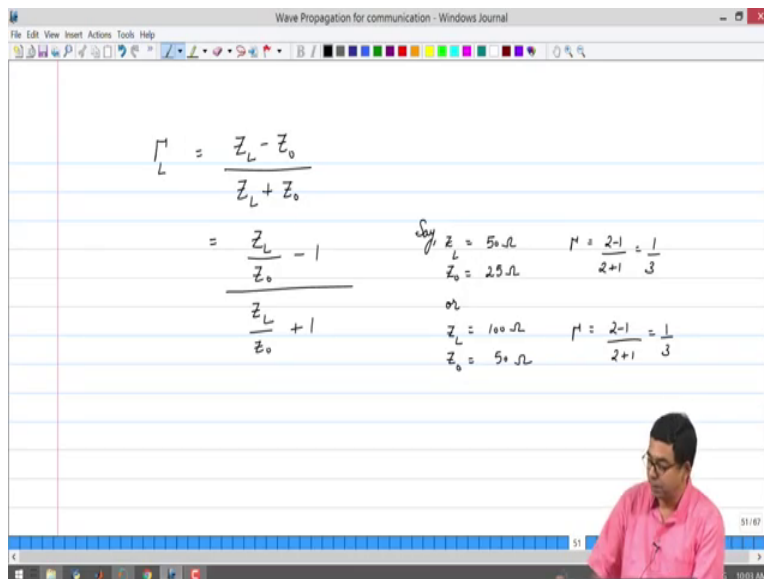


**Transmission lines and electromagnetic waves**  
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**Department of Electrical Engineering**  
**Indian Institute of Technology, Madras**

**Lecture – 11**  
**Graphical Representation of Reflection Coefficient**

We will get started. We are going to be seeing more impedance in transmission lines that is the place where we had last stopped right.

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Just to refresh our memory the load reflection coefficient is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Now, we already know that the reflection coefficient is going to be a complex number. It is going to have a real part and an imaginary part. Alternately you can say that it has some magnitude and a phase. We have seen this from the simulations already.

Now, we are going to touch some aspects on this particular reflection coefficient over here and try to understand the relationship between gamma and Z more closely ok. Now, since this is a ratio of  $\frac{Z_L - Z_0}{Z_L + Z_0}$ .

The first question that we have to put I mean that we have to put to rest is whether the absolute values matter ok. Now, one could always divide the numerator and the denominator by  $Z_0$  ok. So, that would give me

$$\Gamma_L = \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1}$$

ok.

So, it seems like the reflection coefficient depends on the ratio of the load impedance to the characteristic impedance and it does not depend particularly on the absolute values, it is the ratio that matters ok. So, an example could be your load impedance ok say

$$Z_L = 50 \Omega, Z_0 = 25 \Omega$$
$$\frac{Z_L}{Z_0} = 2, \quad \Gamma_L = \frac{2 - 1}{2 + 1} = \frac{1}{3}$$

If you take another absolute value, say

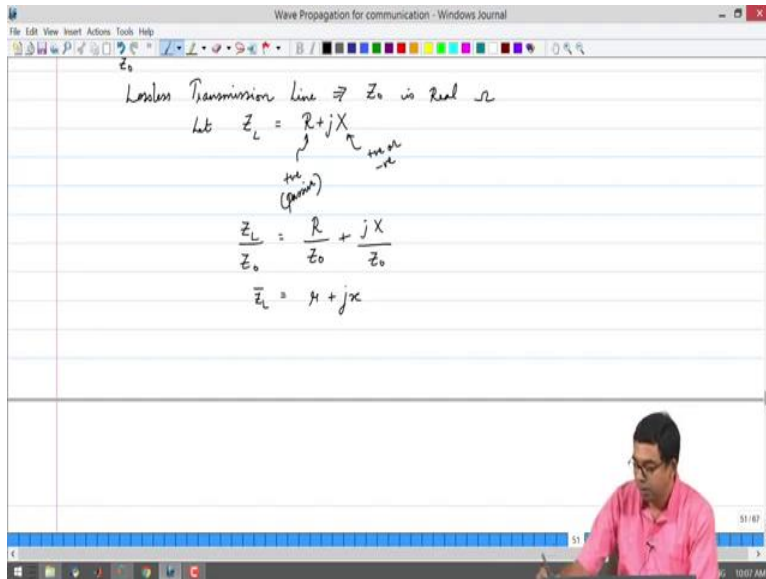
$$Z_L = 100 \Omega, Z_0 = 50 \Omega$$

I have taken the initial values and doubled the value of load impedance and the characteristic impedance,

$$\Gamma_L = \frac{2 - 1}{2 + 1} = \frac{1}{3}$$

So, clear that the absolute values of the impedance does not matter. What matters is the load impedance to the characteristic impedance ratio ok.

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Now, this ratio we refer to as normalized impedance is ok.

Student: (Refer Time: 03:14).

Ok we can say that  $Z_L$  by  $Z_0$  is the normalized impedance. We will begin with some simple cases to understand what this relationship actually means and the simple case that we are going to be considering is that the transmission line is lossless ok. When the transmission line is lossless ok.

Student: (Refer Time: 03:39).

Ok the lossless transmission line ok ok will imply that  $Z_0$  is real ok it is going to be purely resistive. We are going to begin with this simple case and try to understand the relationship between the reflection coefficient and your load impedance ok. Now, the way we are going to represent the normalized impedance is that suppose here  $Z_L$  is not real, it has a real and an imaginary part ok. So, the ratio  $Z_L$  to  $Z_0$  will have both real and imaginary parts ok. So, let us say your

$$Z_L = R + jX$$

ok, where  $R$  is the resistive part of your impedance,  $X$  is going to be a reactive part of the impedance ok.

Now, there are two things that we have to make clear before going further right. So far we have drawn the circuit equivalent only for passive circuits and we already know that for passive circuits,  $R$  cannot be negative all right we have in our simulations in the past plugged  $R$  to be negative all right, but that is just to prove a point that gain can occur in your medium; however,

in passive circuits R is going to be only positive ok. So, let us make that clear over here right ok. I wrote the same thing, the same passive ok.

However, the X could be positive or negative depending upon whether you have a capacitive reactance or an inductive reactance ok. So, for that inductive reactance it is going to be positive and for the capacitive reactance it is going to be negative ok. So, these are the cases, which are possible for  $Z_L$ .

Now, once you normalize with this  $Z_0$  and  $Z_0$  is real, it means that we are going to be having

$$\frac{Z_L}{Z_0} = \frac{R}{Z_0} + \frac{jX}{Z_0}$$

$$\underline{\underline{z_L}} = r + jx$$

Now, this is your normalized load impedance ok. So, correspondingly here I will just make it  $Z_L$  because a capital  $Z_L$  and small  $Z_L$  look identical I will just put a bar on the top to indicate that it is actually normalized to with respect to the characteristic impedance of the transmission line ok. So, it is  $r + jx$  ok.

Now, you can go back to the expression for the reflection coefficient and substitute for

$$\frac{Z_L - Z_0}{Z_L + Z_0}$$

So, you can substitute the numerator and denominator. So, we can go ahead with  $\Gamma_L$  ok.

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The screenshot shows a Windows Journal window with the following content:

Wave Propagation for communication - Windows Journal

File Edit View Insert Actions Tools Help

$$\Gamma_L = \frac{\bar{z}_L - 1}{\bar{z}_L + 1} = \frac{r + jx - 1}{r + jx + 1} = \frac{r-1 + jx}{r+1 + jx}$$

$$\bar{z}_L = r + jx$$

$a + jb$

$\text{Imag}(\Gamma_L) = b$

$\text{Real}(\Gamma_L) = a$

52/61

10:14 AM

So, we have just substituted the normalized load impedance in the expression for the reflection coefficient ok. This means that  $\Gamma$  itself will be a complex number ok it is going to be having some real part and some imaginary part. We have assume the lossless transmission line case so,  $Z_0$  is purely real, but we are assuming  $Z_L$  to be anything could be anything which means that the most general case  $\Gamma$  is going to be having a real part and an imaginary part could write this as  $a + jb$  ok.

What we are interested in now knowing is how  $\underline{z}_L$  influences  $\Gamma_L$  ok. Specifically the points that we would like to see now are if I substitute for a value of  $\underline{z}_L$  I know that there is going to be a corresponding unique value of  $\Gamma_L$  alright and there is a good chance that I want to keep the resistance fixed and change the reactance of  $Z_L$  just to see how  $\Gamma_L$  changes or I could keep the reactance fixed, vary the resistances to see how  $\Gamma_L$  changes. These are the two things that we would like to understand in this particular lecture ok.

So, I am going to have

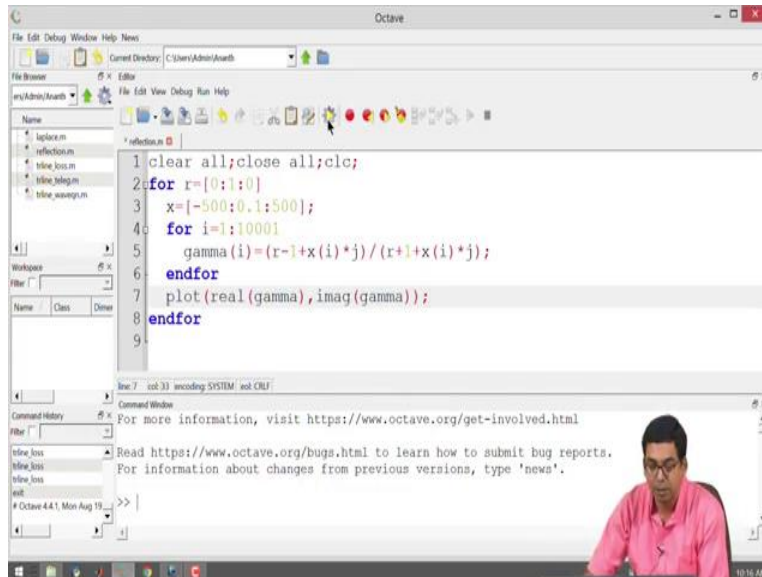
$$\underline{z}_L = r + jx$$

and I would like to understand what happens for fixed values of  $r$  with varying reactances and fixed values of  $x$  with varying resistances. These all I want to find out are ok. And, the method in which I am going to be doing is I am going to be fixing the value of  $r$  and  $x$  and then I am going to be calculating the value of  $\Gamma_L$  this is going to be having a real and an imaginary part and I am just going to draw a graph ok.

The graph is going to be like this. It is going to have a vertical and a horizontal axis. The horizontal axis is going to be the real part of the reflection coefficient at the load in this case alright. It could be done for any location in the transmission line, but we are keeping it simple for the beginning. We just calculate the reflection coefficient at the load right. So, I am just saying that real part of  $\Gamma_L$  and in our lecture this is going to be denoted by  $a$ . So, you could also say that this is equal to  $a$  and the vertical axis is going to be corresponding to  $jb$  or the imaginary part of  $\Gamma_L$ .

And, I am going to take fixed values of  $r$  and  $x$  and I am going to calculate what is the value of  $\Gamma_L$  and I am going to make a plot, that is all I want to do and any other inferences I will draw once I have completed the plot ok. This is the exercise. So, I am going to fire Octave ok I am going to do this using Octave ok.

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```
1 clear all;close all;clc;
2 for r=[0;:0]
3   x=[-500:0.1:500];
4   for i=1:10001
5     gamma(i)=(r-j*x(i)*j)/(r+j*x(i)*j);
6   endfor
7   plot(real(gamma),imag(gamma));
8 endfor
9
```

Now, I am going to start with a simple program to do that. So, I am going to. So, here what I am just saying is that  $r$  is equal to  $0 : 1 : 0$ ; that means, I have fixed  $R$  to be 0 ok later on. I would like to add a series of  $r$ . So, that is why I am writing the program using a For loop. Normally one could fix the value of  $r$  to a 0 to be constant ok I want to understand the effect of  $x$  ok. So, I have another loop in between alright which will compute the value of gamma.

Let's say that I want to have an insanely wide range of  $x$ . So, I am going to take going from the normalize, these are the normalized values, mind you ok. These are divided by  $Z_0$  ok. So, I am going to go from - 500 ok just pick a number; you could pick 50, you could put you could pick a 5000. I will show the consequences of using all of them in the second. So, I am going to have the normalized reactance going from - 500 to 500, but my  $r$  value is going to be fixed at 0 ok and all I want to do now is ok.

The reason I have picked a variable  $i$  equal to 1 to 10001 is because I have - 500 to 500 in steps of point 1, alright that corresponds to 10001 points alright and for every reactance value I would like to calculate  $\Gamma_L$  that is all ok. So, I am just going to use the formula we had. I am going to build a gamma vector that is equal to or let me explain this in a second alright.

So, the formula that we have is

$$\Gamma_L = \frac{Z_L - 1}{Z_L + 1}$$

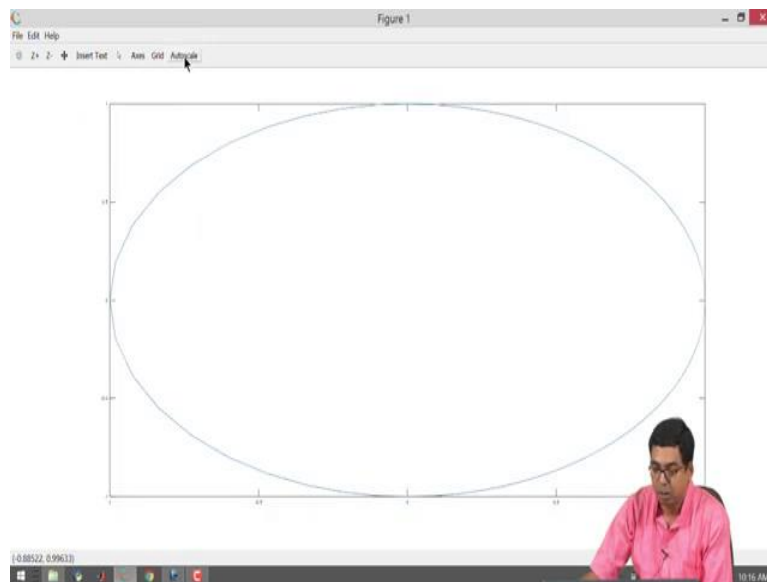
You could also substitute for  $z_L$  you could say that this is

$$\Gamma_L = \frac{r + jx - 1}{r + jx + 1}$$

I will go back to my Octave program ok and I will complete this formula for calculating the reflection I mean the value of a voltage reflection coefficient gamma. So, it will be  $r + 1 +$ . So, the reason why we are putting  $x(i)*j$  is Octave accepts this for the imaginary part. You have to put the variable name first multiplied with j, j square root of - 1 here, i is the index of x for which you are going to calculate the value of gamma ok. This is just a detail.

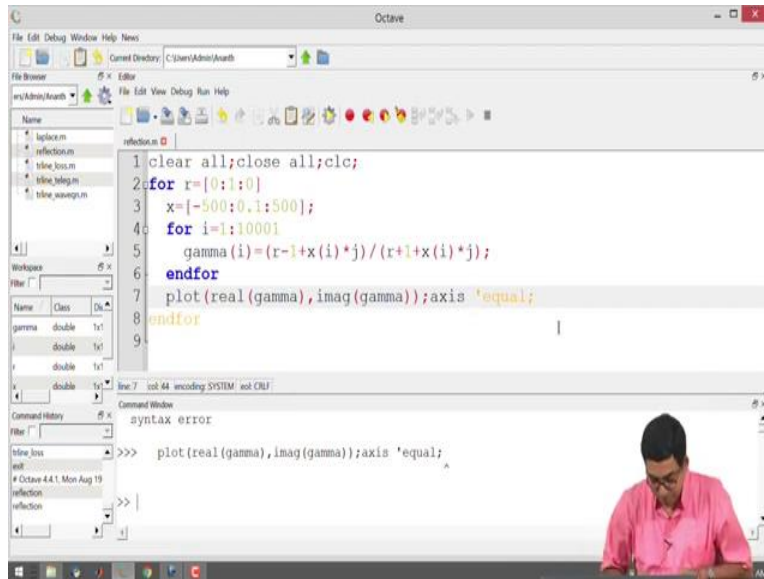
So, now I have my resistance fixed and I am calculating a value of gamma for a variety of reactance and I want to understand the effect of this on the gamma and I want to make a plot. So, let me go ahead and make a plot ok. I simply want to plot the real part of the gamma with the imaginary part of the gamma right. As I have described this is going to be otherwise known as complex gamma plane ok the complex gamma plane I am just using the real part, I am going to be taking it as the x axis. Imaginary part is going to be the y axis and I am going to place a point corresponding to the calculation ok and I am just going to run this right, ok.

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Now looks oval.

(Refer Slide Time: 15:43)

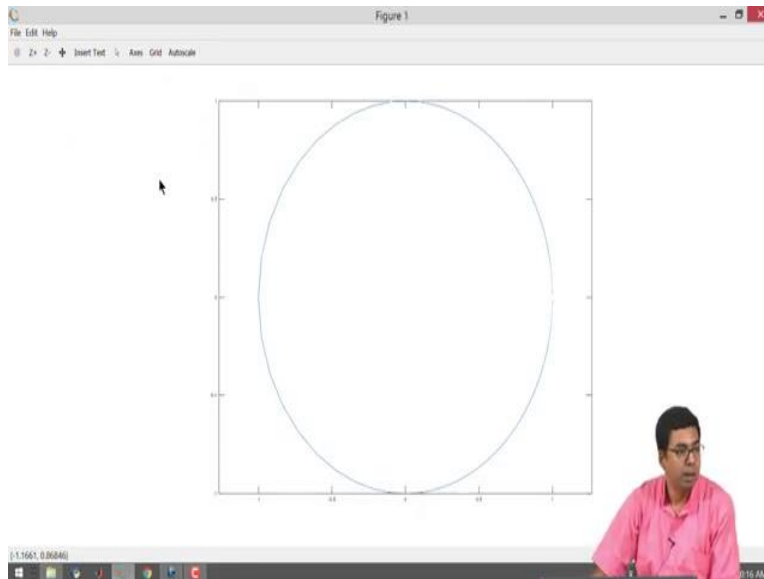


```
1 clear all;close all;clc;
2 for r=[0:1:0]
3   x=[-500:0.1:500];
4   for i=1:10001
5     gamma(i)=(r-1+x(i)*j)/(r+1+x(i)*j);
6   endfor
7   plot(real(gamma),imag(gamma));axis 'equal;
8 endfor
9
```

Command Window  
syntax error  
>>> plot(real(gamma),imag(gamma));axis 'equal;  
>>>

So, I am just going to use a command to just make this a little bit more easy to interpret ok.

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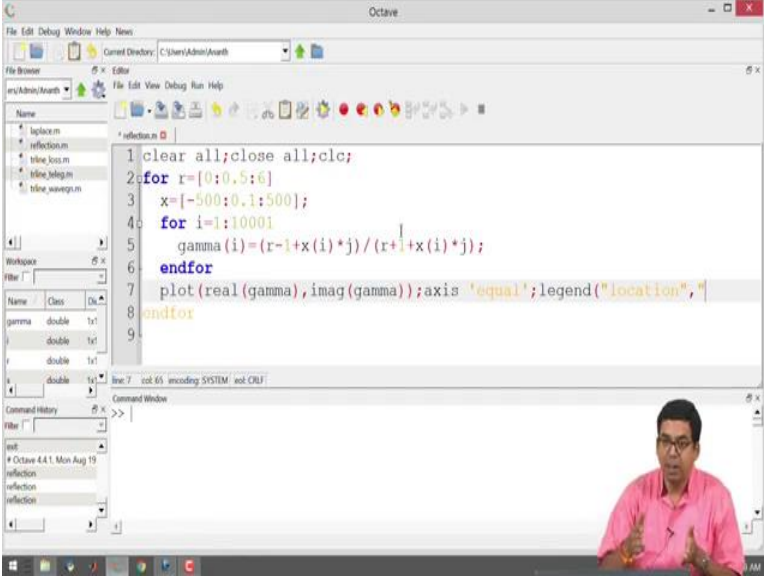
Now, the locus of all the points is a circle ok. What this means is the x axis corresponds to the real part of gamma, y axis corresponds to the imaginary part of gamma and keeping r fixed at 0. I am taking x going from - 500 to + 500 and I am trying to plot what the value of gamma would be and it turns out that it sweeps a circle ok. So, what this means is that the center of the circle



here that we observe is (0, 0) alright and on the right hand side I notice that it is having a radius of 1 alright. So, it is a unit circle and that is it alright.

Now, I would like to draw more of these and try to understand what is exactly going on ok. So, for now we know that it is going to be a circle. Now, this circle is known as a constant resistance circle ok because you have fixed  $r$  to 0, but you have just varied  $x$  ok.

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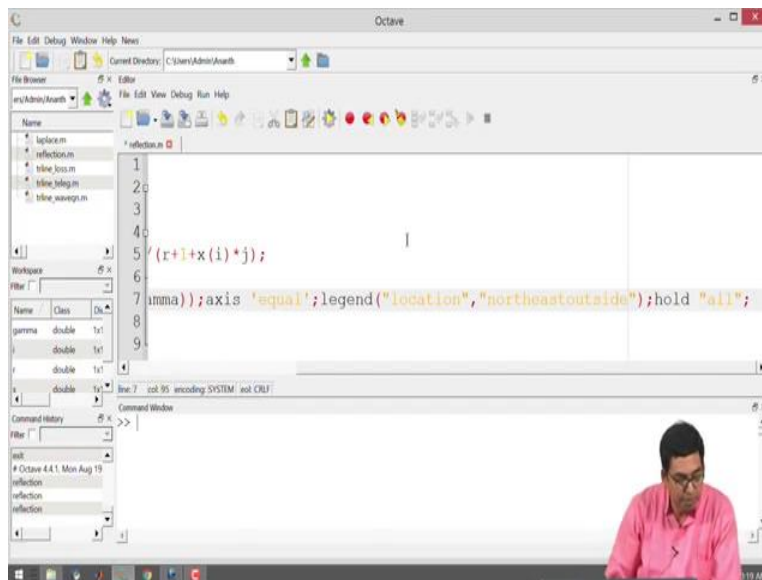


```
1 clear all;close all;clc;
2 for r=[0:0.5:6]
3     x=[-500:0.1:500];
4     for i=1:10001
5         gamma(i)=(r-i*x(i)*j)/(r+i*x(i)*j);
6     endfor
7     plot(real(gamma),imag(gamma));axis 'equal';legend("location", "
8 endfor
9
```

Now, I am going to go ahead and make  $r$  go from 0 to 0.5 in steps of main 0 to 6 in steps of 0.5 ok. And, I want to draw the plot ok and I want to figure out what is going on alright. I will just modify my program a little bit to make it a cleaner graph at the end because currently it will draw with the same color alright. I want to draw it with different colors to analyze what is going on. On top of that, I would like to have a legend to tell me which color corresponds to what value of resistance alright.

So, I am just going to add a couple of commands over here. I would like to have a legend ok and I would like to have the legend outside the graph ok. So, I am specifying a location. So, typically, I would like to have the legend on the north east side of the graph and I would like it to be outside of the graph.

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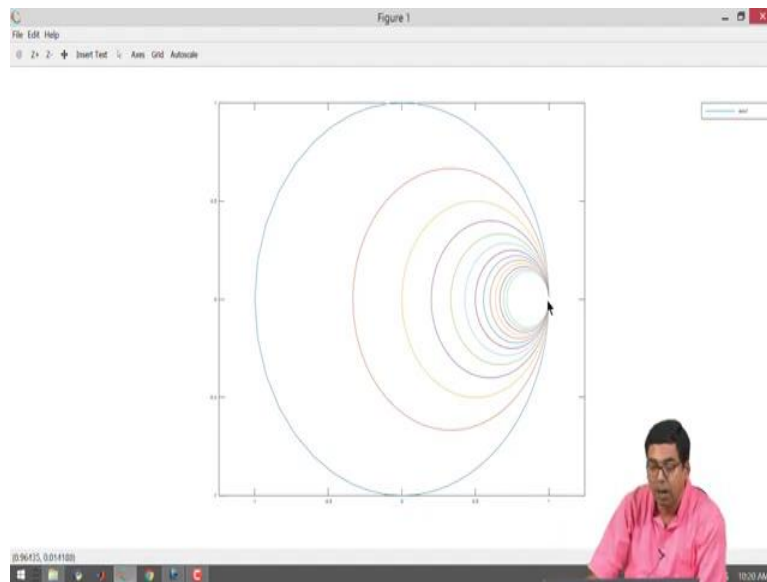


```
1  
2  
3  
4  
5 / (r+1+x(i)*j);  
6  
7 gamma); axis 'equal'; legend("location", "northeastoutside"); hold "on";  
8  
9  
Name Class Ds  
gamma double 1x1  
double double 1x1  
double double 1x1  
double double 1x1  
Command History  
# Octave 4.4.1, Mon Aug 19  
reflection  
reflection  
reflection
```

So, you know there are a lot of parameters, but you will get used to it. So, the legend I am going to place adds a location which is on the north east side of the graph that is top right and I do not want the legend to come inside the plot at all. I wanted to be completely outside. So, I am just saying north east outside just syntax in Octave. And, I am using the command hold all; it means that when I plot is drawn it will retain that plot and then it will draw on top of it.

For example, if I draw for r equal to 0 when it draws for r equal to 1, I do not want it to erase the previous plot. I want it to hold it in place and draw on top of that ok. So, hold all is just saying that hold all the parameters of the previous plot as it is and then redraw the new plot on top of it ok. So, these are the two commands that I am adding to it.

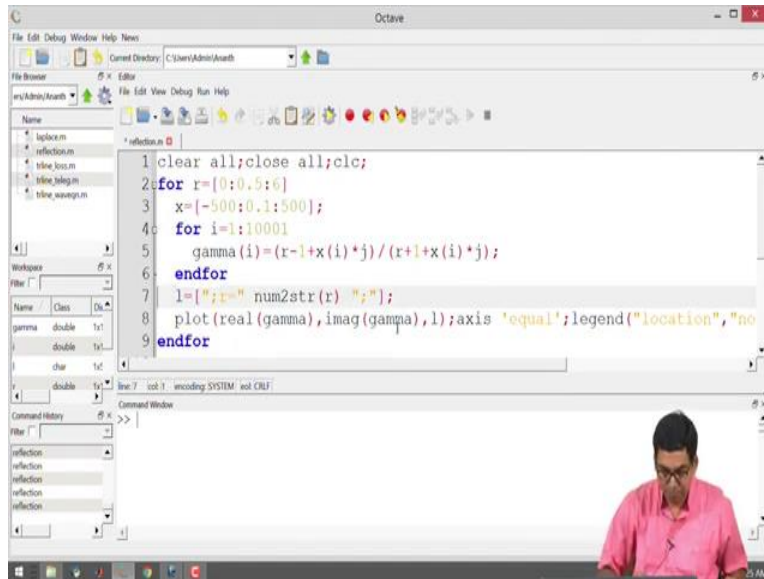
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So, I will run it now alright. Now, I am having a series of circles. The only thing is my legend is not showing up correctly. I will fix that in a second. However, I know for a fact that  $r$  equal to 0 was a unit circle in the gamma plane alright. So, I know for a fact that this is  $r$  equal to 0 and I call to tell you that as  $r$  increases it keeps reducing the radius of the circle and it also moves the center of the circle towards the right ok.

So, if you keep increasing your  $r$  all the way to infinity your circle will reduce to a point on the right extreme alright. So,  $r$  equal to infinity will reduce to a point because the center will keep moving to the right side and the radius will shrink abnormally low and it will become a point ok. So,  $r$  equal to infinity is going to have a point on the right side.

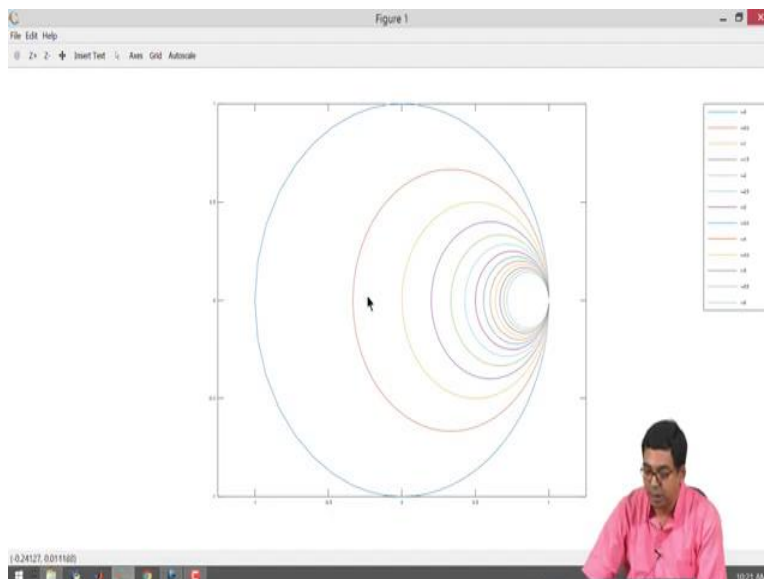
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```
1 clear all;close all;clc;
2 for r=[0:0.5:6]
3     x=[-500:0.1:500];
4     for i=1:10001
5         gamma(i)=(r-1+x(i)*j)/(r+1+x(i)*j);
6     endfor
7     l=[";r=" num2str(r) ";"];
8     plot(real(gamma),imag(gamma),l);axis 'equal';legend("location","no
9 endfor
```

So, just to be safer I will just correct this plot legend ok. So, I will just include a string, which I will create for drawing the legend. So, I am just saying r equal to in text ok. I need a legend string. This is Octave syntax for a legend string. I just want to have r equal to for each and every time it draws right, everything else can be understood later ok ok.

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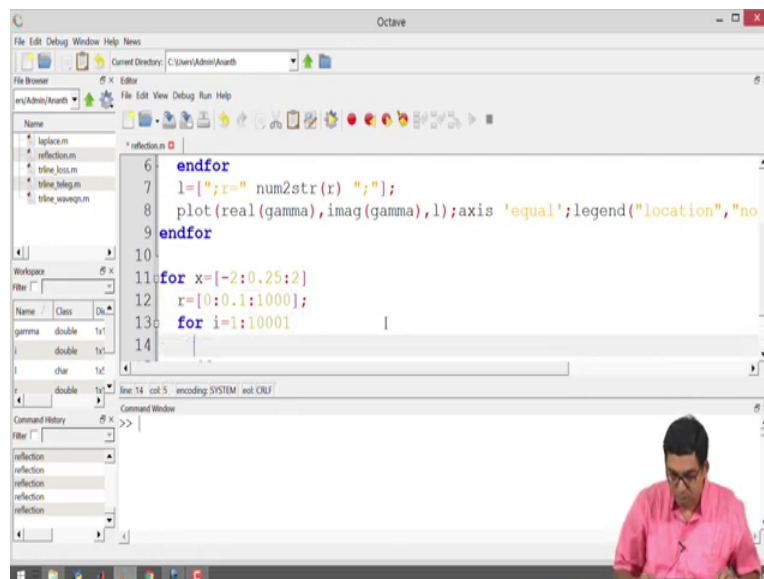


So, now it is abundantly clear, for increasing values of  $r$  the circle is reducing in radius ok and it is the center is moving towards the right and that gives me an indication that at  $r$  equal to infinity it is going to be a point on the right side ok. I guess now you know where we are heading to ok. We are trying to see if we can construct a Smith chart ok and we are going to construct it on our own.

Now, the relationship between  $\Gamma_L$  and the resistance should be clear ok. So, if you keep your resistance fixed no matter what value of  $x$  you take alright;  $\gamma$  is going to sweep a circle, so it is going to have a fixed value of the reflection coefficient. The magnitude is going to be fixed. The only thing that is going to be changing is the phase alright, that means, for a given value of  $r$  alright if you change  $x$  the value of the reflection coefficient the magnitude is going to change. But I mean magnitude is going to be fixed, but only the phase is going to be changing because it sweeps a circle in the complex plane ok.

Now, I would like to do a similar thing for the imaginary part. So, I am going to go back to the program right.

(Refer Slide Time: 22:42)



```
6   endfor
7   l=[";r=" num2str(r) ""];
8   plot(real(gamma), imag(gamma), l); axis 'equal'; legend("location", "no
9   endfor
10
11  for x=[-2:0.25:2]
12    r=[0:0.1:1000];
13    for i=1:10001
14      |
```

And, I am going to keep this a little clean. So, I am going to write another for loop going to say for  $x$  going from say  $-2, 0.25$  to  $2$  ok. So, I want to vary  $x$  from  $-2$  to  $2$ . I am doing this because the resistance we were keeping from  $0$  to a positive number because we are considering here only passive circuits is ok, but I know that  $x$  can go from negative to positive. So, I am just taking a

value - 2 to + 2 and I am going to go in steps of 0.25 ok and I want to build a vector for the resistor 0 to 1000 in steps of 0.1 all the way up to thousand.

The program structure for this part will be similar to this part, just the values are slightly changed. The values are changed because you will notice that if you change the values something will become not clear. I will explain that in a second once we complete the program. So, now, I will have it equal to 1: So, this is about 1000, 10000 and ok right.

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```

9   endfor
10
11  for x=[-2:0.25:2]
12    r=[0:0.1:1000];
13    for i=1:1000
14      gamma(i)=(r(i)-1+x*j)/(r(i)+1+x*j);
15    endfor
16  endfor
17

```

So, r going from 0 to 1000 and my i value is going from 1 to 1000 and I will just calculate the value of gamma ok

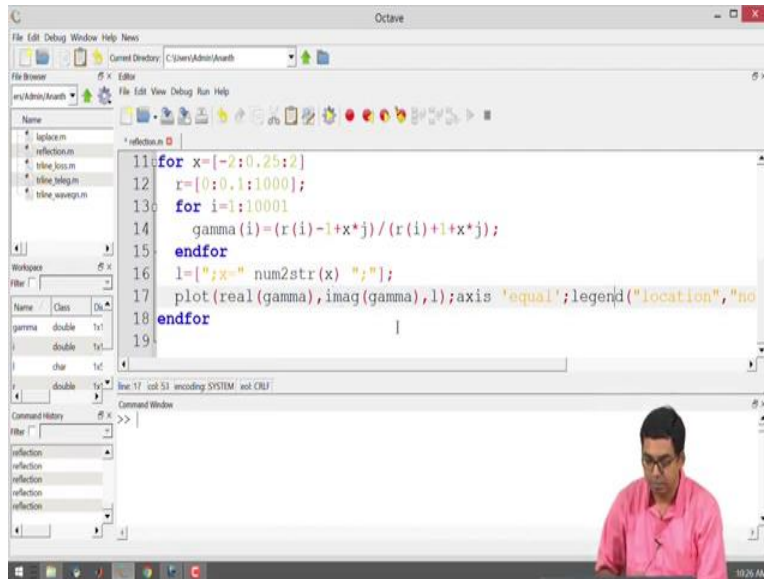
$$\gamma(i) = (r(i) - 1 + x * j) / (r(i) + 1 + x * j)$$

The formula is similar to the case that we had written in the previous loop just that because I am using variable reactance in the first loop and variable resistance in the second loop the subscript or the array indices are coming in different positions, but the form is same it is

$$\frac{r - 1 + jx}{r + 1 + jx}$$

And I want to be able to draw this. So, I am going to be copying the plot commands from here.

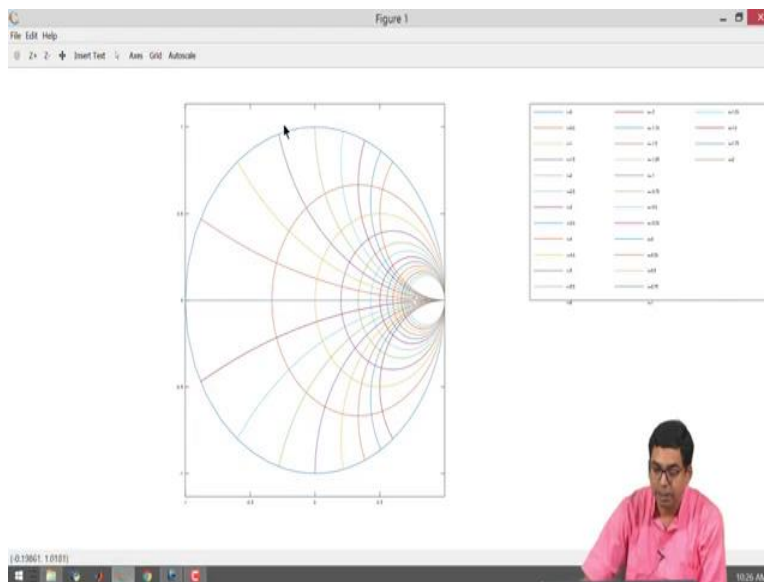
(Refer Slide Time: 25:21)



```
11 for x=[-2:0.25:2]
12     r=[0:0.1:1000];
13     for i=1:1000
14         gamma(i)=(r(i)-1+x*j)/(r(i)+1+x*j);
15     endfor
16     l=[^;x=num2str(x) ^];
17     plot(real(gamma), imag(gamma), l); axis 'equal'; legend("location", "no
18 endfor
19
```

And, I want to be able to plot the real part of the gamma, the imaginary part of the gamma with some string. Here the value of x is changing. So, I am changing the legend string to reflect that ok.

(Refer Slide Time: 25:50)



So, I am going to run this program. This is what it looks like alright. Previously, we had the circles, which were shrinking in radius and moving to the right for fixed values of resistances with varying values of reactance. So, the blue circle outermost corresponds to  $r = 0$  with  $j$  going from one value to another ok.

Now, we are having some additional plots that are coming into the picture ok. Each of them deals with a different value of reactance ok. So, if you look at the legend that is going to tell you for example,  $x = -2$  is this purple plot and  $x = +2$  is this orange plot. So, you can always see where this purple plot is. So,  $x$  equal to  $-2$  is coming here alright and  $x$  equal to  $+2$  is going to the top right.

So, it tells you that  $x = +2$  is creating something here  $x = -2$  is creating something here, alright and this representation is simply known as a Smith chart. So, all you are doing is taking different values of your load impedance that is normalized to the characteristic impedance of your transmission line and you are plotting the root locus of these points and we are plotting the variation of real and the imaginary parts right on the complex gamma plane.

Each and every point here is telling you the value of gamma; the gamma is going to be having a real part and an imaginary part ok. So, this is known as the Smith chart, but we have to be very clear about the kind of Smith chart. The kind of Smith chart that we are seeing here some impedance Smith chart ok because we started with

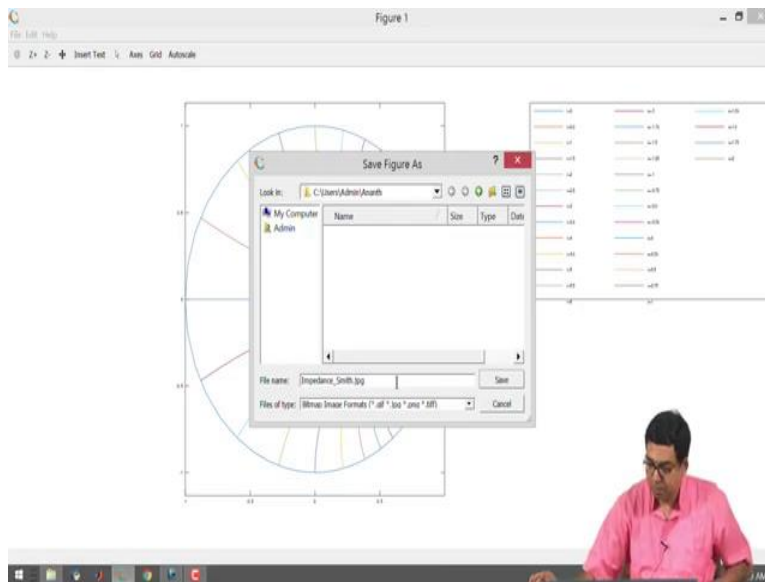
$$\frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1}$$

So, it is an impedance Smith chart.

For a minute let us look at this impedance Smith chart and try to see what it actually means right.

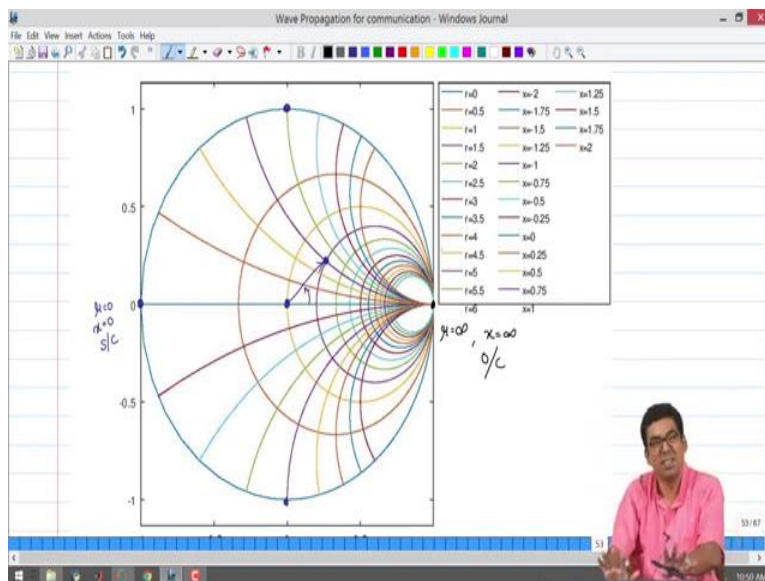


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So if possible, I will save this graph as a jpeg and I import that over here ok.

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There we go, ok. Now, I will mark a few points on these charts ok. So, the right hand side corresponds to this point and corresponds to  $r$  equal to infinity. It also corresponds to  $x$  equal to infinity ok. The  $x$  equal to  $-2$  curve lies over here I mean  $x = -2$  is here  $x = +2$  is here alright as

you increase the value of  $x$  it keeps going towards this side alright. So,  $r$  equal to infinity  $x$  equal to infinity is the point on the right alright.

Now, let us try to figure out what that physically would mean or equal to infinity  $x$  equal to infinity will mean that you are having an open circuit alright. So, we mark this region as an open circuit. Now, we will also look at this point. This point is on the complex gamma plane alright. We can also tell what the value of the reflection coefficient at the load is going to be. The value is going to be equal to 1 because the angle it makes with respect to the real axis is 0 as no phase.

So, we can go back to the derivation that we made for the reflection coefficient for an open circuit we would have got

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 1$$

alright. So, the analytical way of finding out the reflection coefficient is by using the formula

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

The computational way you are already aware of the telegrapher's equations and try to see what is going back alright this is the third method, which is the graphical way ok.

Now, you know that on the right hand side of this chart  $r$  is equal to infinity,  $x$  is equal to infinity. So, somebody tells you that the load impedance is an open circuit. You can quickly go to the Smith chart, mark this point and say that the reflection coefficient is going to be one. Let us go to the left most point ok the leftmost point ok is an interesting point because  $r = 0$  ok. However, there is a straight line that goes over here. The straight line corresponds to  $x = 0$  ok because the radius of the circle becomes infinite that expression looks like a straight line ok.

So,  $r$  equal to 0 and  $x$  equal to 0, we already know what the physical interpretation would be: it is going to be a short circuit on the left hand side. So, the short circuit is represented on the impedance Smith chart on the left hand side of the complex gamma plane. If somebody says that the load resistance is short ok you can say that yes, the reflection coefficient is going to be - 1 ok. You can look at the axis values over here right alright it is going to be - 1 and the reflection coefficient is going to be - 1.

So, given a load impedance and given a transmission line characteristic impedance and if the transmission line is lossless and we are given a value for the a impedance of the transmission line characteristic impedance, you should be able to go to the Smith chart, quickly identify the points, immediately say what the value of complex reflection coefficient is going to be ok.

Now, there are also two other points that we have to remember: ok there is a point going upward alright this point right where your  $x$  is positive ok where your  $x$  is positive, but the resistance is 0, it is a circle which corresponds to  $r$  equal to 0 and  $x$  is positive. So, that corresponds to pure reactance right, it is an inductance.

And, at the bottom, you are having pure capacitive load. We know that if your transmission line is going to be having loss it is going to be lossless it is having a characteristic impedance in ohms if you connect an inductor or a capacitor to it the reflection coefficient magnitude is still going to be equal to 1, only the phase is going to change. And, we already know from our circuits that in pure inductors and pure capacitors, the phase change between the voltage and the current is going to be 90 degrees.

So, if you want to measure the angle you could go from the horizontal axis and reach any of these points you will end up getting an 90 degree right, but here we have to be a little careful with the interpretation because  $\Gamma_L$  is voltage reflection coefficient. It is not the same as the phase between your voltage and current [laughter] it is the voltage reflection coefficient. So, it is  $\frac{V^-}{V^+}$  ok. So, it means that the reflection voltage is going to be 90 degree sort of phase alright with respect to the incident voltage for the top and bottom points; these are the important points in your Smith chart ok.

Now, given some value of load impedance ok and suppose, it has a real and an imaginary part the first step that you will do is normalize the given load impedance with respect to the transmission line characteristic impedance to get small r and small x. You will then try to identify the circle corresponding to small r ok and then once you have identified that circle what you will do is you will identify the circle corresponding to the x. The value of x that you have in your normalized transmission line load impedance, you will try to see whether it is this way or whether it is that way and then you will take the intersection of those two circles and make a dot.

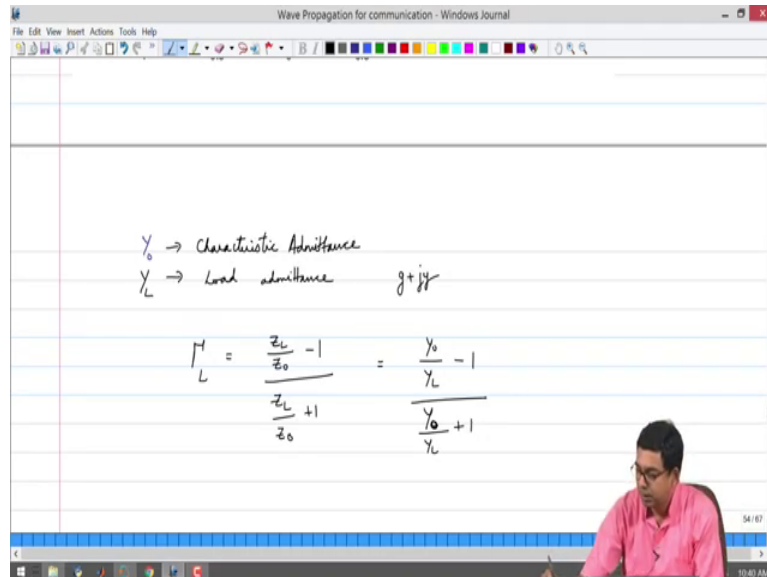
Say for example, I will make a dot over here. This is going to be your load impedance marked on a Smiths chart ok is going to be a load impedance marked on the Smith chart and what that point tells you is a location of gamma in the complex gamma plane. So, if I want to calculate for an arbitrary load impedance what the reflection coefficient is going to be I can just take the center of this circle and I could calculate the Euclidean distance from here to there alright.

In other words, I could take the real part, imaginary part and express gamma to be a real + j times some imaginary part. I could also do this in magnitude arguments form. I could take the distance and say that this is not gamma and I could take the angle from here and say that that is the angle of the reflection coefficient that I am going to get ok. So, this is how you construct a smith chart. So, construction of the Smith chart should be clear right.

Now, in most of the cases in transmission lines one would use actual admittance instead of impedance because many times you would connect something parallel to a transmission line to achieve what is known as impedance matching which is where we headed towards. Many of the times we do not cut the transmission line and insert something in the middle to achieve impedance matching. So, you will have a transmission line and you will connect something in parallel to it.

So, when you connect something in parallel it is easier to deal with admittances ok. So, one could also do the same for admittances, but are there going to be any significant differences ?

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Let us have a look at that once again ok. So, if I say that my characteristic admittance is  $Y_0$ , instead of  $Z_0$  I am having  $Y_0$ , this is the characteristic ok. And, suppose the load admittance given as  $Y_L$  is ok and let us say that we are dealing with a lossless transmission line which means that  $Y_0$  is going to be having only a real part.  $Y_L$  could be anything, could have a real and an imaginary part and you could write this to be say  $g + j$  you know  $y$  something previously we had  $r + j$ . So, you could write this as  $g + j y$  right.

So, you could go back to your reflection coefficient formula right. So, we had

$$\Gamma_L = \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1}$$

So, we could always write this down as

$$\Gamma_L = \frac{\frac{Y_0}{Y_L} - 1}{\frac{Y_0}{Y_L} + 1}$$

This will be the way to calculate the reflection coefficient given the admittances ok ok. So, now, I am having

$$\Gamma_L = \frac{\frac{Y_0}{Y_L} - 1}{\frac{Y_0}{Y_L} + 1}$$

And, we could also do something else we could say that  $Y_0/Y_L$  seems a little weird ok. We have normalized impedance with respect to transmission line impedance, we would like to do the same thing alright.

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The image shows a handwritten derivation of the reflection coefficient  $\Gamma_L$  in a Windows Journal window. The window title is "Wave Propagation for communication - Windows Journal". The text in the journal is as follows:

$Y_L \rightarrow$  Load admittance  $g + jy$

$$\Gamma_L = \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1} = \frac{\frac{Y_0}{Y_L} - 1}{\frac{Y_0}{Y_L} + 1} = \frac{Y_0 - Y_L}{Y_0 + Y_L}$$

$$= \frac{1 - \frac{Y_L}{Y_0}}{1 + \frac{Y_L}{Y_0}} = \frac{Y_0 - Y_L}{Y_0 + Y_L}$$

$$= \frac{1 - \frac{Y_L}{Y_0}}{1 + \frac{Y_L}{Y_0}} = \frac{-(\frac{Y_L}{Y_0} - 1)}{(\frac{Y_L}{Y_0} + 1)} = \frac{-(g + jy - 1)}{(g + jy + 1)}$$

We could simply write this down as

$$\Gamma_L = \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1} = \frac{\frac{Y_0}{Y_L} - 1}{\frac{Y_0}{Y_L} + 1} = \frac{Y_0 - Y_L}{Y_0 + Y_L}$$

$$\begin{aligned}
 &= \frac{1 - \frac{Y_L}{Y_0}}{1 + \frac{Y_L}{Y_0}} = \frac{1 - \underline{y_L}}{1 + \underline{y_L}} \\
 &= -\frac{(\underline{y_L} - 1)}{(\underline{y_L} + 1)}
 \end{aligned}$$

Let us write a simple program and try to plot the different possibilities for gamma for different values of real and imaginary parts of  $Y_L$ . So, just like the previous program first we are going to take  $g$  to be some fixed value say 0, 1, 2, extra. If  $g$  equal to 0 we are going to sweep the  $Y$  from going from some negative value to positive value and try to see what is going to happen. Then we will keep  $y$  you know  $Y$  to some fixed values and then vary  $g$  and draw these curves ok, but the formula for calculating gamma will be now this formula that we have created ok.

(Refer Slide Time: 41:30)

```

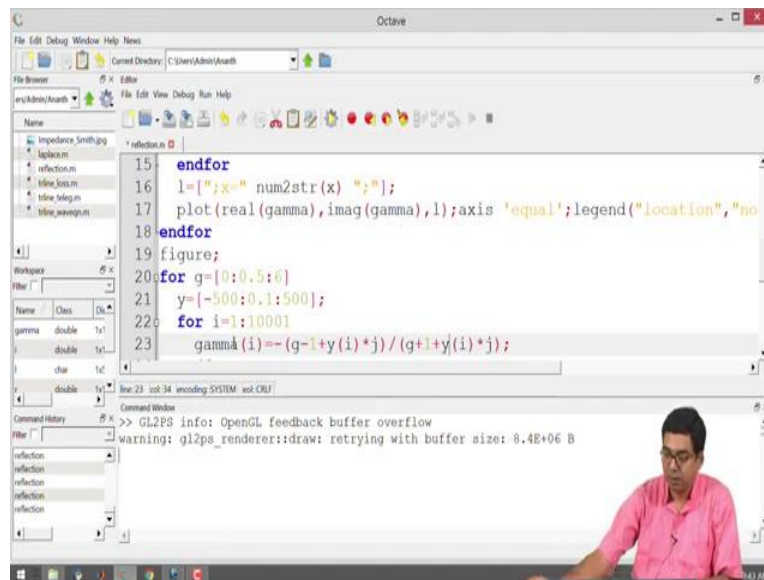
11 for x=-2:0.25:2
12   r=[0:0.1:1000];
13   for i=1:1000
14     gamma(i)=(r(i)-1+x*j)/(r(i)+1+x*j);
15   endfor
16   l-1";x=" num2str(x) ";";
17   plot(real(gamma), imag(gamma), 1); axis 'equal'; legend("location", "no
18 endfor
19 figure;

```

Command Window  
>> GL2PS info: OpenGL feedback buffer overflow  
warning: gl2ps\_renderer::draw: retrying with buffer size: 8.4E+06 B

So, I will go to the program and I will create a new figure for doing this. All I do now is I will copy the previous code entirely and I will start replacing things  $r$  with  $g$  alright. I will do some manipulation so that I can get the impression that I have right.

(Refer Slide Time: 42:05)



```
15   endfor
16   l=[";g=" num2str(x) ";"];
17   plot(real(gamma), imag(gamma), l); axis 'equal'; legend("location", "no
18   endfor
19   figure;
20   for g=[0:0.5:6]
21     y=[-500:0.1:500];
22     for i=1:10001
23       gamma(i)=- (g-1+y(i)*j) / (g+1+y(i)*j);
```

So, I am going to go ahead. So, this is going to be ok. So, I want to go for g equal to 0 to 0.5 to 6. Let us say that I am not going for negative values of g because we are dealing with passive circuits again ok and x is going instead of x, I am going to use the variable y right is going to my from -500 to +500 keeping everything as the same alright. Now, the formula that we had was

$$\gamma(i) = -\frac{g - 1 + y(i) * j}{g + 1 + y(i) * j}$$

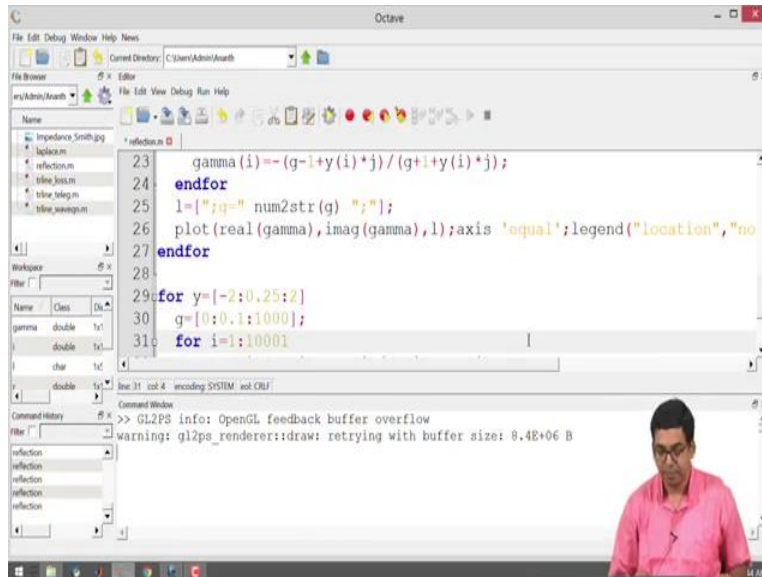
This is the formula we had written ok.

So, just to go back and write this in terms of ok you could write down the real and the imaginary parts together. So, we will have

$$\Gamma_L = \frac{-(g + jy - 1)}{(g + jy + 1)}$$

That is the exact formula that I am putting in the program ok.

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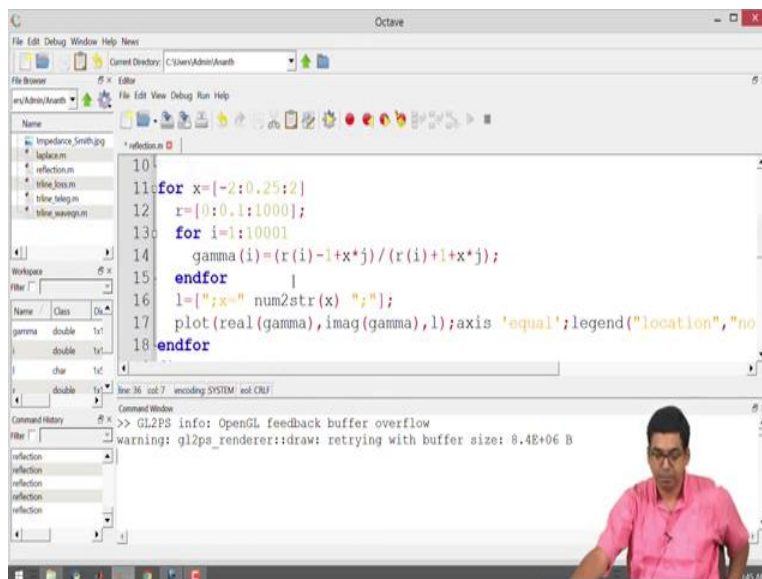


```
23 gamma(i) = -(g-1+y(i)+j)/(g+1+y(i)+j);
24 endfor
25 l = [";g=" num2str(g) ";"];
26 plot(real(gamma), imag(gamma), l); axis 'equal'; legend("location", "no
27 endfor
28
29 for y = [-2:0.25:2]
30 g = [0:0.1:1000];
31 for i = 1:1000
```

Command Window  
>> GL2PS info: OpenGL feedback buffer overflow  
warning: gl2ps\_renderer::draw: retrying with buffer size: 8.4E+06 B

This is the value of gamma that is calculated using admittances ok and, I would like to say that this is for g equal to ok everything else remains the same and then I want to make this for the next loop for y; y goes from say - 2 to + 2 this becomes g.

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```
10
11 for x = [-2:0.25:2]
12 r = [0:0.1:1000];
13 for i = 1:1000
14 gamma(i) = (r(i)-1+x*j)/(r(i)+1+x*j);
15 endfor
16 l = [";x=" num2str(x) ";"];
17 plot(real(gamma), imag(gamma), l); axis 'equal'; legend("location", "no
18 endfor
```

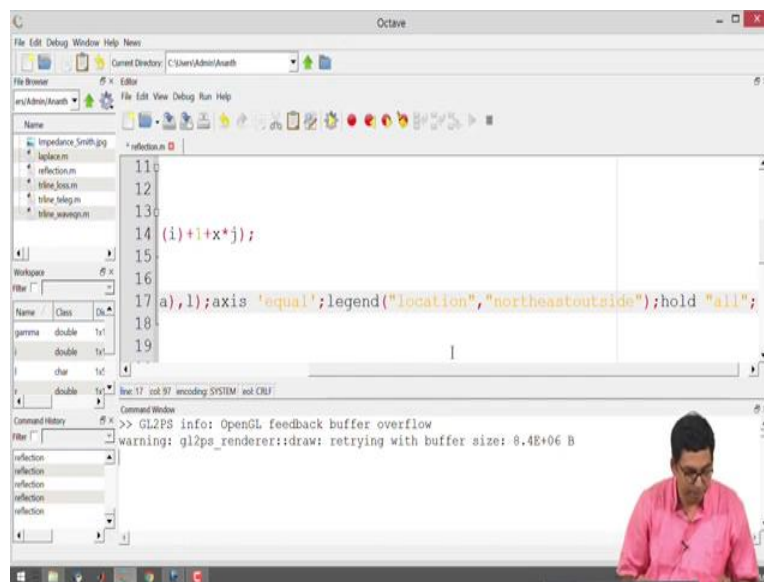
Command Window  
>> GL2PS info: OpenGL feedback buffer overflow  
warning: gl2ps\_renderer::draw: retrying with buffer size: 8.4E+06 B



So,  $r$  is being replaced with  $g$ ,  $x$  is being replaced with  $y$  that is all ok and then the formula for  $\gamma$ ; obviously, is different ok. So, we have  $\gamma$  formula is  $-(g(i) - 1 + y*j) / (g(i) + 1 + y*j)$  again I want to make this  $x$  to  $y$  to  $y$  ok.

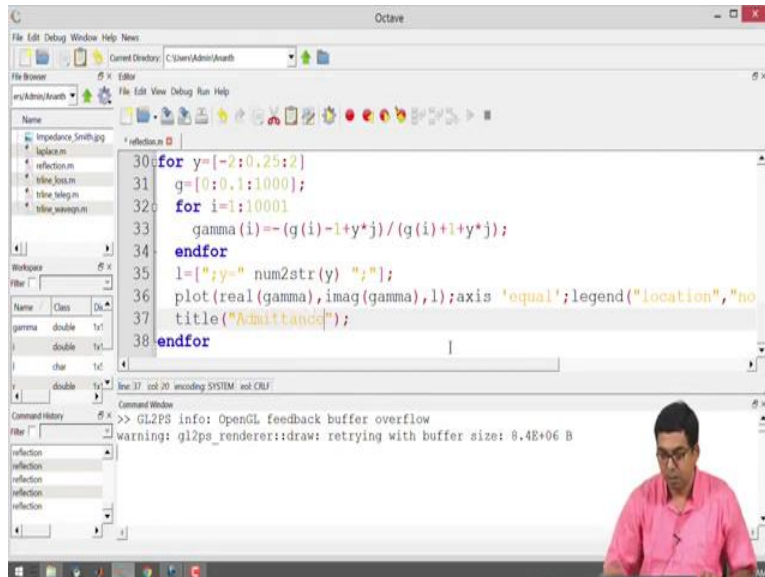
So, all I have done is copied the program, changed the formula for  $\gamma$ , used admittances. Just change the variable names to reflect the real and the imaginary part of the admittances ok and I am going to have this in a different figure ok and to be clear what we can do is the first figure we can give a title.

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So, I will just give a title to the first figure. I will just call this as an Impedance right.

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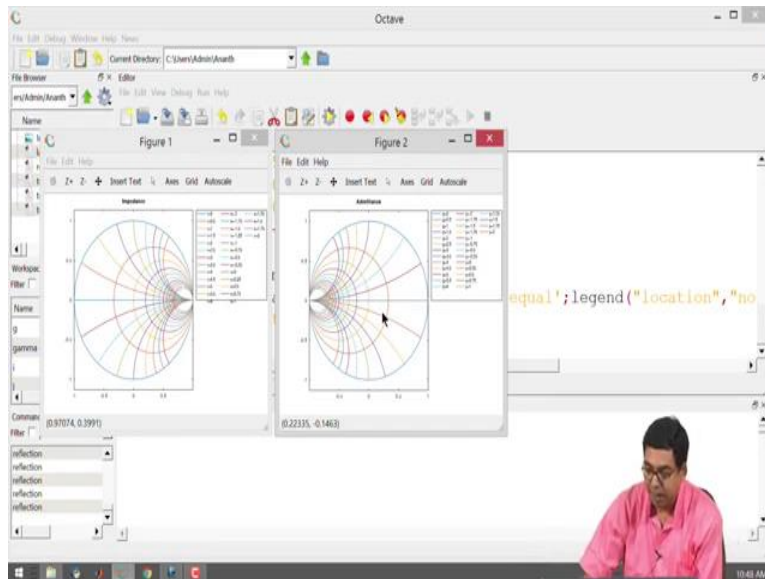


```
30 for y=[-2:0.25:2]
31     g=[0:0.1:1000];
32     for i=1:10001
33         gamma(i)=-((g(i)-i*y*j)/(g(i)+i*y*j));
34     endfor
35     l=[^;y=- num2str(y) ""];
36     plot(real(gamma), imag(gamma), l); axis 'equal'; legend("location", "no
37     title("Admittance");
38 endfor
```

I am going to call the second one as Admittance. I am going to run the program.

Ok.

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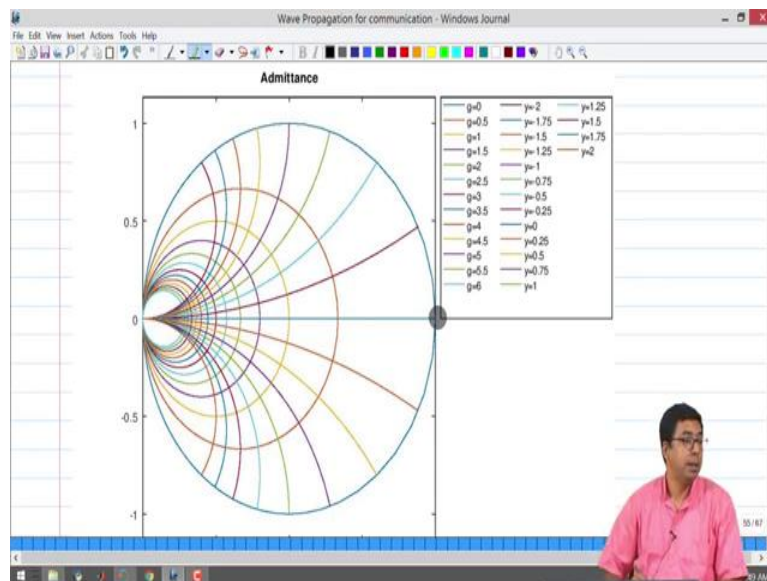
So, now I have two graphs. Let us take this, restore it a little bit and add a scale, put figure 1 and figure 2 side by side and you should be able to see something that is happening alright. Both of these are Smith charts because you are just representing impedances or admittances on the

complex gamma plane. X axis is going to be your real part of gamma, y axis is going to be the imaginary part of gamma.

The purpose of doing this is suppose we are given a load resistance or a load impedance and the transmission line characteristic impedance or admittance, you should be able to quickly go to the Smith chart find the intersection of two cycle corresponding to that and then say this is the value of the reflection coefficient that is the objective ok.

Now, we can clearly see that if you are going to deal with admittances, there is a different figure that is popping up ok. We notice that the right side figure is a mirror version of the left side figure ok. So, the x axis is flipped ok; on top of that if you see in detail the y axis is also flipped ok. The x axis is flipped the, y axis is also flipped ok. So, this is going to become confusing for people; that means, the interpretation is going to be different.

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So, if we take this curve right and we make a version of this ok. Now, we have to start marking some points. The left hand side is corresponding to g going from 0 to infinity ok. As your g keeps increasing you go all the way to the left hand side point right. Similarly, we will have to see what is going to happen to the right hand side alright and then the interpretation of what is short circuit and what is an open circuit will change right. It will not be the same as your impedance chart. So, one has to be a little careful ok.

But, I have never seen somebody use an admittance Smith chart like this. Usually when people talk about Smith charts they usually have a way of orienting the Smith chart before starting any problems, they always do the orientation in this manner. Alright that is as you go to the right the circle should become smaller and more closely spaced alright and your reactance constant

reactance circle should go from the right side branching outside should be flaring out from the right hand side. This is the normal orientation of the Smith chart that people use.

What they do for admittances is that they flip the axis. So instead of having the positive axis they will call mark it the negative axis and simply continue to do the problem, but there are some nuances to that and we will have to see them in detail. But, for now you should know not only how to use the Smith chart the objective is that you make your own smith chart so that you get a feeling for what that chart actually is. It is just a gamma plane and you are marking different load impedances and load admittances ok. So, the first effort is to make your own Smith chart then only you understand what is going to be happening with the respect your admittances and reactances.

Now, what we are going to be doing is we are going to use this to do impedance matching using some different techniques ok. So, we will meet in the next class for this.