Transmission lines and electromagnetic waves
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Lecture – 10
Voltage Reflection Coefficient and Standing Wave Ratio

I think we will begin. There are a few more terms that we have to define ok, after that we are ready for working out some problems. So, the couple of things that we are going to define very clearly is impedance in the transmission line when excited with an AC. And, we are also going to define a term that we have not seen before in the DC excitation, which is standing wave ratio or voltage standing wave ratio the significance of these two terms.

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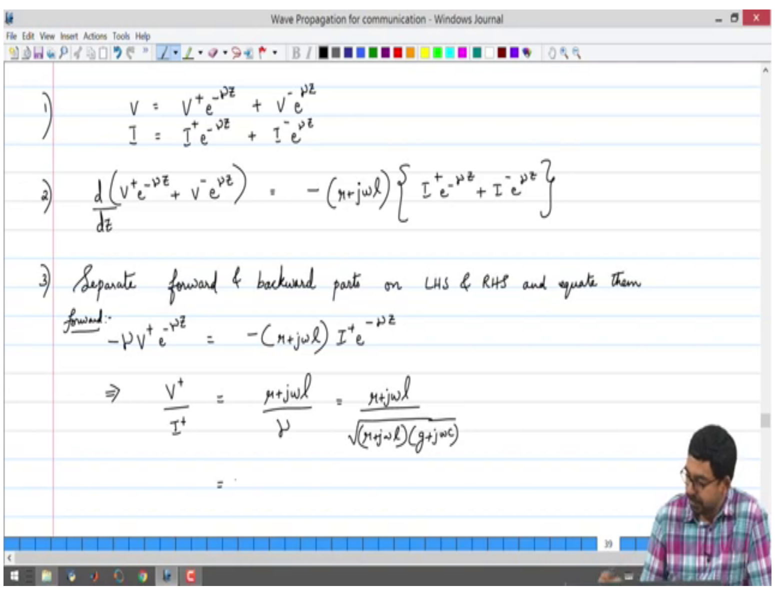

So, I will begin with the general solutions to the voltage in a transmission line. So, we had

$$V = V^{+}e^{-\gamma z} + V^{-}e^{\gamma z}$$

The first term will be the forward voltage and the second term is the backward voltage right. So, I am just writing down the general solution for the current in the transmission line that is going to be

$$I = I^{+}e^{-\gamma z} + I^{-}e^{\gamma z}$$

So, the first thing that we can do is we can use the telegrapher's equation to arrive at an expression for the impedance in the transmission line. So, we need a ratio of V to I ok. So, we can always do

$$\frac{d}{dz}(V^+e^{-\gamma z}+V^-e^{\gamma z})$$

So, this is the left hand side of the first telegrapher's equation dV/dz ok.

On the right hand side, we will have in this case since we have written γ, that means, that we are assuming that r and g are going to be non-0and present, we are writing the most general case over here. So, we will have

$$-(r+j\omega l)\{I^+e^{-\gamma z}+I^-e^{\gamma z}\}$$

These are the general solutions that we have obtained before and we are substituting back into the first telegrapher's equation. And, then one can go ahead differentiate the left hand side and take the terms on the right hand side, and the first thing one can do is separate the forward and the backward parts ok.

Separate the forward and the backward parts on the left hand side and right hand side and we equate them all right. So, all we are doing is we are going to separate the forward and the backward part on both the left and the right hand side. And, we are going to equate them. The idea is to find some relationship between the forward voltage, forward current right, backward voltage and backward current and in that process we will be able to find out what the impedance is going to look like right. So, for the a forward part if I take

$$\frac{d}{dz}(V^+e^{-\gamma z})$$

That is the forward part.

So, I am going to take this term and then equate it on the right hand side to the forward term

$$-(r+j\omega l)\{I^+e^{-\gamma z}\}$$

So, if I take the derivative I will have

$$-\gamma V^+e^{-\gamma z}=-(r+j\omega l)\{I^+e^{-\gamma z}\}$$

And, from here the quantity that we are interested in is V plus divided by I plus.

$$\frac{V^+}{I^+}=\frac{r+j\omega l}{\gamma}=\frac{r+j\omega l}{\sqrt{(r+j\omega l)(g+j\omega c)}}$$

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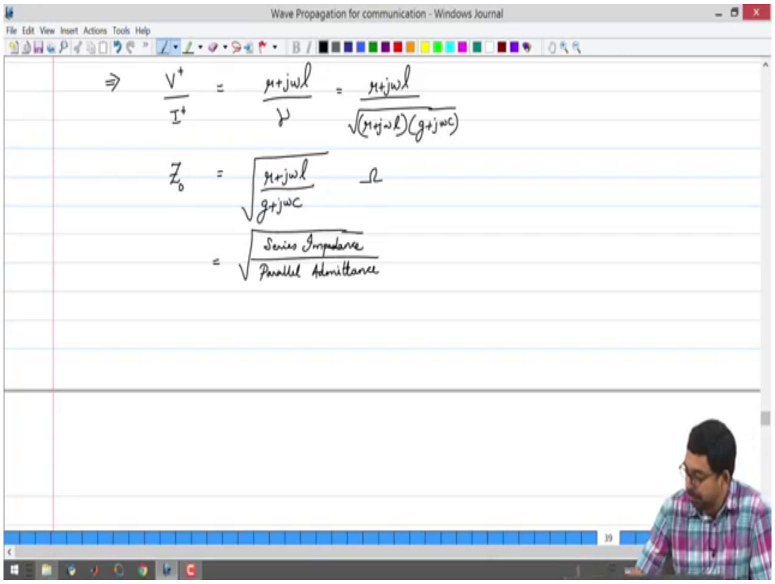

So, in other words, the left hand side has to correspond to the impedance in the transmission line all right. So, this is the characteristic impedance of the transmission line all right.

So, you can mark this as some Z_0 or Z_c depending upon the convention that you are following right. Z_0 we will use it to represent the characteristic impedance right. Is going to be

$$Z_0 = \sqrt{\frac{r + j\omega l}{g + j\omega c}}\ \Omega$$

Another way of remembering this is by using series impedance in your circuit model divided by the parallel admittance in the circuit model, the series we had r and l connected. So, we can have series impedance divided by parallel admittance, $g + j\omega c$, this is your characteristic impedance Z_0 ok.

Since, we have equated the forward terms let us also go ahead and equate the backward terms and try to see what we are getting all right. So, we go ahead, ok.

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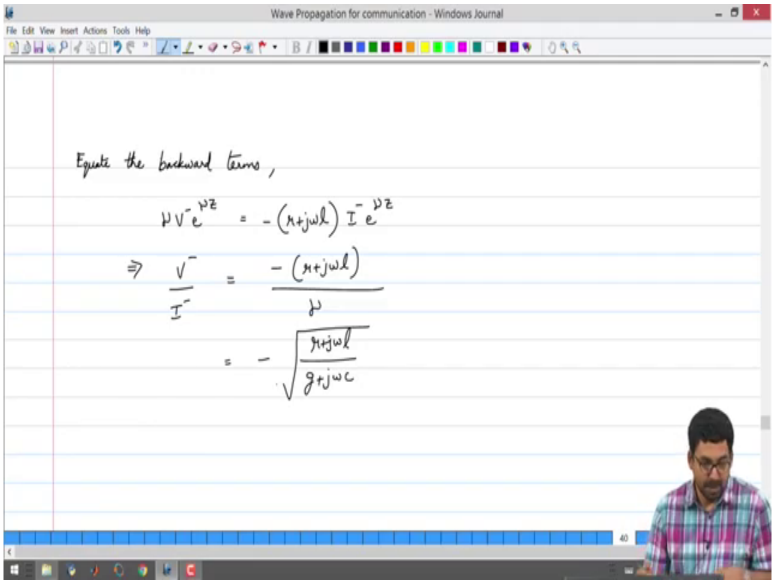

So, when we equate the backward terms, the left hand side I am having

$$\gamma V^{-} e^{\gamma z} = -(r + j\omega l) I^{-} e^{\gamma z}$$

$$\frac{V^{-}}{I^{-}} = \frac{-(r + j\omega l)}{\gamma}$$

$$= -\frac{\sqrt{r + j\omega l}}{\sqrt{(g + j\omega c)}}$$

Now, one has to realize that we have still found the characteristic impedance, but the negative term appears over there all right. It should not be mistaken for some active circuit, which is providing a negative impedance of some kind. It just signifies that the energy is travelling backward all right. So, characteristic impedance negative right is occurring because you are taking the backward voltage divided by the backward current ok.

However, the characteristic impedance as we conventionally, as you know denote in all the problems that we will be solving we will be having only positive values, but you should be aware that if you are measuring backward voltage divided by backward current. It is not as you know unusual to get negative characteristic impedance, because you have to get signifies the direction of energy transfer ok. Now, once that definition is clear, ok. Then, we can go for the derivation of this standing wave ratio ok. For this, we will draw the transmission line model with a slight change compared to what we had done before all right.

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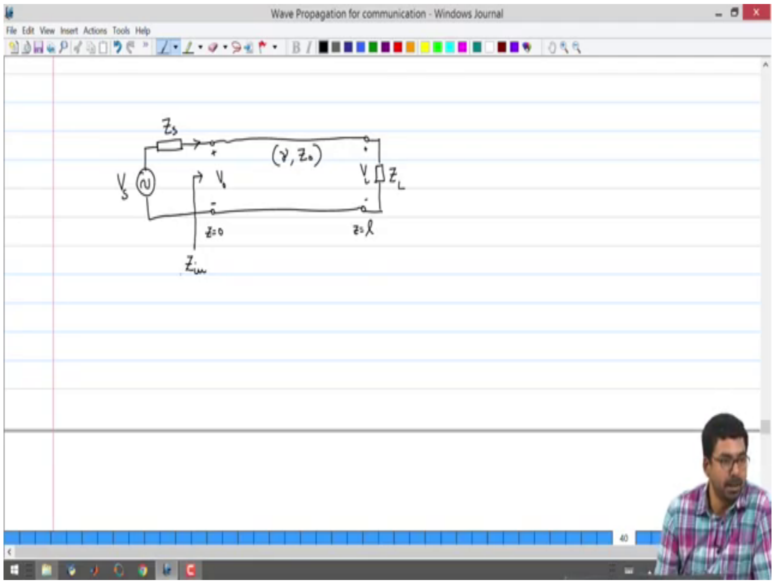

So, I will start with the source voltage, which is an alternating source. So, I am having a V_s the series impedance of this source is denoted by Z_s following that I am having a transmission line section ok. So, I am going to mark this transmission line section like so and a load impedance. Let us connect it on the other end.

So, it is denoted by Z_L. In order to say that this is a transmission line, we provide some parameters in the circuit diagram. We say that the propagation constant is γ characteristic impedance is Z_0 ok.

We also want to mention something about the length of the transmission line. So, to denote that very clearly we take a coordinate axis, we say that Z = 0 and Z = l. Previously we had taken a different coordinate system, where the load side was 0 and the left side was -l. I just wanted to show that you can also use any other coordinate system all right, which is as long as you are consistent, you will be ending up with the same result.

Sometimes, I found that the students find this coordinate system easier while going from left to right it is going from 0 to l.

So, if you feel this to be comfortable, you can stick to this coordinate system for the rest of the course and just base all your derivations on this coordinate system you will hit no problems ok. And to just instill that confidence, I just want to reverse the coordinate system as I mean a you know which is not in the textbook, textbook usually uses the other coordinate system ok.

So, I am having a current that is supplied by the source right. Since, this is the closed circuit. I am going to also mark some values in this circuit diagram. Just going to say that the voltage at the source end of the transmission line that is the left hand side end, which is Z = 0 is V at Z = 0 and I am going to denote that with V_0 ok.

So, V_0 is going to be the voltage at Z = 0 ok. The voltage **at** the load end of the transmission line is going to be V_L ok. On top of that to be a little bit more precise can also say, that by looking into the transmission line from the source side ok, while you are looking into the transmission line from the source side, the impedance that you will measure is Z_{in}. It is an effect of the characteristic impedance and the load impedance put together all right. And, we will find out about all these things in just a minute ok. So, this is the circuit diagram based on which we are going to start the derivation for reflection coefficient and voltage standing wave ratio ok.

So, we will begin by writing down the expressions for voltage and current.

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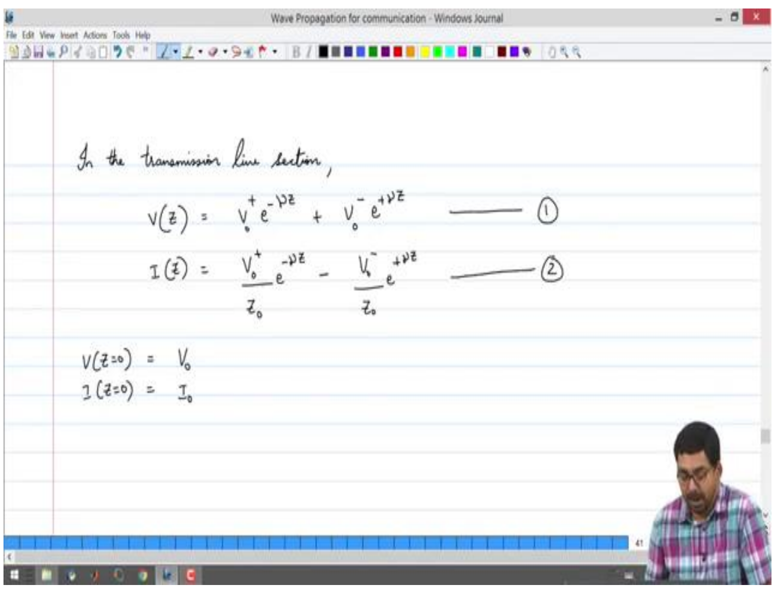

So, we will start with the transmission line section ok. So, in the transmission line section at any position Z, the voltage is given by

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \text{ ……… } 1$$

This is the general solution to the transmission line equations that we have seen before right. The current ok,

$$I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z} \text{ ……… } 2$$

So, along with the convention that we are using,

$$V(z=0) = V_0$$

$$I(z=0) = I_0$$

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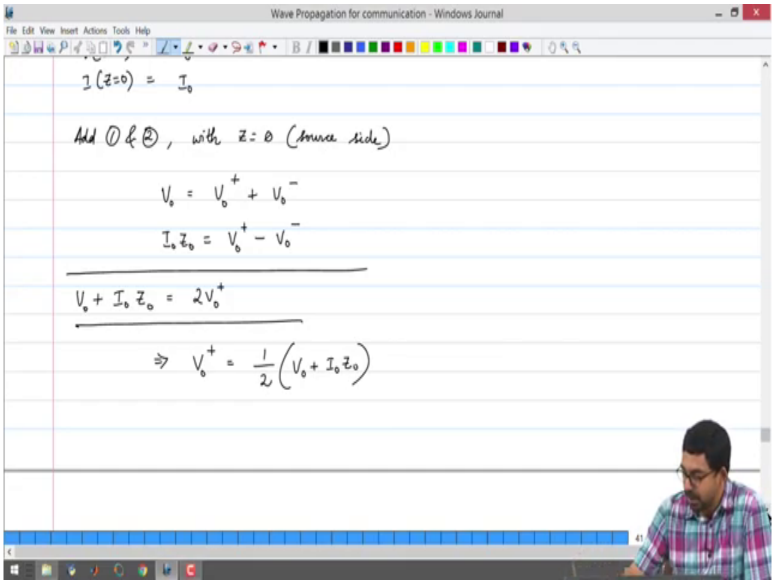

Now, I will have the sum of equations one and two all right along with the condition that I am going to look at Z = 0 which is the source side. So, I am going to add one and two people will be confused, why are we adding voltage and current and how can we add voltage and current? Simple answer to that is I will be bringing the denominator Z_0 to the left hand side, then they become dimensionally stable and you can add the two equations ok ok.

So, I am adding equations one and two, but I am not adding voltage and current, I am just bringing the impedance from the denominator to the numerator making it dimensionally stable and then I am adding ok. So, this means that I will be having

$$V_0 = V_0^+ + V_0^-$$

$$I_0 Z_0 = V_0^+ - V_0^-$$

$$V_0 + I_0 Z_0 = 2V_0^+$$

$$V_0^+ = \frac{1}{2}(V_0 + I_0 z_0)$$

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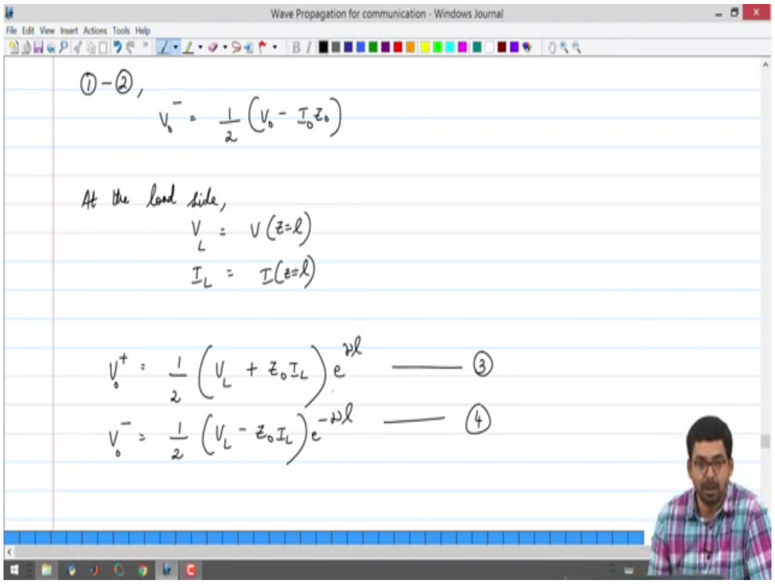

And, in order to find V_0^- you can take the a you know equation number one. And, subtract two from it. So, one minus two can be done. And, there you will be ending up with

$$V_0^- = \frac{1}{2}(V_0 - I_0 z_0)$$

On the source side this is how one will write down what is the forward and what is the backward, there can always be some questions as to what is exactly V_0 all right.

So, if we look at the circuit diagram one has to be a little careful, the way we had done bounce diagrams, for the DC case just to understand, what is a time based evaluation of voltage? We started with an example where we did not have this series resistance in that particular example. So, whatever voltage was applied was appearing as V_0 and then you are calculating the bounce diagram.

Now, after that example we have progressed a lot more all right. We know that, if we are looking into the transmission line from the source side there is going to be an impedance of Z_{in} all right. So, V_0 if you want to calculate, we have to take the source voltage alright and we have to do a voltage division rule between the series impedance Z_s and Z_{in} ok.

And, then you will have to figure out what is V_0^+ and then you will have to figure out what is V_0^-, and then you have to start drawing bounce diagrams ok. So, to be clear the V_0 is coming because of voltage division between the series impedance and the input impedance seen over here ok.

So, the first step you will do is assume this transmission line entire thing up to the load being represented as a single lumped parameter whose value is Z_n, then you will calculate V_0 and then you will determine what is V_0^+ and V_0^- ok. That is what this so far it means ok.

So, at the source side it is clear what we are doing ok at the load side ok, we can use

$$V_L = V(z = l)$$

$$I_L = I(z = l)$$

So, we can always write down at these positions right what my V_0^+ and V_0^- would look like. So, we can say that V_0^+ can also be written in terms of V_L and IL.

So, I can take the equation that I got for V_0^+ ok. So, I can write this down as

$$V_0^+ = \frac{1}{2}(V_0 + I_0 z_0)e^{\gamma l} \dots \dots \dots \dots 3$$

ok, that, simply, because the voltage on the load side and the voltage at the source side have a difference in the form of an exponential to the power I mean exponential γ l ok. So, all we are doing is we are writing down V_0^+ in terms of V_L,I_L the only change that you will do to your expression is you will account for the propagation loss and the phase constant of your transmission line in the form of $e^{\gamma l}$ok.

Similarly, you can write down V_0^- as

$$V_0^- = \frac{1}{2}(V_0 - I_0 z_0)e^{-\gamma l} \dots \dots \dots \dots 4$$

You can also mark these equations as 3 and 4 they give you the exact same information, means that the unknown quantity as on the left side you are trying to find out V_0^+ and V_0^-. If you know some quantities on the right hand side all right, if you know V_L, IL. And, if you know the length of the transmission line the propagation constant γ, then you will be able to find out what is exactly the voltage that was travelling forward at the source side.

What is the voltage that was going backward at the source side extra ok? This just gives you additional forms of finding the same unknowns ok. During the quiz, you may not need to memorize all of this ok. A, If, the formula is required it may actually be given. So, that you do not have to worry about memorizing this, you can worry about the application of what we are doing other than just memorizing all these things. I know that sometimes it can become overwhelming. So, you do not have to worry about it. If the question needs something that you need to remember it will be provided with the questions.

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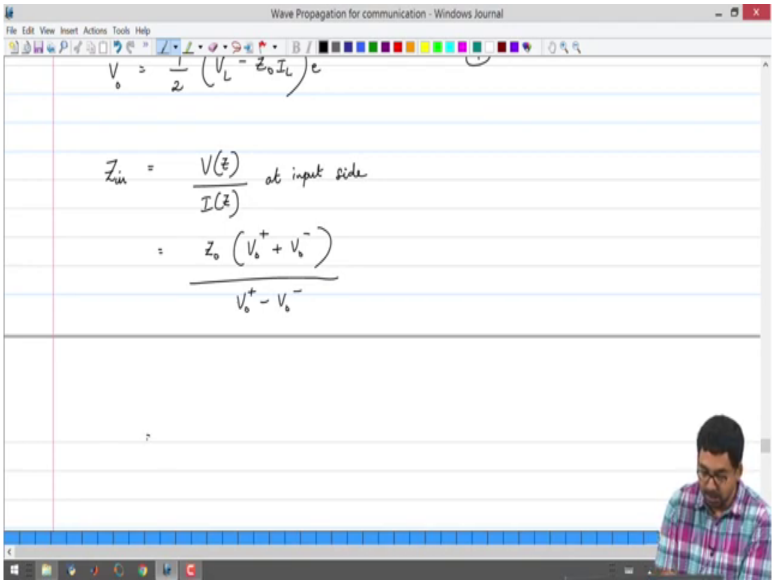

So, another thing is we have marked Z_{in} in our transmission line. Now, the way we have marked Z in our transmission line, yes it is going to be a ratio of voltage divided by current ok.

And, it is going to be at the input side or the source side ok at the source side.

$$Z_{in} = \frac{V(z)}{I(z)} \; at\ input\ side$$

$$= \frac{Z_o (V_0^+ + V_0^-)}{V_0^+ - V_0^-}$$

This is what is your source a I mean input impedance from the source side for a transmission line that is terminated by the load impedance of Z_L. However, one can also make this a little bit more elaborate ok. From equations 3 and 4 ok, we are able to see that given the load side parameters ok. Only the load side parameters, load side parameters will be voltage at the load, current passing through your load impedance length of the transmission line and some propagation constant, you will be able to estimate what is the forward voltage that was launched on the source side ok.

So, using load side parameters you are able to estimate this same way, using the load side parameters you are able to estimate, what is the backward voltage reaching at the source side? Ok. So, you may very well make use of this information all right and plug it into the expression for the Z_{in} and that will make it look a little bit tedious that is all.

$$Z_{in} = \frac{Z_o(V_0^+ + V_0^-)}{V_0^+ - V_0^-}$$

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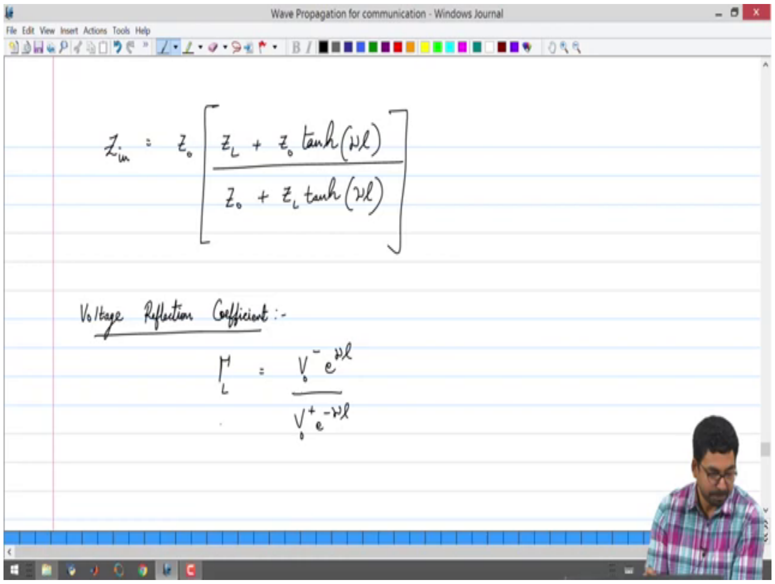

So, you can always express this in terms of load side parameters. So, which will mean that you will make use of equations 3 and 4 ok and you will substitute for them here and then you will arrive at an elaborate expression.

I will write the expression directly because there is no magic in between steps ok. So, you will end up getting, since we are dealing with γ to be a complex propagation constant, it has a real and an imaginary part ok. So, when you take the exponentials, you take the sum, you take the difference, you will end up with hyperbolic quantity stuff. Say sin or you will be having sinh, instead of cosine you will be having cosh, instead of tan you will be having tanh, that makes it a little bit more you know an elaborate that is it. So,

$$Z_{in} = Z_0[\frac{z_L + z_o \, tanh \, tanh \, (\gamma l)}{z_0 + z_L \, tanh \, tanh \, (\gamma l)}]$$

The reason this was done is previous lectures, we have seen what Z_{in} would be in the case of a lossless transmission line. In the case of lossless transmission line, we had a very similar expression we just had

$$Z_{in} = Z_0[\frac{z_L + z_o tan\beta l}{z_0 + z_L tan\beta l}]$$

So, this is the expression that we had for the lossless case. I just wanted to make sure that you had it for the lossy case also. And, if you remember this you can always, substitute γ to be you know $\alpha = 0$ and then the sanity check can be whether you get back the same expression appears that you will get back the same expression ok.

Now, there are a few more definitions that we will see once again with the ac excitation right, the voltage reflection coefficient ok. We can start with γ at the load end ok, it is going to be your backward voltage divided by your forward voltage as we had seen in the prior lectures. So, we are going to write down the exact same thing.

So, backward voltage is going to be $V_0^- e^{\gamma l}$. Note that everything I am writing in terms of V_0 and some parameter ok, that is how we are denoting everything divided by $V_0^+ e^{-\gamma l}$.

Ok. So, the way of representing your load reflection coefficient is

$$\Gamma_L = \frac{V_0^- e^{\gamma l}}{V_0^+ e^{-\gamma l}}$$

One could also go one step further and say that V_0^-/V_0^+ is your reflection coefficient at the source side. And, then if you multiply that with you know another quantity $e^{2\gamma l}$, you will be ending up with the reflection coefficient on the load side. This just now gives you a relationship between reflection coefficients at different places. If you know the source side reflection coefficient and the length of your transmission line and say the propagation constant, you can estimate what the reflection coefficient at the load side is going to be all right and vice versa, that is all ok.

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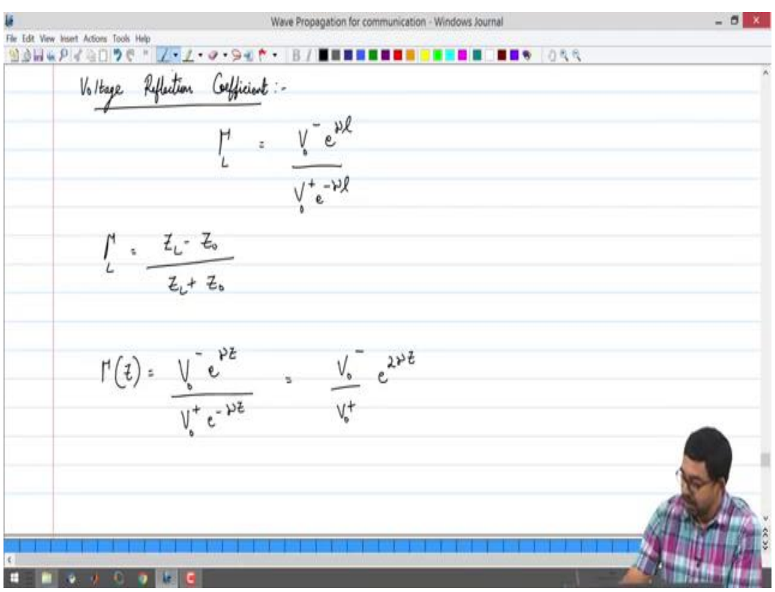

The expression that we are already aware of is we cannot remember all this right, we can always say that we we do know that it is going to be

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

This we are already aware of from the previous lectures, you are free to use this to find out the load reflection coefficient. If you know Z_L, Z_0 you will be able to find out what is going on with your load reflection coefficient.

Now, one of the things that we need to understand now is that you could calculate in the bounce diagrams the reflection coefficient on the load side ok. And, then the reflected wave will come to the source side, you will calculate another reflection coefficient and then you will start marking the bounce diagrams again, that is how we drew the bounce diagrams in the prior classes, which means that the reflection coefficient is a function of position.

It is the quantity that changes continuously along the position of the transmission line, that means, that γ can exist anywhere in the transmission line, and every section of the transmission line can have its own reflection coefficient that is what it means all right. So, in general you can write down the reflection coefficient γ to be a function of position Z all right. And, you can always take a hint from this expression over here right. And, then write down that in general

$$\Gamma(z) = \frac{V_0^- e^{\gamma z}}{V_0^+ e^{-\gamma z}} = \frac{V_0^- e^{2\gamma z}}{V_0^+}$$

If, Z is = 0 in our case, the way we have defined now corresponds to the source side, you will end up getting

$$\Gamma(z = 0) = \frac{V_0^-}{V_0^+}$$

So, it is correct if Z is = l, you will end up getting what we have got here all right. So, it is correct. So, at any position Z you can have a reflection coefficient ok.

Now, this reflection coefficient in the case of ac excitation also means that you will be having the superposition of the forward and a backward wave, and some instance of time you will be having the amplitude of the voltage going higher than the forward all right. And, the currents could be higher than the forward current extra.

Which means that, we also saw from our simulations that there were some places, where the voltage was pegged at 0 and some other places where it exhibited standing wave characteristics like it was increasing with respect to time, but then going back? So, if we go back to your simulations and use a sinusoid, you say open a circuit or a short circuit boundary condition, you will be able to see some standing wave patterns, there will be some nodes some places, which are antinodes and you will be able to see what is happening?

So, this means that at certain places the voltage is always pegged at 0, ok. Now, let us look at the other place, where it is you know going between a maximum and some minimum all right. So, in the case of our simulations we use only two types of boundary conditions: short circuit or open circuit. In both these cases, the backward voltage went I mean the superposition of the forward and the backward went to 0 at some instance of time ok. Not at the places where it is nodes, but in the antinodes with respect to time the voltage was increasing and then it was going to 0 and then it was increasing again and it was going to 0, extra.

So that means that there was a maximum voltage in the transmission line and there was also a minimum voltage in the transmission line barring your antinode places ok. So, since we had only two boundary conditions ok, it will be tough to imagine this, but you could always have terminations with different values of load impedances ok. In these cases, the reflection coefficients would be different right, it would be different. In our cases for the ideal simulations that we have done, the reflection coefficient was either plus one or minus one ok.

But, here it means that you can have a fraction all right and it could also have a face associated with it. So, it becomes a little bit more complicated all right. So, nevertheless you will end up having, say , a maximum voltage in your transmission line. And, you will also be having some minimum voltage in the transmission line, that could be non-zero, it need not be zero, zero happens only because the forward and the backward voltage are exactly equal and they cancel off each other at specific points. But, we know that the reflection coefficient need not be plus or minus one. So, the backward voltage need not be equal to the forward voltage.

You can end up with some residual voltage ok. So, V_{max}/V_{min} in these cases is known as the voltage standing wave ratio ok.

$$\frac{V_{max}}{V_{min}} = VSWR$$

$$\frac{V_{max}}{V_{min}} = \frac{1 + |\gamma L|}{1 - |\gamma L|}$$

V_{max} depends on the reflection coefficient all right. And, the time it will be maximum you want to have the positive component coming from the back backward voltage and adding up your forward voltage that will be the maximum voltage, minimum depends upon the difference between the two. So, we will be having $1 - |\gamma L|$.

This ratio is referred to as a voltage standing wave ratio. And, it is important in ascertaining the kinds of terminations your transmission line is having. In our case previously, we had seen that we were having open and short circuits for the simulations ok. So, γL was either equal to +1 or -1 ok. Suppose, it was +1, γL = +1 ok, if γL = +1. So, we will just mark this with some short form as VSWR Voltage Standing Wave Ratio ok.

So, I will have

$$VSWR = \frac{1+1}{1-1} = \infty$$

The minimum voltage in my transmission line was zero, because of which my voltage standing wave ratio is infinity. So, I am having huge standing waves in my transmission line that is what it means all right.

If, γL = -1, this does not change you will still have perfect standing waves in your transmission lines. So, the difference between short circuit and open circuit is not easy to ascertain by just looking at VSWR all right. So, you will be having the ratio

$$\frac{V_{max}}{V_{min}} = \infty$$

So, if you measure a standing wave ratio to be infinity that means that you have some extreme termination problems in your transmission line, maybe it is an open or a short circuit ok.

But, it is also quite possible that the load reflection coefficient is zero, if that is the case, we call this transmission line to be having a matched load at the other end of the transmission line.

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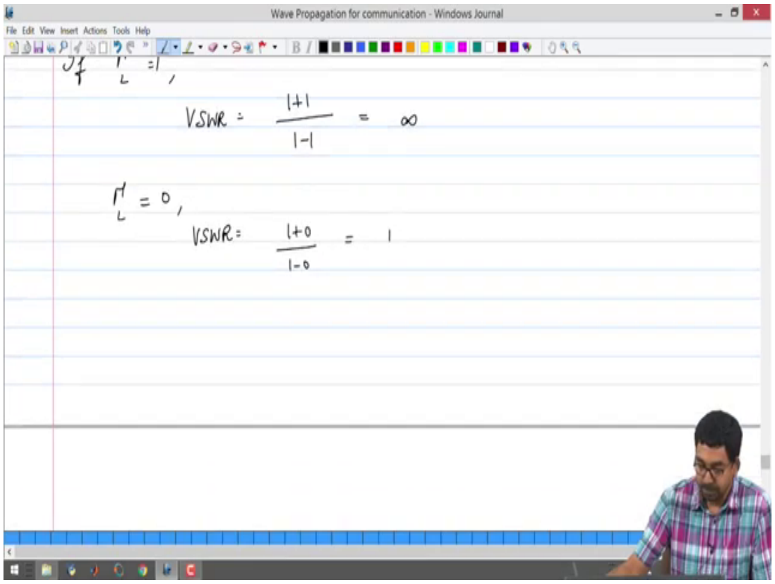

If, the load is matched and the load reflection coefficient is 0

$$VSWR = \frac{1+0}{1-0} = 1$$

So, the standing wave ratio goes between one and infinity. In the case of matched impedance, you will have a standing wave ratio of one. In the case of extreme terminations like open circuit and short circuit the standing wave ratio will be infinity.

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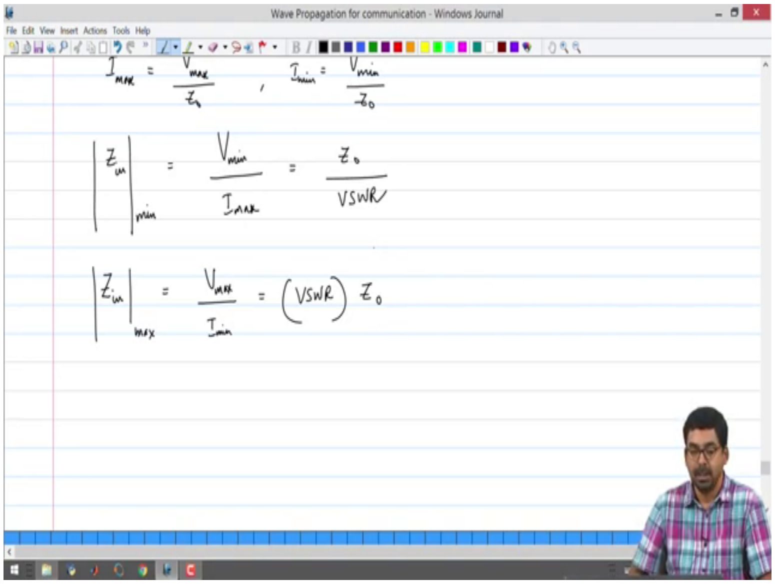

Ok. And, one could also think about what will happen to your maximum current, we now know that the maximum voltage standing wave ratio is telling you about the voltage. One could always use ohm's law and try to find out what the maximum current would be.

So, this should we say I_{max} in your circuit could simply look like

$$I_{max} = \frac{V_{max}}{Z_0}$$

and

$$I_{min} = \frac{V_{min}}{Z_0}$$

So, this would be the maximum and the minimum currents in your transmission line ok. So, this gives rise to some thoughts. We have talked about the impedance measure from the input side ok.

So, we talked about the impedance in the transmission line looking from the input side, which is known as Z_{in} ok. So, now, we have to think about what Z_{in} could look like right. So, we can write down

$$Z_{in}|_{min} = \frac{V_{min}}{I_{max}}$$

What is the minimum impedance that you can measure? Which means that I will be having minimum voltage on the numerator, divided by the maximum current in the denominator ok? And, since

$$I_{max} = \frac{V_{max}}{Z_0}$$

I can substitute for I_{max} and I will just get

$$Z_{in}|_{min} = \frac{V_{min}}{I_{max}} = \frac{Z_0}{VSWR}$$

Now, if I know what my standing wave ratio is going to be I can clearly calculate what the minimum input impedance could be right. It is going to be Z_0 divided by VSWR now you can notice that the VSWR goes from one to infinity ok.

So, the minimum Zin that you will be measuring is Z_0 maximum is 0 ok. Now, let us also look at what the maximum value of Z_{in} is going to be ok, Z_{in} max will be maximum voltage on the numerator divided by the minimum current at the denominator ok. I can always substitute for I_{min}

$$Z_{in}|_{max} = \frac{V_{max}}{I_{min}} = Z_0(VSWR)$$

This means that your maximum input impedance will go between one and an infinity ok, I mean Z_0 and infinity ok ok. So, these are the ways of looking at what your minimum impedance and maximum impedance in the transmission line can be, what current maximum currents they can be?

Usually, they revolve around finding out the voltage standing wave ratio. These quantities are important, because if you are dealing with transmission lines and if they have to carry some current, you want to make sure that electrically and mechanically it is going to be stable, you want to make sure that the current rating for the materials, that you are using is going to be supporting this levels of current all right. And, when you are talking about impedances you need to know the range of impedances unfortunately for the open and short circuit the impedances are not. I mean the range is not small at all right.

You can go anywhere from 0to infinity in the case of impedance that you are measuring from the input side. So, if you are going to measure the impedance of a transmission line from the input side all these equations tell you if it could be anything ok. So, it is not easy to arrive at a conclusion just by measuring impedance on the source side, you need to have a lot more additional information in order to figure out what is going on in the transmission line.

The second thing that we should remember is during the design phase of the transmission line, we need to estimate what the standing wave ratios could be for the extreme termination cases.

Then we have to figure out what the maximum current, the minimum current in your transmission line could be maximum voltage, and the minimum voltage should be, then based on that you can design your insulation for your transmission line to support that voltage level. And, you can also support I mean make sure that the dissipation of the heat in your transmission line is taken care of when the maximum current is flowing without melting any part ok.

So, from a design point of view, calculation of VSWR is very important. From a diagnostic point of view VSWR can tell you, what kind of load termination is present, whether it is an open circuit short circuit or anything in between it could tell you that ok? So, with that we will stop.