit more interesting, ok.

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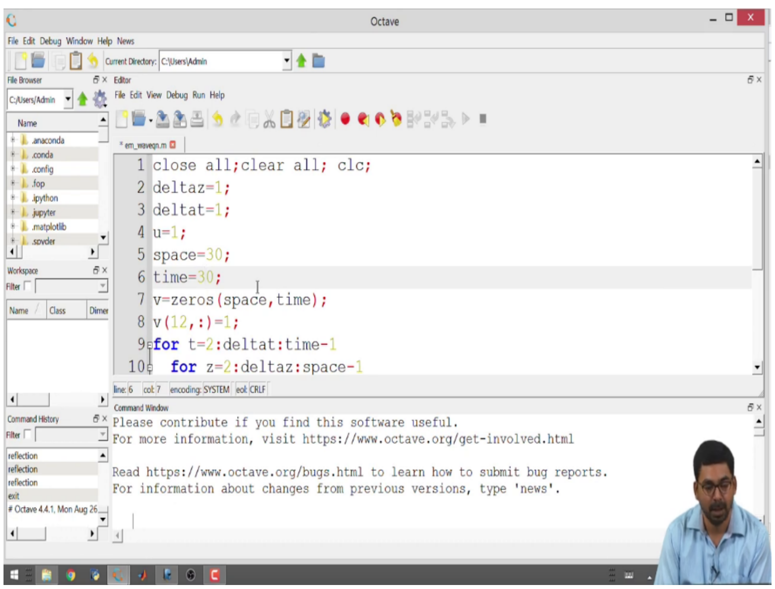

So, I am having the previous file that I have used in one of the lectures for the transmission line, I am just going to make a copy of it.

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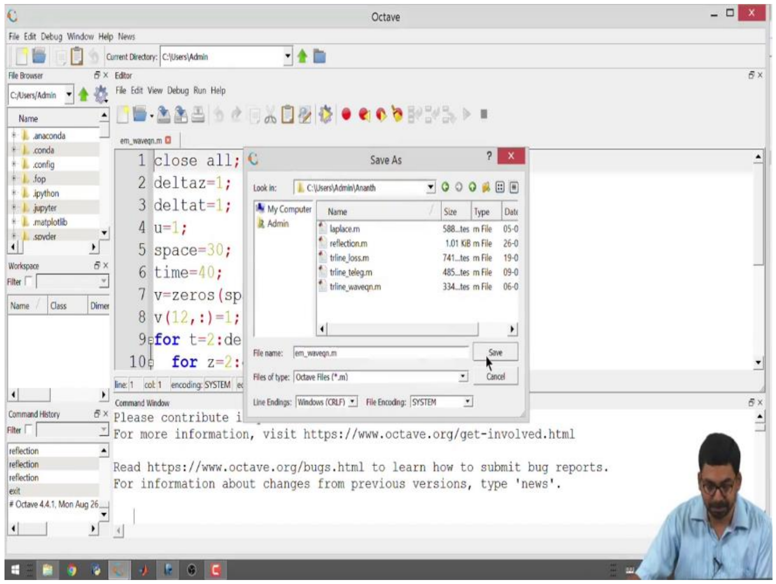

So, I am going to save it as I will just call it the e m wave equation, right ok. The steps involved are nearly the same, but we will make some small changes here and there, all right.

So, I am having the space to be 30 units, time to be 30 units, first of all I am not making use of any absolute numbers. For example, I am not saying this is 30 μm or 30 nm, 30 m extra, these are 30 units, ok. So, what we are doing here is, writing this in some dimensionless space right. So, if you consider the velocity to be equal to 1, then in 30 units of time, it will travel 30 units of space that is all, ok.

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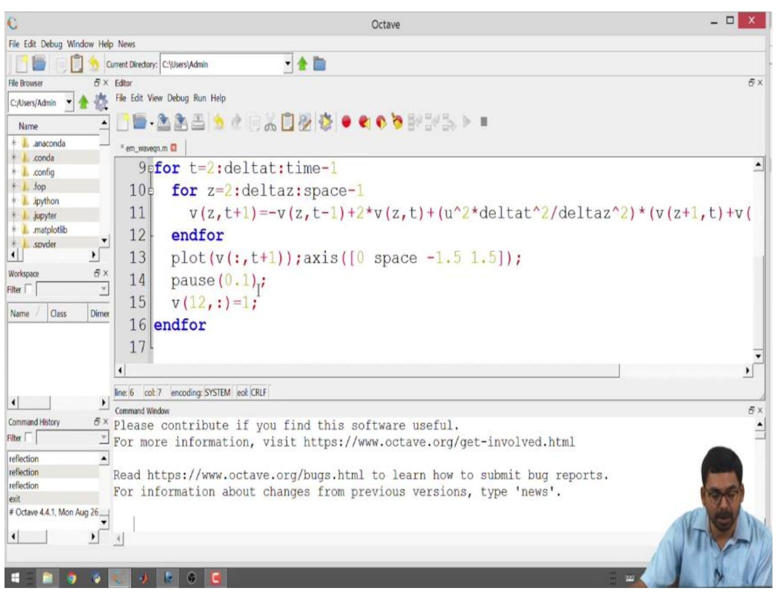

So, it is easier for us to think in this way, right. So, we continue to do that, all right.

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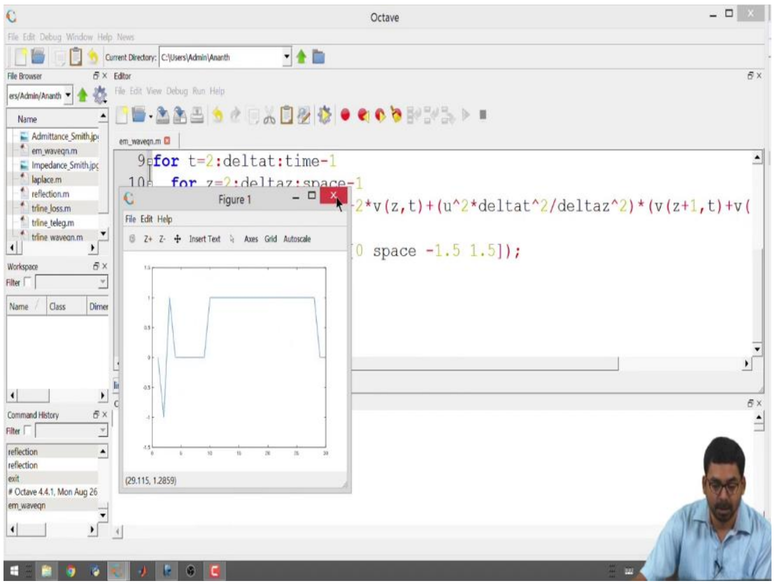

And just to be sure that we are on the correct program, I will just run this once, ok.

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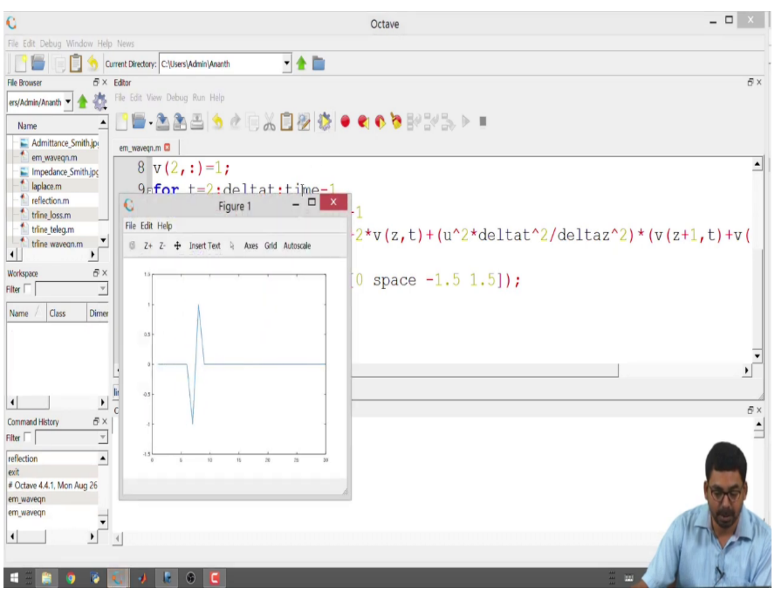

Going back to the original version, start with an impulse because that is easier to explain, ok.

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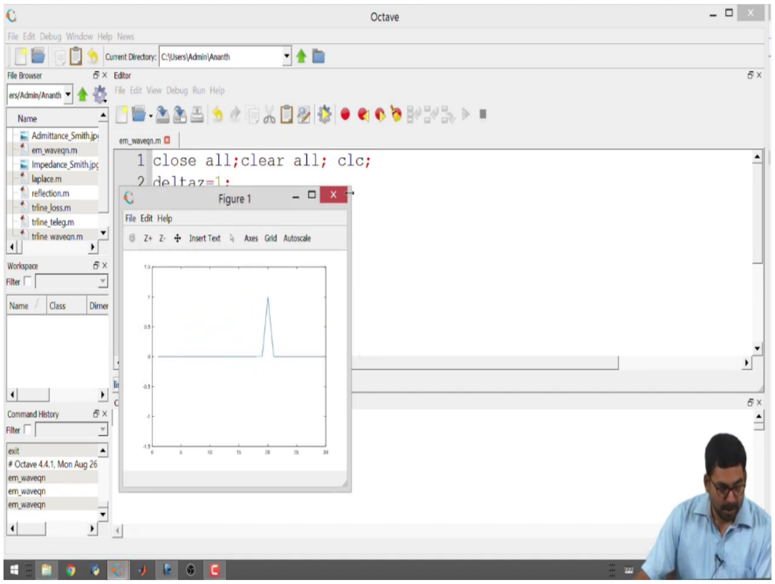

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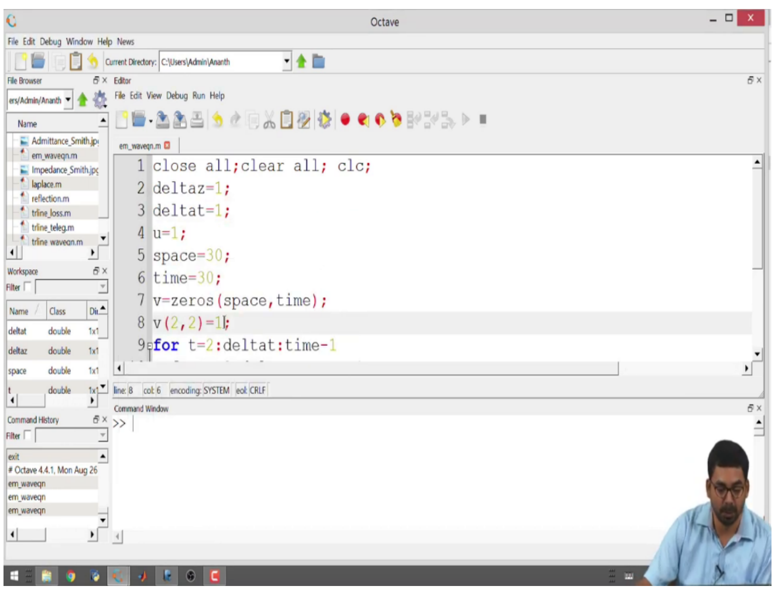

So, I just made the source at a particular instant of time to be equal to 1, all right.

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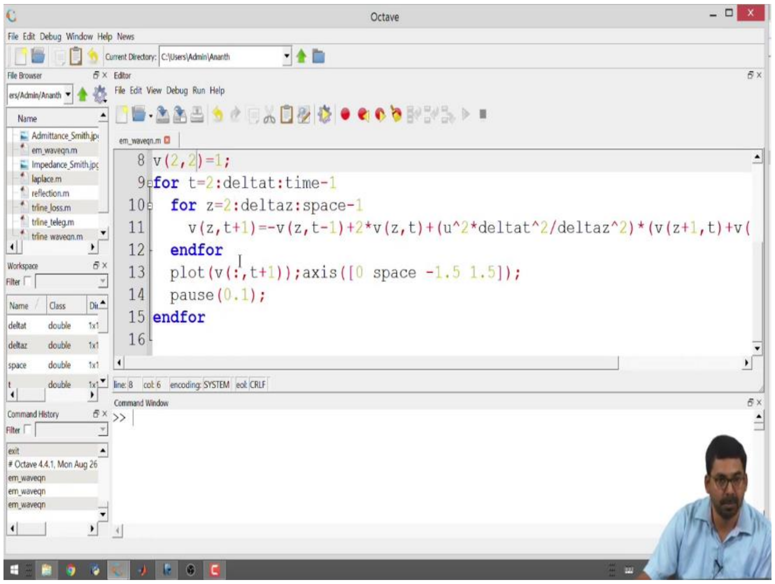

And then I have the loops over here right and this is for the transmission line, where we are calculating the voltage. All we now need to do for this program is change some parameters, Δz can remain as Δz as because the propagation direction that we assumed in the yesterday's class is z, Δt can remain as Δt ok, space can remain as space, time can remain as time, u was changed to c, ok.

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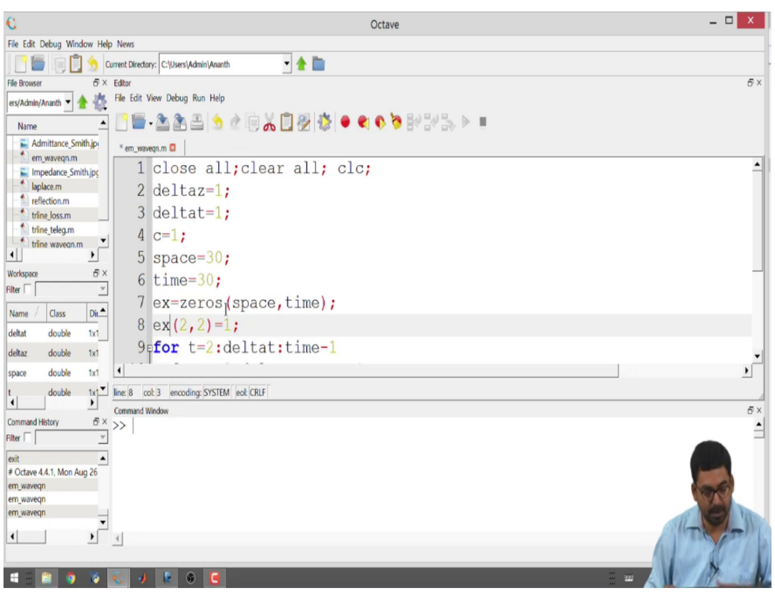

So, I will make that change, c is equal to 1, V was changed to Ex, ok.

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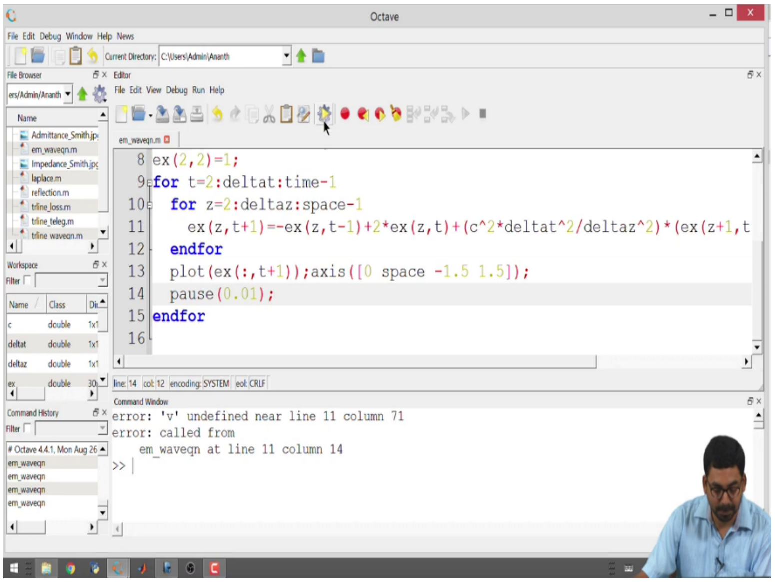

Just to be sure that I have replaced everything correctly, I am running this once ok, there is a V somewhere, so just going to change that, ok. So, I have a working program, all right.

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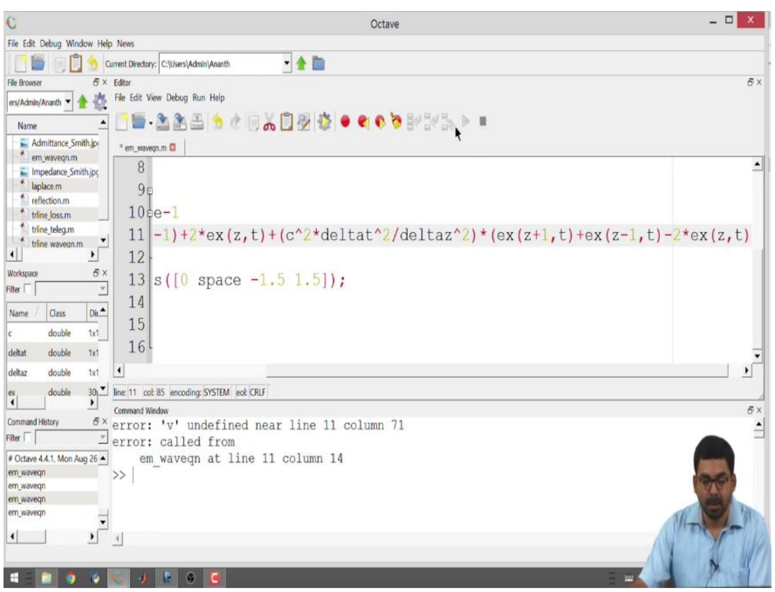

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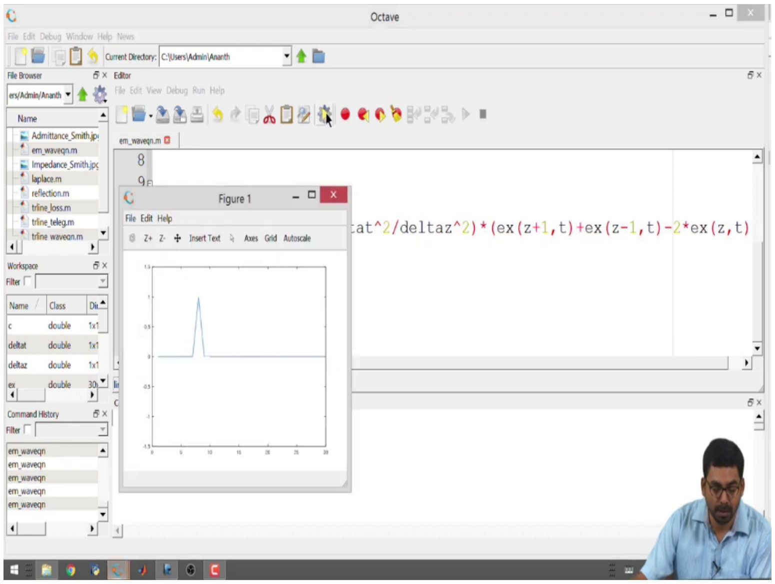

And it is a correct solution to the wave equation, because all we have done according to the mathematics is change the variable names, nothing significant here.

But since we are now in an advanced stage, let us rewrite this in a slightly better way, ok. One of the things that we notice here is that, as we are trying to calculate for all instances of time, for all points in space the value of Ex is dictated by the wave equation all right and there are two for loops. Generally when you have large matrices, for loops are not encouraged, because you are manipulating only one value of the array at a time.

So, now since we are in an advanced stage, we can try to write them with fewer for loops and by using what is known as vector updates in octave or MATLAB. Vector updates allow you to update the contents of an entire array in one shot and internally it handles how to do the mathematical operation for the entire array by itself, all right. So, it may calculate a number of elements at the same time and then give you a value, rather than a loop where each value is updated once and then you go around.

So, since we are now in an advanced stage, I will rewrite the for loop inside ok, which is with respect to the space has a vector update, ok. And then we will see that, we will increase the size of the space and time and see how much time it takes to execute with the vector update and with the for loop, ok.

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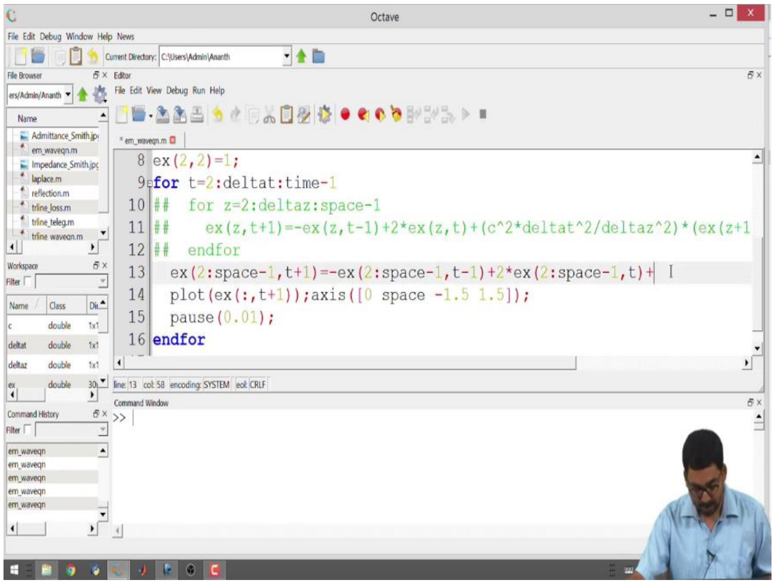

So, first step what I will do is, I will comment on this portion, ok. And I am going to write this for loop in one line of code, ok. So, first of all I notice that the loop goes from 2 to space -1, which means that, whenever I am having z array subscript all right, means that I have to substitute instead of z, 2:space-1, all right.

So, suppose I have, I make the first sentence, the first term Ex of instead of z, I will just put 2 to space -1 is equal to, on the right hand side I have -Ex z comma, oh I missed the t+1. Z, once again instead of z, I will substitute whatever is present in the definition of the for loop for z. So, I will just take this and I will plug it in over here. So, comma, then I have a time indicator of t -1 ok plus 2 times Ex of z comma t, z is being substituted with 2 colon space -1.

And comma t plus I have c square delta t square by delta z square. So, I am just going to copy this term, ok.

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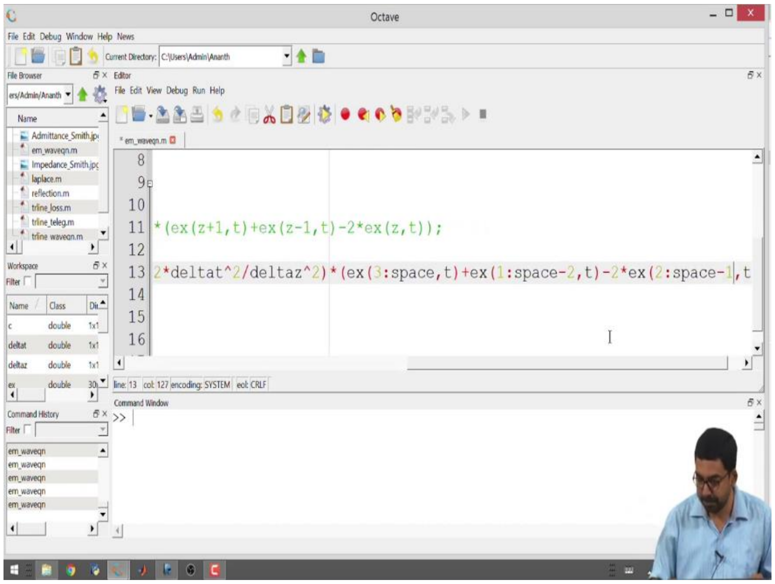

And then I have some terms over here. So, I will copy these and start replacing for z, ok. So, z plus 1 ok, z is given as 2 to space -1, z plus 1 we will be adding 1 to both the left and to the right side of your colon. So, that means, it will become 3 colon spaces, that is, all right.

So, we were here. So, it becomes 3 colon space comma times and then z -1, it just means that you will subtract 1 from the definition. So, you will be subtracting 1 from here, so that becomes 1 and then space minus 2, ok. And then I have z, just going to be 2 colon spaces -1, right.

So, now what we have done in essence is, we are making some calculations on the entire array all right for all points in space for that instant of time. I said we have removed this need for, we have removed this for loop completely, now we are not manipulating one element at a time, octave will figure out a way to manipulate this entire thing for all points in space in some efficient manner, ok.

We do not want to go into the details, but this is the basics of vectorization, all right. And we can run this program and you will notice some results, all right.

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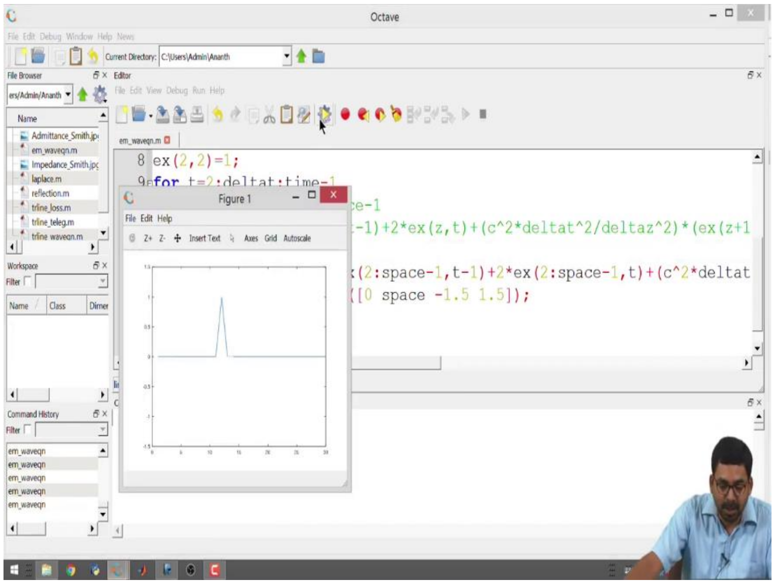

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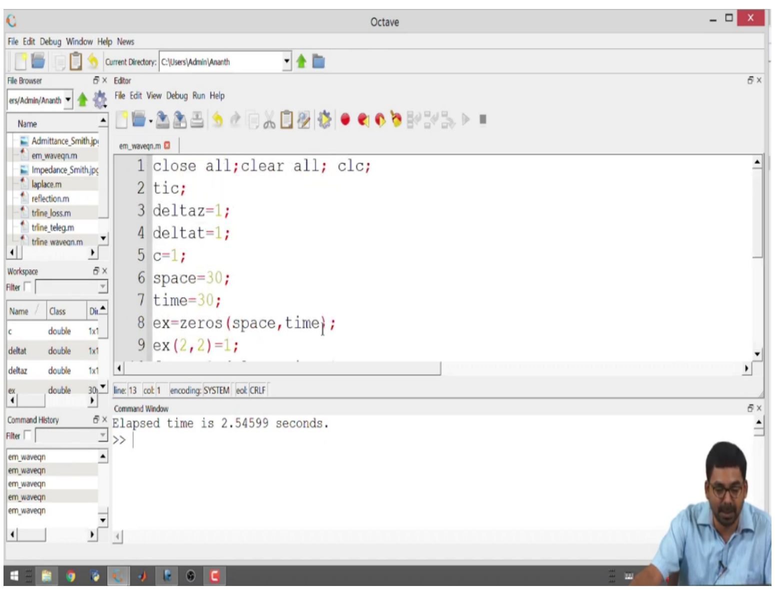

For small sizes of space and time right, you may not notice a significant difference. So, I will just go ahead and introduce a command called tic, allow me to start a clock ok, tic will allow you to start a clock and at the end of the program, I will do what is known as toc, all right.

So, it is tic, toc right, the beginning it will start the clock, at the end it will stop the clock and display the amount of time it has taken, right. So, the elapsed time is 2.53568 seconds, which is what it shows over here, right?

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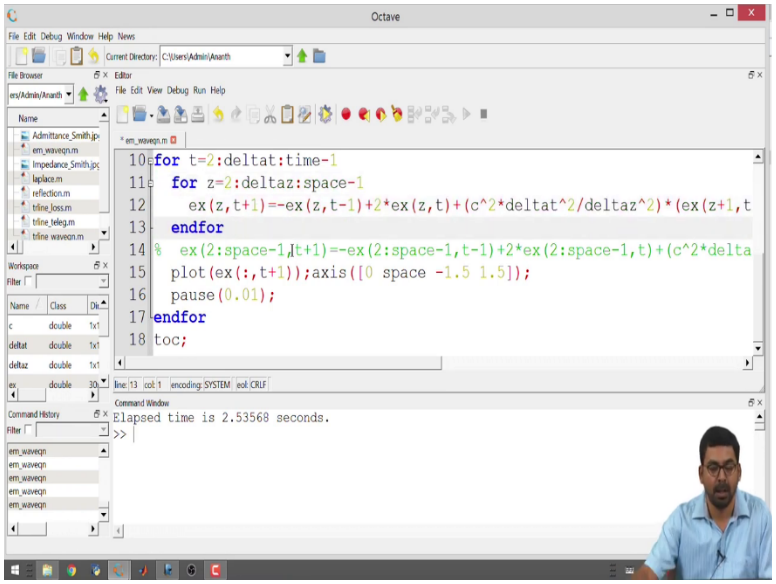

So, one could always comment on this portion, ok.

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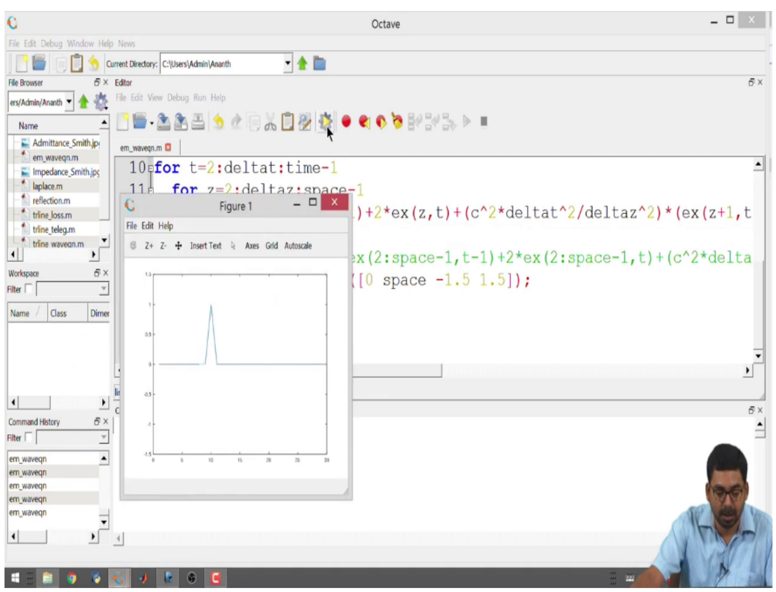

So, it is roughly the same for small values of space and time it is the same.

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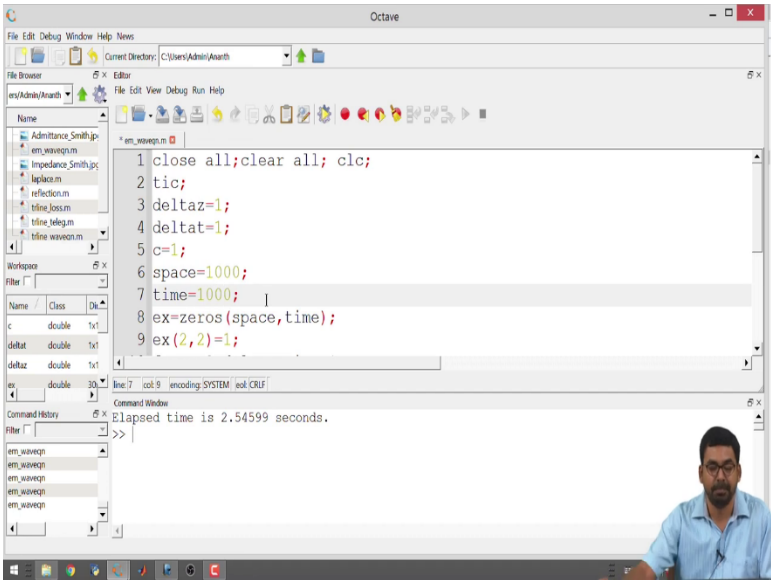

So, I will make the space as 1000 ok and I will make the time as 1000 ok and we will run the program with the for loop.

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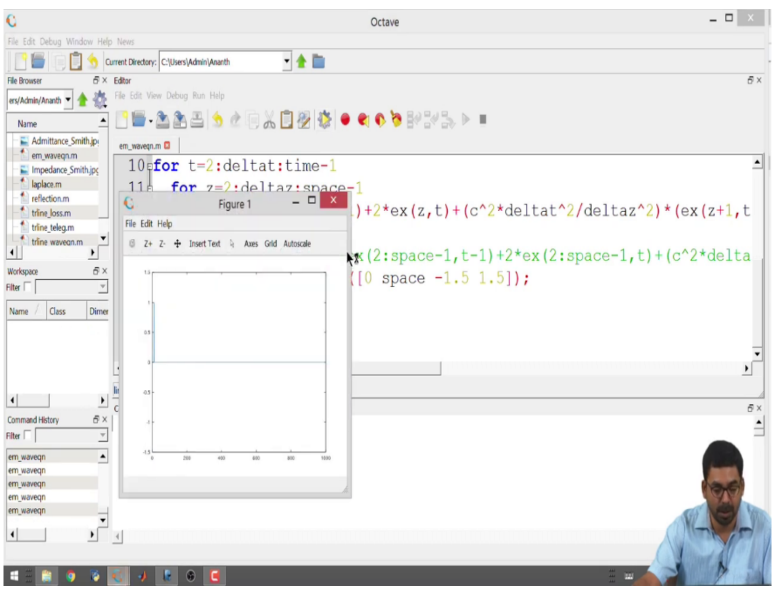

And we will just note down the value, ok.

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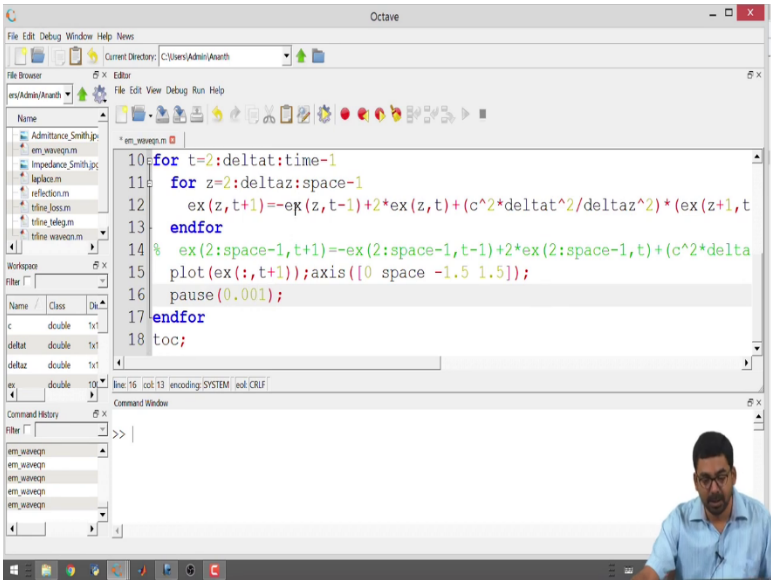

So, I think I am going to reduce the time in between two consecutive, does not help much but let go.

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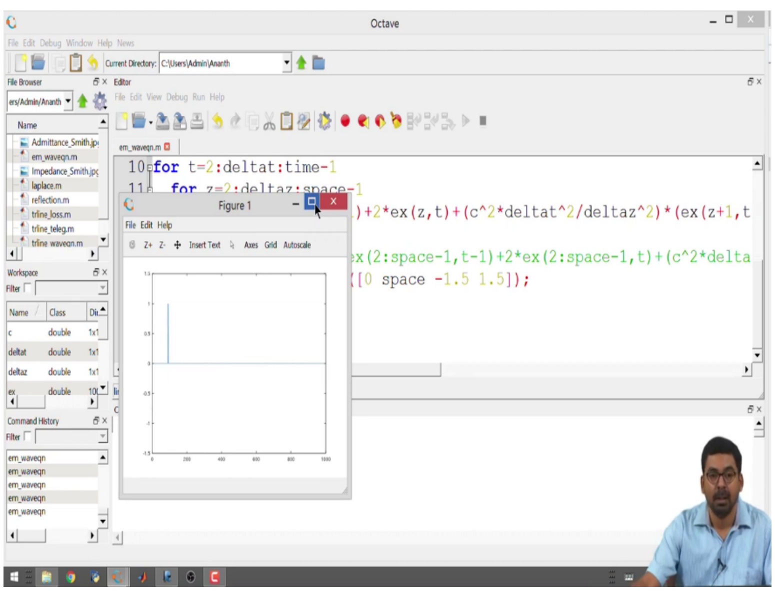

Maybe I am not so patient, so I will just go ahead and make this 300 that is good enough of value, not the speed of light.

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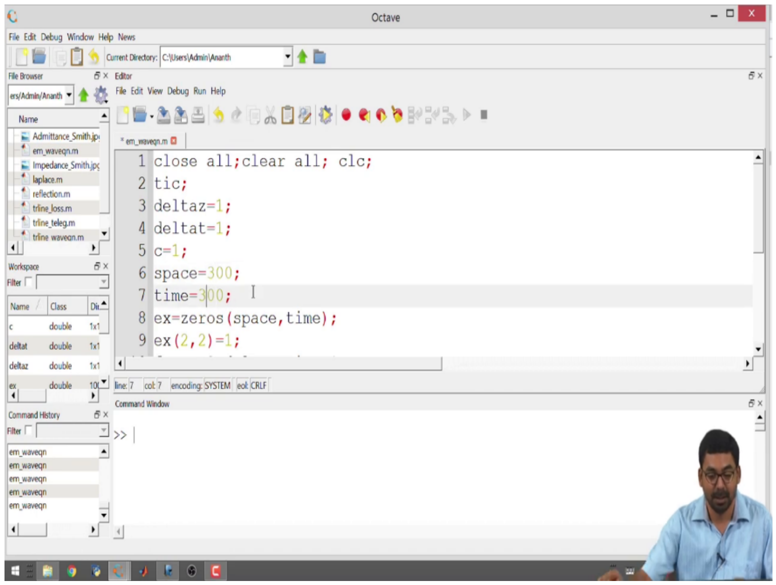

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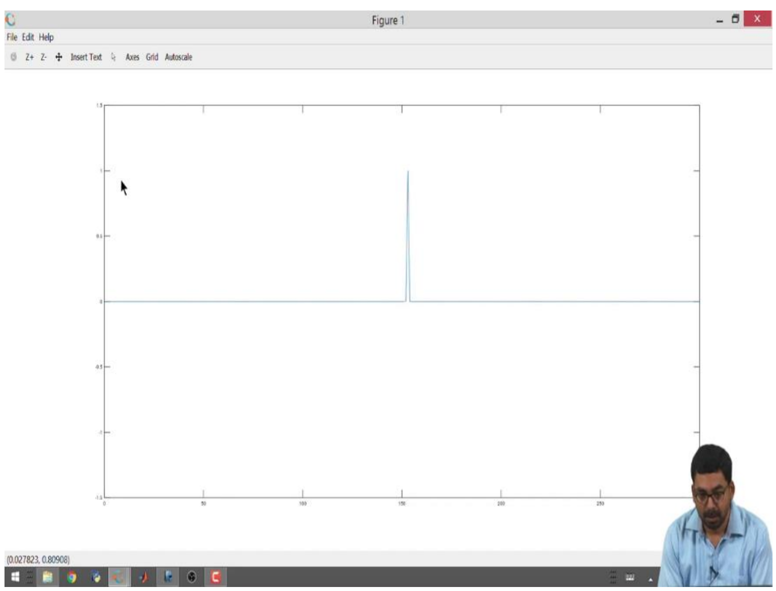

Actually most of the time in the execution is taken for drawing this plot, the computation was actually fairly fast. We are delaying it because of our pause program, that cannot be managed with vectorization all right, but you will see some improvement, right.

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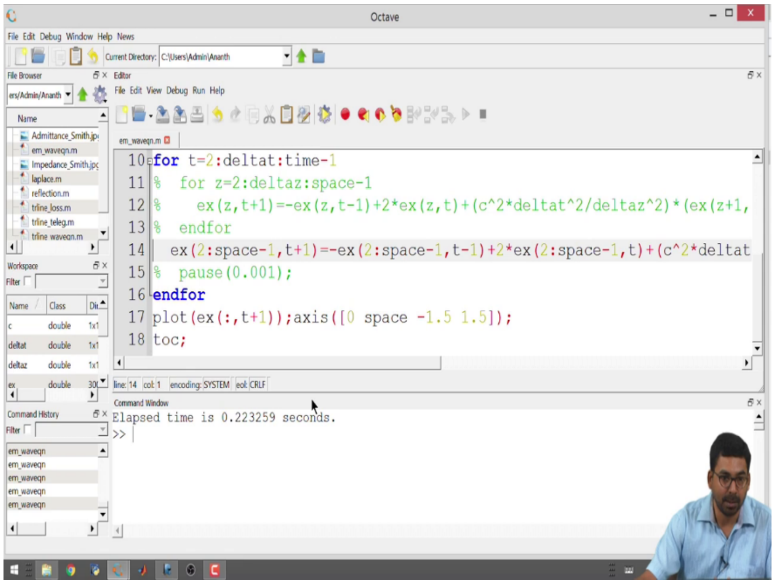

So, one of the things that we could always do is, bring this plot command below all right, comment this say that, I want to have a plot only at the end of all the calculations, I think that makes a slightly better way to approach this right, all right it is much better. So, it is about 2.7 seconds with the for loop, right. So, about only 0.2 seconds with this, ok. So, that was again a 10 time speed just because you removed a for loop and you are calculating for all points in space in one shot.

So, you will notice a significant amount of speed up if you choose to use this vectorization. I know that some of you will want to run many trials with the programs. My suggestion is to replace the inner for loop ok with the vector update, that will give you a significant boost in time, ok. But now in this class, we are interested in seeing what happens as it is happening, so I will move this back inside, ok.

Now, let us increase the amount of, so I will just go back to 30 and the amount of time that we will run this for is 40, ok.

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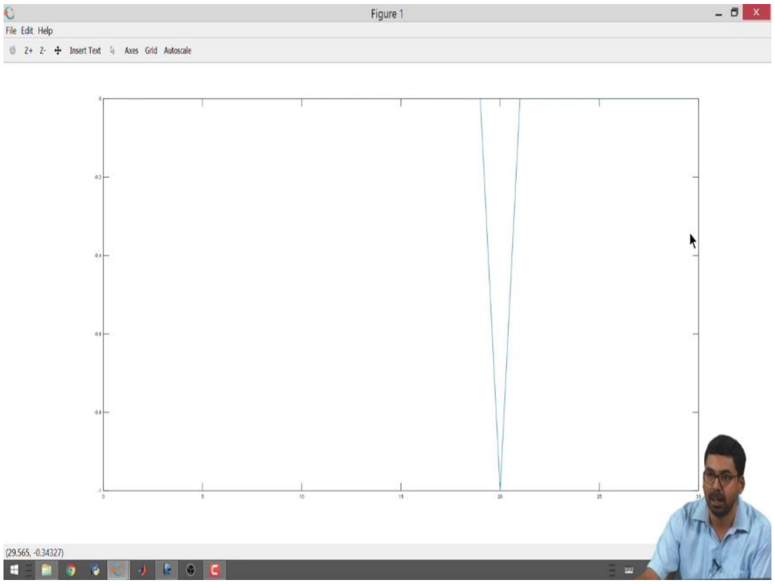

So, there are some things that we noticed, the first thing that we noticed went to the right, flipped and then it came to the left, that means that, there was a reflection, there was otherwise an impedance mismatch.

And this impedance mismatch yesterday we saw that we are talking about characteristic impedance of the medium. So, on the right side it hit a wall and it then bounced back with a change in the sign, ok. So, the electric field flipped. So, if we look at what we are doing in the program we will be able to notice that, on the right hand side which is space, which is called you know space is what we are calling it. So it is number 30 all right in the way we have defined it is the number of points that you have is 30, at the 30th point on the right side is your right extreme of the graph, ok.

We are not doing any update at all, ok.

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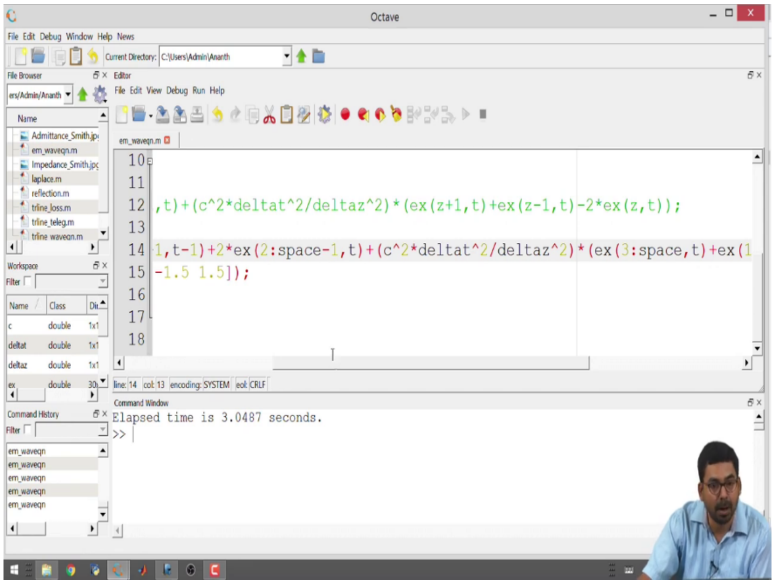

So, space -1 is given by space -1, space you know you do not do any update all right, because the maximum that this one will go to is only 29, so it does not have an update on the right side most edge.

So, the right side most edge in your simulation is going to be having Ex equal to 0 ok, which is consistent with the previous transmission line, you know the discussion that we had, ok.

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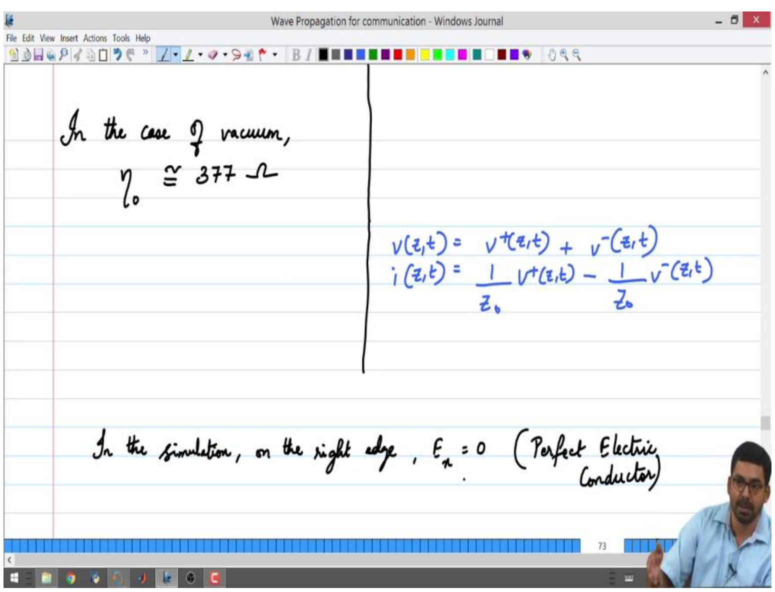

So, on the right edge ok, Ex is equal to 0 all right is what we had, Ex is equal to 0 all right means that, there is no electric field present in that location, ok.

In other words there is no electric field present in that location and we call this as a perfect electric conductor, ok. In the case of transmission line, you would have had V equal to 0, V equal to 0 was equated to a short circuit ok, in this case you call it as a perfect electric conductor. In the case of Laplace equation solutions when we are doing with the program ok, you have a plate on the top and a plate at the bottom and then you are calculating for the voltages in between.

The plate on the top remained at 10 volts, the plate at the bottom remained at 0 volts, so each of these plates did not have any fields inside of them, because there was no gradient of potential in the plates. So, in other words those plates were actually perfect electric conductors ok, but we did not do that simulation in time right, it was just a spatial update of all the voltages in the capacitor right and calculating the electric field.

So, now we are doing it in time. So, Ex equal to 0 is a perfect electric conductor, it means that, if you apply a voltage at any point on that plane, all the other points will get updated to the same value of voltage, so that the net field that you have there is going to be equal to 0, ok.

Remember that Ex equal to 0 does not mean voltage is equal to 0, it just means that, the gradient of voltage is going to be equal to 0, you have a fixed value of voltage, ok. So, you have to remember that e equal to 0 does not correspond to V equal to 0, it means that you are having a constant voltage, ok. So, this is a line of constant voltage in other words, ok. Let us also go ahead and do a few other things, ok.

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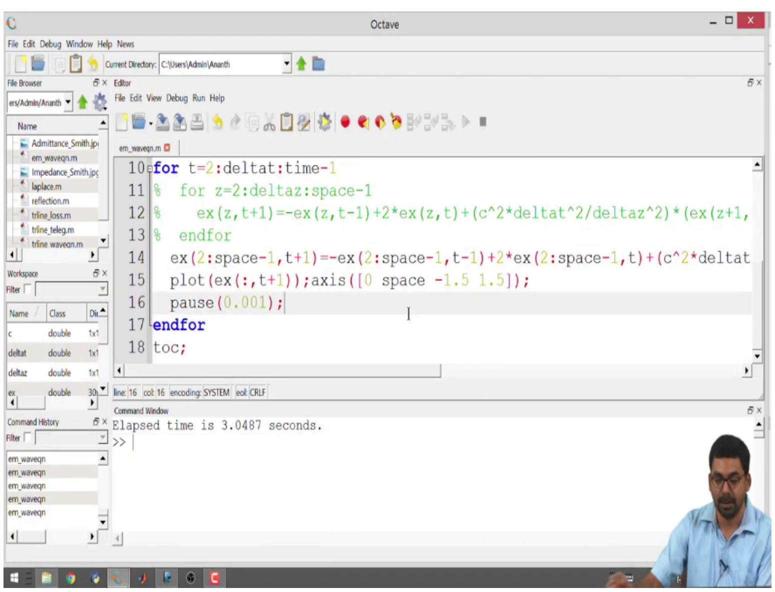

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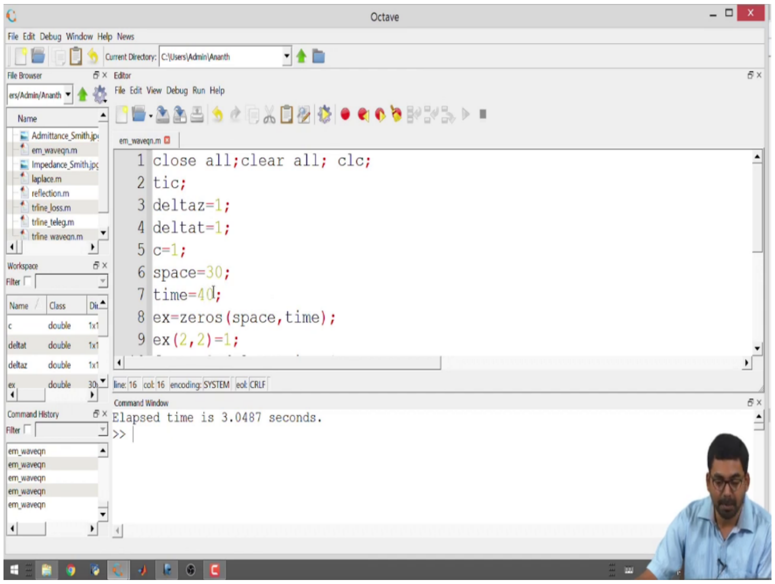

Let us go ahead and we already have the time to be at 40. So, it went to the right extreme, bounced and came back, ok. In the case of transmission lines, when we had dealt with the transmission lines originally, we started with this program, later on we incorporated r and g in the program and we saw that you can have decaying values of voltages and current extra, ok.

Now, the question that can be asked over here is, suppose I want to simulate an infinitely long medium, right. A medium in which there is no reflection of any kind, no matter how much time you run it for, that is it should go from left to right, it should never make a pass from right to left. It should appear like the impulse is starting from here going to the right and after that it just vanishes, it does not enter your system back, ok.

In the previous examples what we would have done is, we would have added some space with the loss r and g in your transmission line and made it exponentially decay all right, the voltage or the current would have exponentially decayed, which means that the amount of reflection that is coming is very small.

Whatever is being reflected again passes through that region with r and g as positive, all right. Once again it gets attenuated and a very small quantity enters the main simulation region again all right, that is one way of looking at it. But there is also another way of looking at it which we will start using now, all right. Ex is equal to 0 was your boundary condition for perfect electric conductor, ok. And previously we have seen some boundary conditions with the Laplace equation we have seen Dirichlet condition, where you have the value to be constant. So, Ex equal to 0 is a Dirichlet condition, all right.

Neumann condition would be some derivative of having the electric field, in this case equal to 0 right, but are there more boundary conditions? There are more boundary conditions. In the case of a transmission line, you use the region to absorb the wave ok, so that can be some kind of an absorbing condition, but it is not really a boundary condition. It did not happen at the boundary, it happened over a region, ok.

Can we make a wave disappear from the simulation region from one side to another side without it coming back? This needs a little bit of imagination, all right. So, since we are concerned only about the point at the right extreme right, we can write down a piece of code that looks like this all right and then try to analyze what it is actually doing, right.

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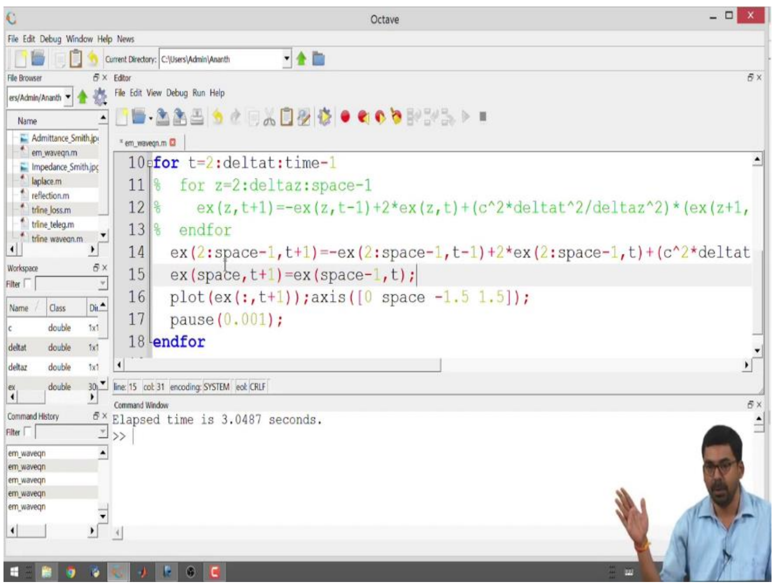

Ex ok at the position space which is here equal to 30 corresponds to the right hand side edge ok, at some instant t plus 1. So, the loop is in t and I am trying to update what is happening with t plus 1, ok ex equal to Ex at the same position, at the position before that ok, at the previous instant of time ok.

Let us just look at what we have written, Ex at space comma t plus 1 is Ex at space -1 comma t. So, what it means is, the new value of the electric field at the right edge is equal to the old value of Ex at the previous spatial location, right. What this means is, if your wave is travelling, it is going to travel one spatial grid point in one unit of time, ok.

Now, in your boundary what you are doing is, you are not using the wave equation to update for the value of the electric field on the right side, all right. Now suppose a wave is travelling all right, a wave is travelling that is the peak of the wave is moving, you know that at the next instant of time, the peak is going to appear at the next spatial location. If that happens, the wave is travelling in one direction, ok.

Implicitly your wave equation is doing that all right, but when it encounters a non uniformity, it also calculates accurately and tells you that right, there is some impedance mismatch. So, it is bouncing back. So, what you do is, you bypass the wave equation for the right side most edge, all right. In order to make it look like the wave is travelling from one point to the next point all you need to do is, in one instant of time you have to move the peak to the next point in space at the next instant of time.

Artificially you have made the wave travel forward ok, which means that, at every instant of time it is just going to take the value which was there in the previous instant of time and keep doing. But it is not going to affect the simulation in any way, because we are not using it for doing anything, right.

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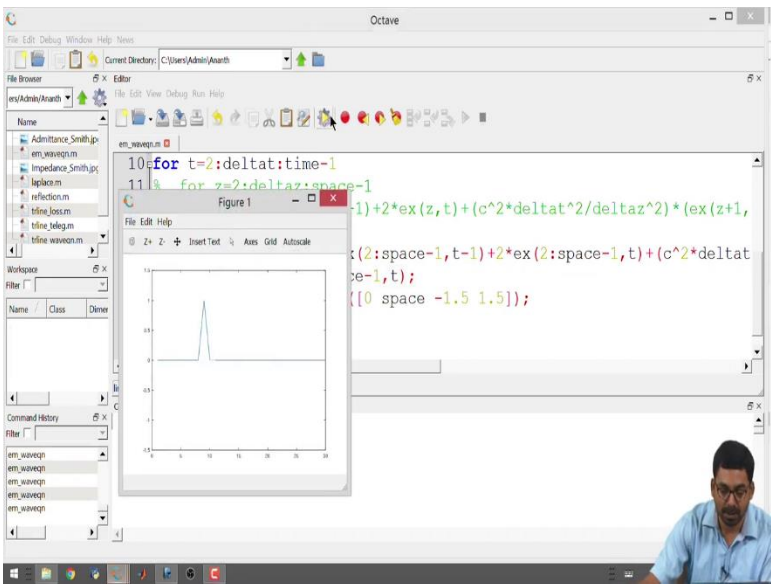

So, if I run this now ok, it just went to the right side and vanished, ok. Maybe a difficult thing to understand, but if you are familiar with waves ok, you can think about some waves and you can think about the waves that are going, you can track the peak of the wave. The wave equation will tell you that, if there is no there is no backward wave it will keep travelling forward again and again.

But if you do not want to use the wave question, at the end point we are not using the wave equation, we are just saying that whatever was there before just put it one time instant later to the right edge. What you are doing is, artificially forcing the wave to travel in the positive direction, ok. So, this boundary condition is known as an absorbing boundary condition, though it is not doing any absorption all right, it is just an absorbing boundary condition.

There are other names that are given to this boundary condition, people call it say transparent boundary condition, people call it infinite boundary condition, extra ok, depending upon the software that you use, they may have different terms for it and slightly different interpretations or names for it, right. So, this is a condition that one could use to make sure that, no matter how small their space is, the wave travels through your system only once, ok.

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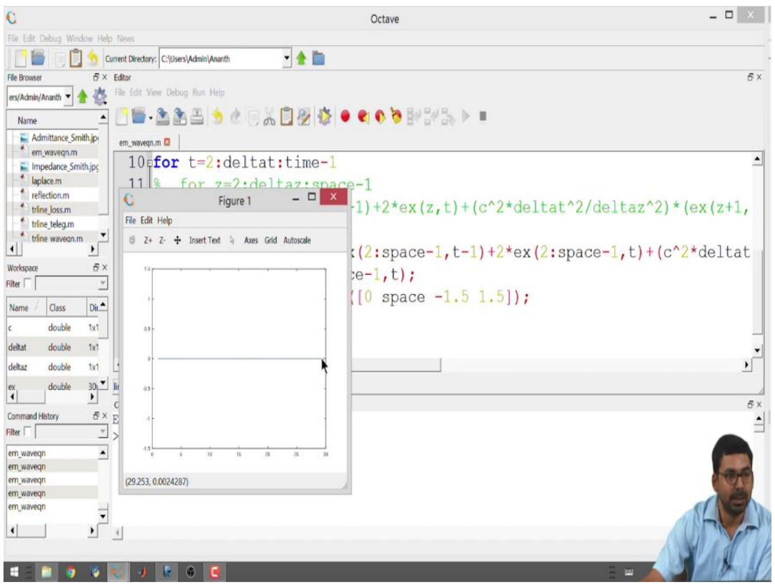

So, once again I will run this, just to show you that it travels only once. It is very important for the wave to travel only once in the majority of the simulations. It is simply because what you are trying to figure out at the end of the simulations is, you can assume that there is a medium or there is a region in between, you are trying to find out the parameters of the region in between.

You can assume this to be some kind of an open loop control system, all right, where you have an incoming signal and you have a box which has a transfer function and then it has an output on the other side. You want the input to pass through the output and not come back, so that you can determine the transfer function edge by taking output divided by input, ok. So, you want to make sure that it happens only once.

So, in most of the simulations at least you make sure that the wave does not travel back, unless the study itself is aimed at figuring out what happens when the waves travel back explicitly, ok. So, this is known as absorbing boundary condition and it can be done in multiple dimensions that we will see about it later, ok. So, this is the first change, a couple of changes that we have made to our program.

Now, we will open our other program all right, which is not solving the wave equation, but actually the telegrapher's equation and then we will update it to a Maxwell solver, right.

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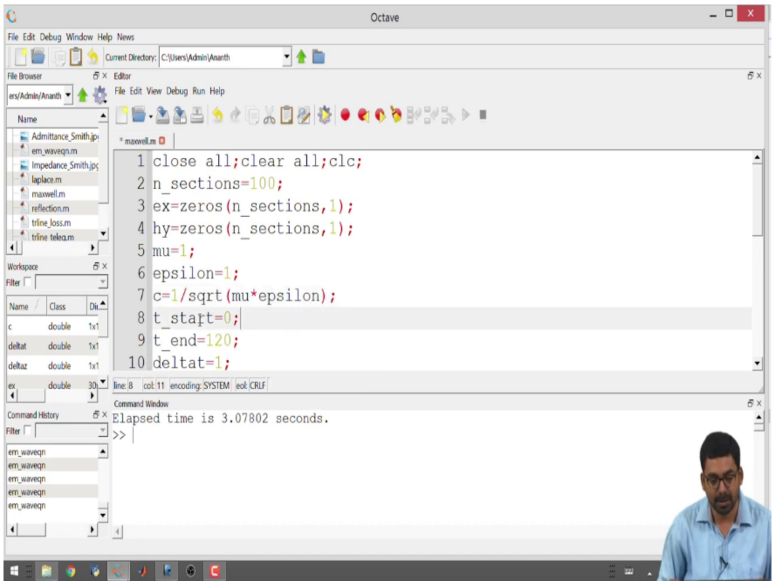

So, I will make a copy of this, ok. And I will change some variables consistent with the lecture. So, we had an electric field in the x direction. So, wherever I had V, I will change it to Ex and I had the magnetic field pointing the y direction, so I will change it to hy, instead of I, we made use of a ϵ a μ sorry and ϵ.

Immediately we will notice that we are plugging in μ as 1 and ϵ as 1, all right. Simply because μ_r and ϵ_r are enough to deal with what is happening. So, most of the time when running these simulations, we are just trying to understand the difference between doing this in vacuum and in a medium, all right. So, a μ_r and ϵ_r are irrelevant, because the velocity becomes 1, all the calculations become easy.

Secondly μ_r is a small value, $4\pi * 10^{-7}$ ok, similarly ϵ is $8.854 * 10^{-12}$, so it is a very tiny number. When you are using these numbers for doing multiplications, divisions extra, there is a possibility that the result will be approximated to zero by the computer, all right.

So, in order to avoid these approximation errors, it is better to do the simulations by considering only ϵ_r, μ_r ok and maintaining the velocity to be equal to 1. And then when you are finished with the simulation, you always try to figure out one velocity corresponds to say $3 * 10^8\ m/s$, but if I get something else, I will scale it accordingly, ok.

So, this kind of simulation is very popular right now, it is also being used in other open source softwares like MEEP right, MIT electromagnetic propagation software, they use normalized units where the velocity is made equal to 1 for vacuum, ok. So, I have

$$c = \frac{1}{\sqrt{\mu\epsilon}}$$

in this case it is equal to 1, ok.

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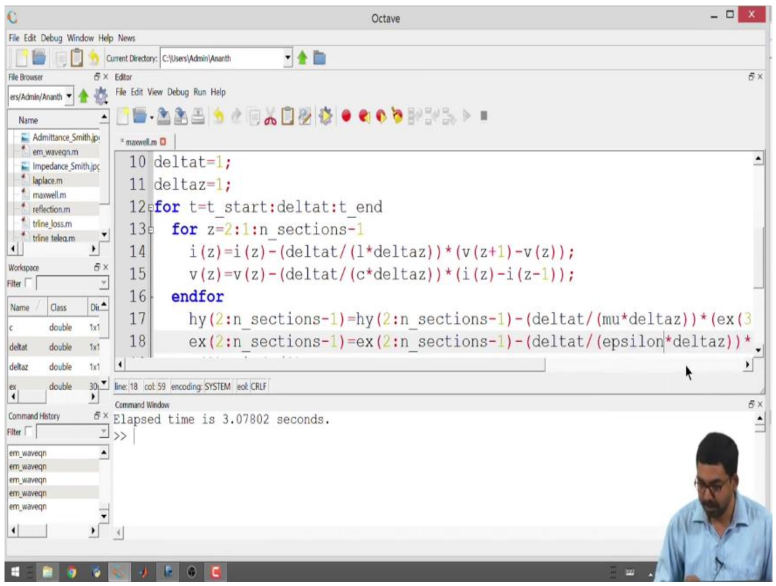

So, I am having two loops, first I am having a time loop, then I am having a space loop.

Since we are now doing something in an advanced format, we can replace the inside for loop with actually just vectorized updates, ok. So, we can go ahead and write that down, right. So, first I am having the update for I's. So, I am going to take this as h of, did I make hy, ok. So, it is equal to the hy of z. So, z in this case is 2 to n sections -1. So, I am just going to make it, I am going to substitute for h directly.

2 to n sections -1 is coming from the definition of the loop itself, so it is in this line, right.

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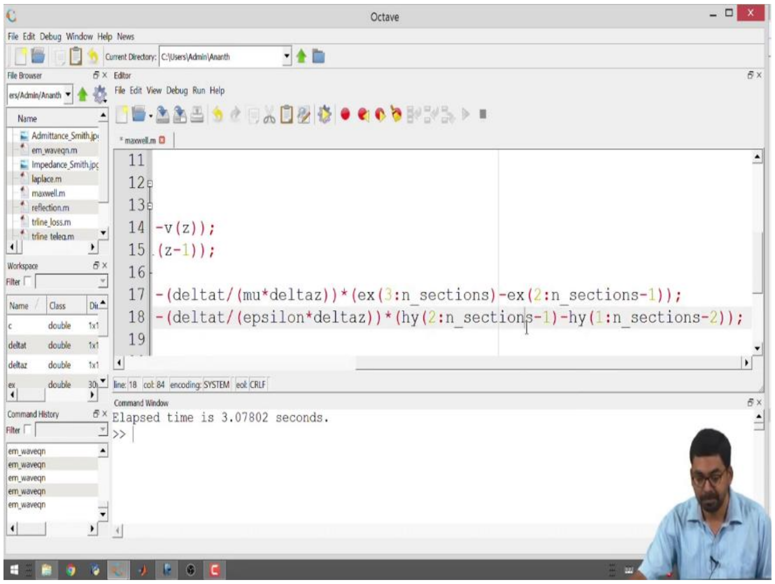

Minus delta t by, delta t by 1 times delta z one is the Curran factor that we did not discuss in this course maybe in a later course. V(z) plus 1 right, so that becomes Ex, z plus 1 becomes 3 colon n underscore sections and z is 2 colon, ok.

We are just replacing z, z plus 1, z -1 extra with the definitions from the for loop directly, ok. So, the goal is to just get rid of this for loop, ok. So, I will copy the second line and I will make similar modifications. So, instead of h y, I will have Ex, and instead of z, I will have 2 colons n sections -1 and l has to be change to mu, c is change to ϵ, i is being changed to h y, z is once again changed to sections -1, ok.

Student: There is one v.

What is that?

Student: There is a one v.

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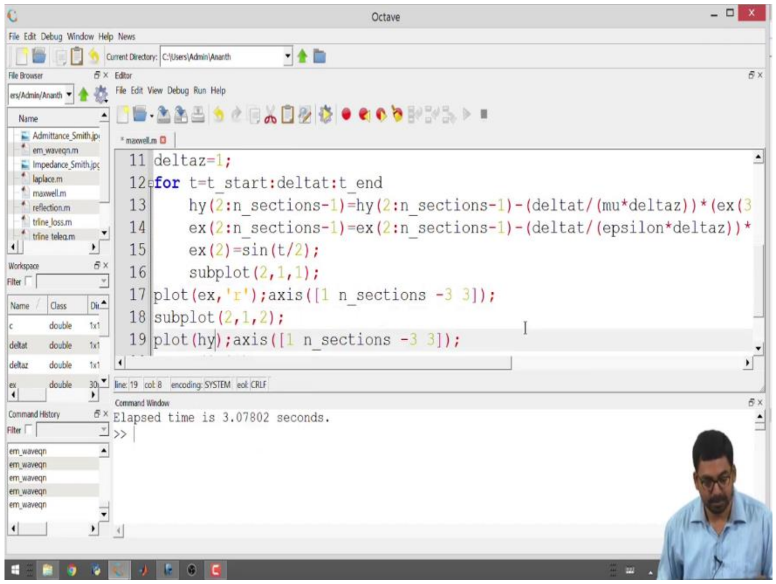

Oops ok, thank you. There are no other errors right, I can just run it, if there are errors I will run it and figure it out, it is not a big deal, all right. So, I am going to have a source Ex having sin(t/2) perfectly fine, I am just making the plot command to be Ex in red, hy in the default colour, ok.

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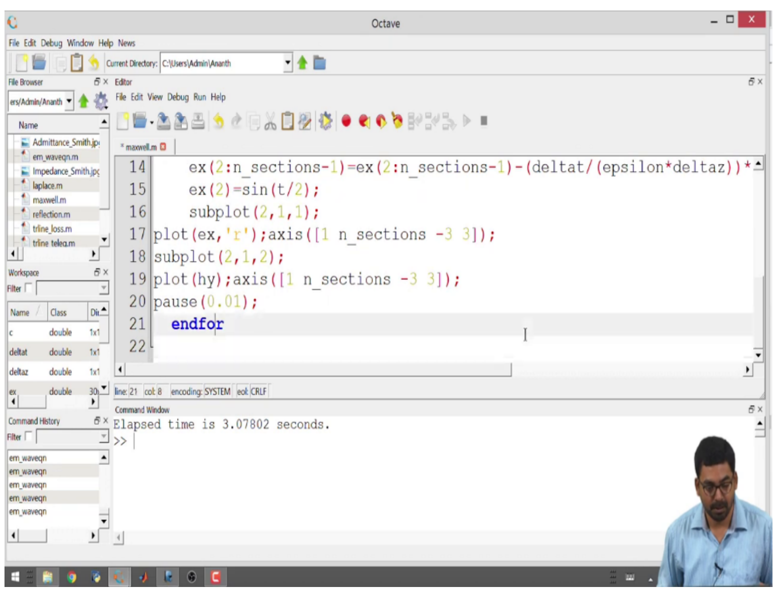

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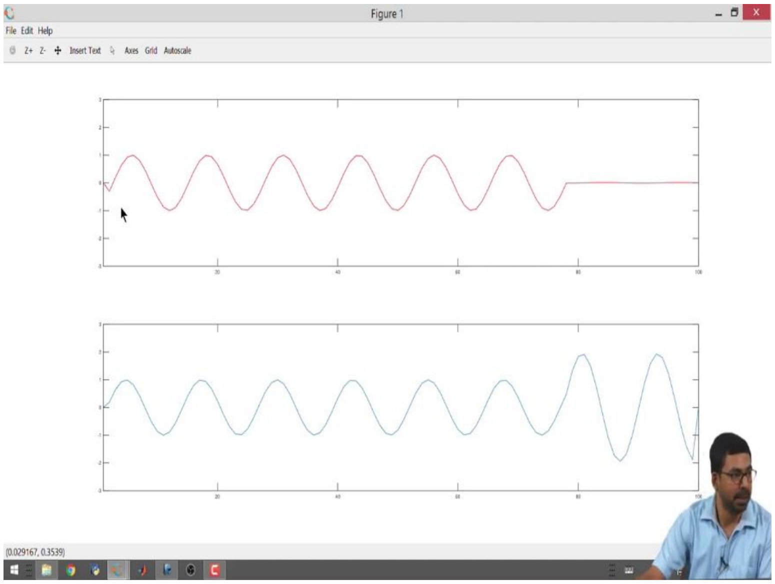

So, now we have written the base equation of our Maxwell solver, but we have not done it in the exact same way as what we have done before, we have made some upgrades.

The upgrade that we made is, we have graduated from the for loop inside to a vectorized step, this is one step at a time, all right. And here also one could write down the command for the right hand side edge all right, to have an absorbing boundary condition that would not actually send the wave back, ok. But I am going to leave that as an assignment, because now you know the command, now you know how to do it, all you need to do is play and figure it out.

With that there is only one more thing that is left. So, there is actually complete equivalence. So, in the previous lecture I showed that the quantities are nearly identical, just need to change the variables, now I showed that in the program also, as there is nothing new over there. As far as one dimensional wave propagation is concerned, it is exactly the equivalent, whether you have wires or no wires Maxwell's equation and the telegrapher's equations are identical, ok.

So, there should be no fear of looking at the vector form of Maxwell's equations at least for one dimensional propagation ok, so they are exactly equivalent. There are some quirks that we may have missed in the previous case, all right.

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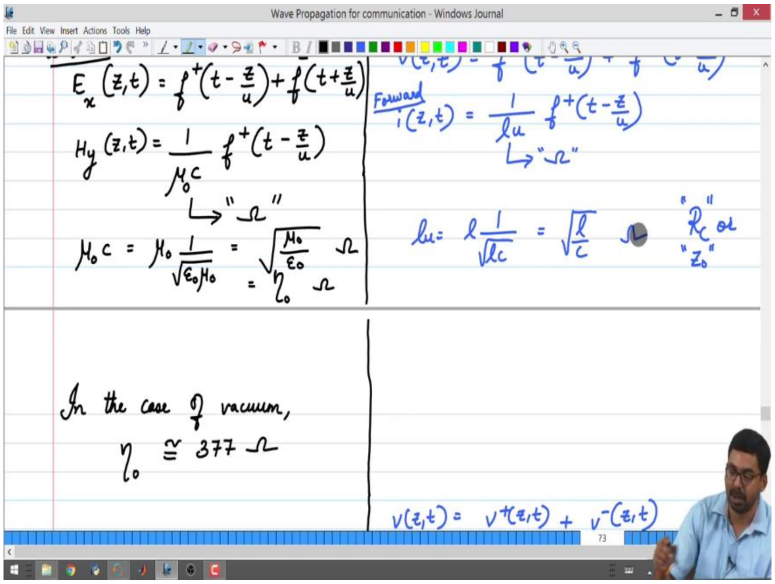

I will just fill that up ok, η_0 in the previous class was mentioned to be 377 ohms, this was calculated from μ_0c, all right.

So, it is

$$\mu_0 c = \frac{\mu_0}{\sqrt{\epsilon_0 \mu_o}} = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

And the equivalent in the transmission line was considered to be

$$lc = \frac{l}{\sqrt{lc}} = \sqrt{\frac{l}{c}}$$

square root of l by c. One of the things that I have noticed is, when we ask for the calculation of the impedance, the students immediately figure out that they had to take the forward voltage divided by the forward current, V^+/I^+ and then they will be able to figure out what is the value of the characteristic impedance, ok. But sometimes in the case of the wave all right, the Maxwell's equation can be a little tricky all right for the people to just imagine.

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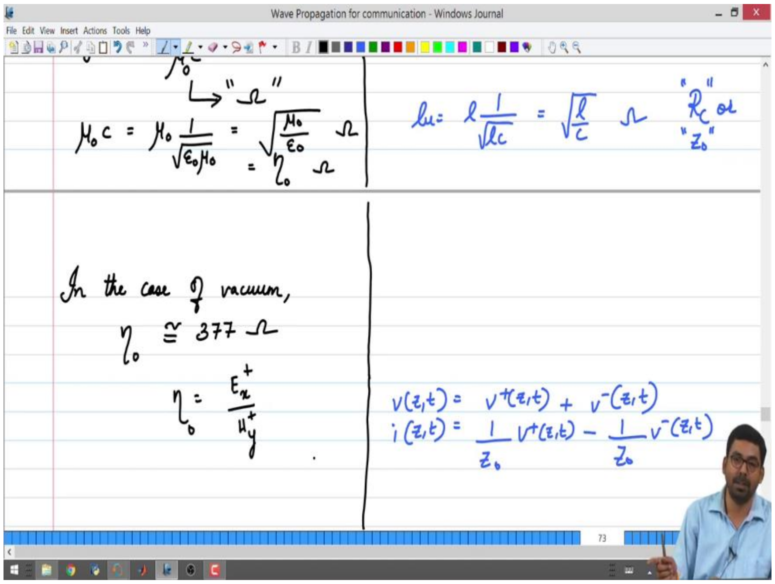

You just have to write this down as $\frac{E_x^+}{H_Y^+}$, ok. This is something that you will have to get used to all right, because people do not immediately associate the characteristic impedance with Ex divided by Hy ok, this is something that you have to get used to, so it is $\frac{E_x^+}{H_Y^+}$.

Now in this simulation all right, I mean in the notes we have seen that the characteristic impedance of vacuum is 377 ohms and I have also mentioned in yesterday's class that, if you have a voltage of one volt in your transmission line corresponding to the resistance, you will be getting some current. The case of vacuum and when you are talking about Maxwell's equations, we have one volt per meter, you will be getting 1/377 A/m.

Which means that, if I did start my simulations with assuming proper values of ϵ_o μ_0 ok, $8.854 * 10^{-12}$ and $4\pi * 10^{-7}$. Apart from the probability that I may end up with some zero instead of a tiny value, which would be an approximation error, there is also a good chance that suppose I start with the electric field of 1 V/m. The magnetic field will be 1/377, so it is a tiny value, all right.

And if I have tinier values of electric field, the magnetic field will lead to an approximation error of zero. So, this is another reason why we try to avoid making use of actual values in vacuum, you just deal with relative permittivity and relative permeability. And at the end you make an analysis as to vacuum it is this, with this material the velocity is this, so the characteristic impedance of this material has to be so much, this is the kind of analysis that you make with comparison, ok.

The other thing that we can look at which we have not looked at in the case of transmission lines, is the effect of ϵ_r and μ_r and some conclusions that we can draw from them, ok.

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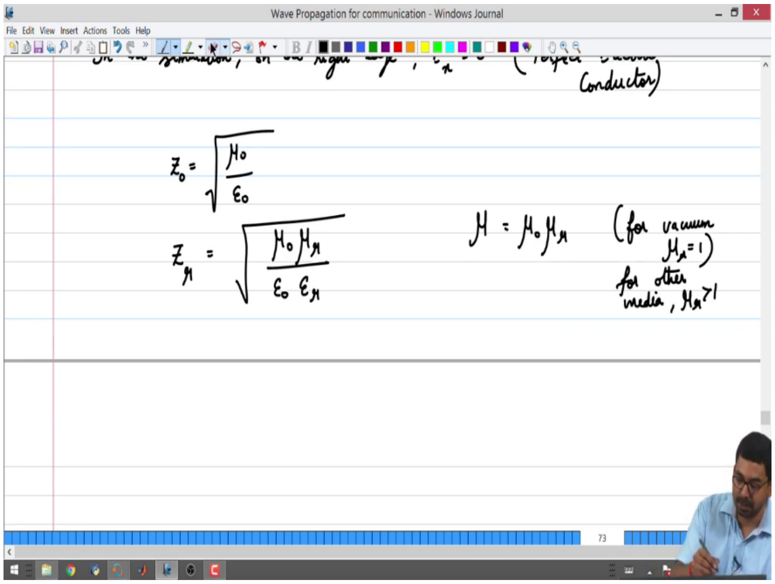

Let us look at the characteristic impedance for the wave, electromagnetic wave, right.

So, I am having

$$z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

t for vacuum. For some medium all right, I will just replace the Z with say Zm to indicate that I am having a medium all right, more common representation is Zr you can use that ok, is going to be

$$z_0 = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}}$$

ok.

Because the μ of any material is going to be $\mu_o \mu_r$, assuming isotropic frequency independent medium here right, it is just μ_0 multiplied by μ_r, right. So, this is your vacuum permeability and μ_r is some number, ok. Usually it is a positive number ok, μ_r is a positive number ok and also μ_r has to be greater than 1 ok, for vacuum μ_r is equal to 1, μ_r is equal to 1, for other media there are some constraints, ok ok.

μ_r greater than 1 is not a you know criterion that is written on the stone, but all you can have is 1 or higher, all right. So, I can also make this a little bit more accurate and say that, could be greater than equal to 1, usually the μ_r tells you whether a material is magnetic or not, ok.

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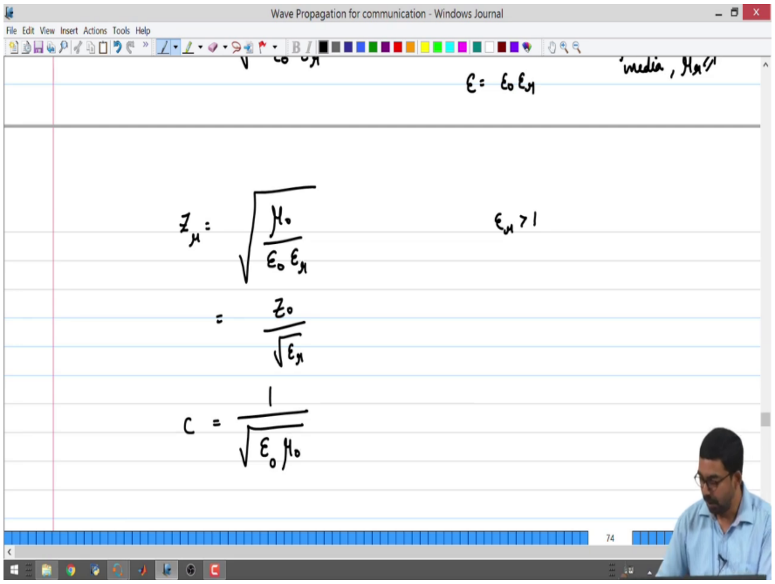

And in the case of a the relative permittivity is ϵ not ϵ_r. Once again for the vacuum ϵ_r is 1, for other media ϵ_r is greater than equal to 1, all right. And we are assuming lossless case here, we have not included any resistance conductance extra, we are dealing with lossless case, so these are the parameters. But most of the time ok, what happens is, not many of the materials are magnetic, not many of the materials are magnetic, ok.

So, what happens is, in majority of the cases you will find that μ_r is actually equal to 1, it just has vacuum permeability, the material also has only vacuum permeability. It is rare that the material has permeability higher than 1, because it is magnetic, but there are materials, but the number of materials is far lower than dielectric materials that you can observe in nature.

So, what happens is, majority of the times μ_r is 1 for optical materials all right or for high frequency, and ϵ_r changes from medium to medium, but it can only be again greater than equal to 1. Which means that, you can say that for non magnetic materials which are very common ok,

$$z_r = \sqrt{\frac{\mu_o}{\epsilon_0 \epsilon_r}}$$

we are saying that ϵ_r say is greater than 1, ok.

What ends up happening is that, for the material medium, you are So, the characteristic impedance of vacuum

$$z_r = \frac{z_o}{\sqrt{\epsilon_r}}$$

This also means that, since your $\epsilon_r$is going to be greater than 1, the square root is also going to be greater than 1, what ends up happening is, the characteristic impedance of a medium mostly will be less than the characteristic impedance of air or vacuum, ok.

So, vacuum represents a special case, where the characteristic impedance is very high, for dielectric materials that are not magnetic, which is the most common case the characteristic impedance will be lower, ok, this is one thing. The second thing is with respect to the velocity, ok. So, the velocity what we have seen for free space is

$$c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$$

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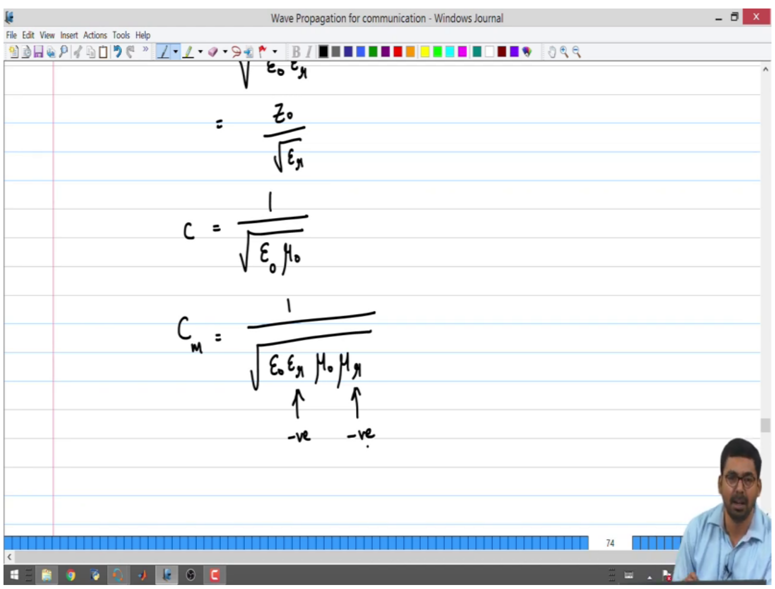

For material media C in some medium right,

$$c = \frac{1}{\sqrt{\epsilon_0\epsilon_r\mu_0\mu_r}}$$

this is greater than equal to 1, this is greater than equal to 1, you can also keep it equal to one for many cases, which means that, in any material medium, the velocity is going to be smaller than the velocity in vacuum. So, vacuum is a special case, its characteristic impedance is very high, velocity is very high, for a majority of the other materials, characteristic impedance is going to be lower, velocity is also going to be lower.

So, always you should expect some kind of reflection to happen between vacuum and some other medium, ok. Always you should expect additional delays while not using vacuum or air, ok. So, if you have any material, you will always get lower speed, we also saw this with the transmission line, there was something known as velocity factor in the transmission line specification. It was written that the speed is 66 percent of the speed of light, it was not written 200 percent ok.

So, the maximum velocity that you will always have in these cases is the velocity of light in vacuum. So, there is always a finite velocity upper bound and any other medium will always have lower velocity, all right. And most likely they will also be having lower values of characteristic impedance if they are non-magnetic. If they are magnetic then μ_r will start to play a role. But it is questionable whether μ_r role is going to be higher than ϵ_r and all that right, those are all special cases, ok.

The other thing that one will notice over here is that,

$$\frac{1}{\sqrt{\epsilon_0 \epsilon_r \mu_0 \mu_r}}$$

The reason I strictly put ϵ_r and μ_r should be greater than 1 all right, is simply because, a if you did discover a material whose ϵ_r and μ_r are going to be less than 1, then you will have propagation at speeds higher than what can happen in vacuum, all right. So, that leads to some mysterious questions. Ok, those are known as super luminal propagations. It is faster than the speed of light right? Those are some research topics that people actively pursue or at least theoretically investigate to see what can happen.

The other thing that people also do is, they take this equation and manipulate it in slightly clever ways. And one of the clever things that people do is, instead of ϵ_r being positive, you make it a negative number, you make μ also a negative number, the product of the two becomes positive. So, these things are all done by people in the current research topics, ok.

So, they try to see if they are able to create materials which have, what does ϵ_r negative mean become a big physics question, what does μ_r a negative mean will become a big physics question. But this much is clear as of now, if I write down the velocity in this format, there are two negative numbers the denominator, they should perfectly give rise to finite positive velocities, so the wave should travel from one side to another side, right. So, these are all some active research topics.

So, as I am going forward, I just wanted to give you these pointers where people are currently working on.

Is it possible to do this, at least theoretically there are some directions, experimentally they are far behind what you know the theory predicts, but this is the simplest theory, we have two negative numbers, multiply you will get positive and then you will have finite velocity. But for the sake of you know, preventing confusion I will just remove all these things right, so that you do not make these things a in the course ok, we will not be seeing all these advance cases in the course, ok.

So, I will stop here, these are some things that I wanted to say, because in the transmission line we did not have the time to talk about negative capacitance, negative inductance and all that. We did see negative resistance and a conductance being equivalent to making gain, we did not. So, I thought I would just mention this at this time, ok.