Computational Electromagnetics: Review of Vector Calculus
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FDTD: Materials and Boundary Conditions
Lecture - 13.17
Sources in FDTD - Currents

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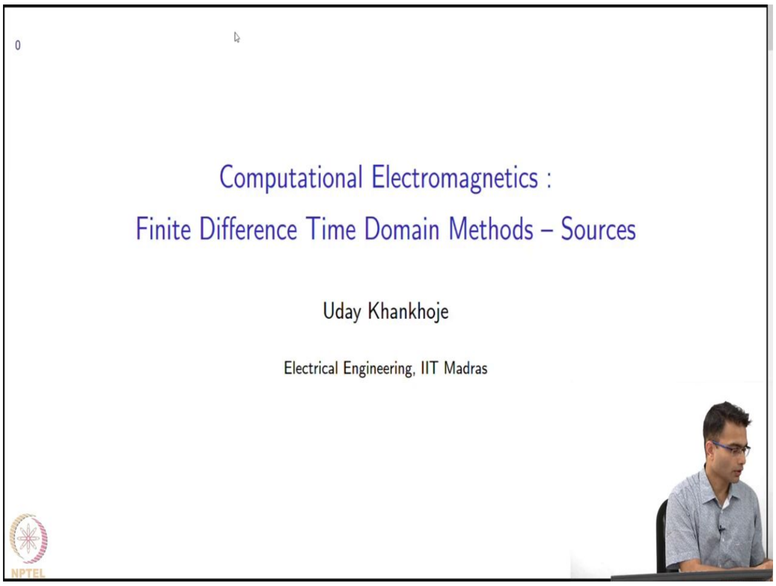

So, the final module to conclude FDTD is we have spoken about everything except how to deal with sources right. If you give me a current source or an incident field, how do I actually get this into the equation right. So, that is the sort of final module here in FDTD right.

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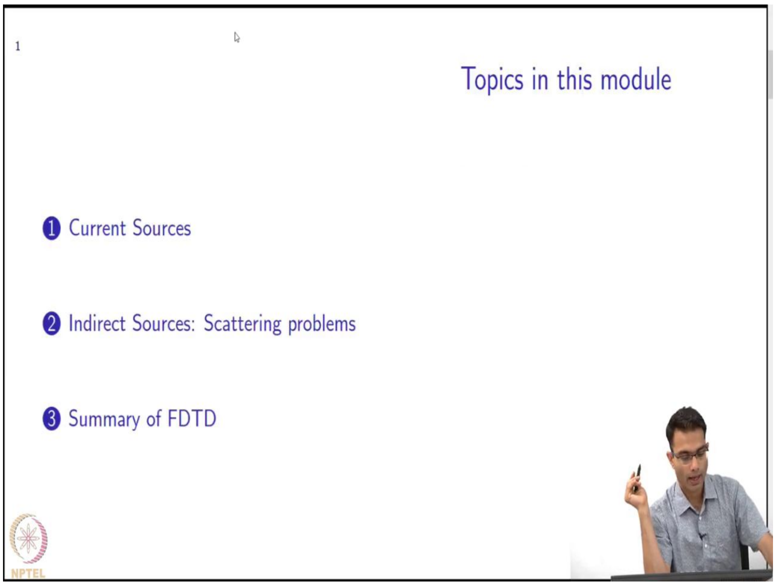

And it turns out that there are two different classes of problem that we have to worry about ok. It is almost like two different research communities, one is like the antenna community where you specify the current source. You say here is the current source this much value so many amps is there at this place in time and space.

And the second one are the people who do scattering problems. So, when I am calculating the radiation cross section the radar cross section of an aircraft, the illuminating antenna is so far away that I am not going to incorporate it in my simulation domain, all I know is that there is an incident field right. So, these are two different approaches and we will talk about how to handle both of them for FDTD ok.

So, we will deal with the current sources first right.

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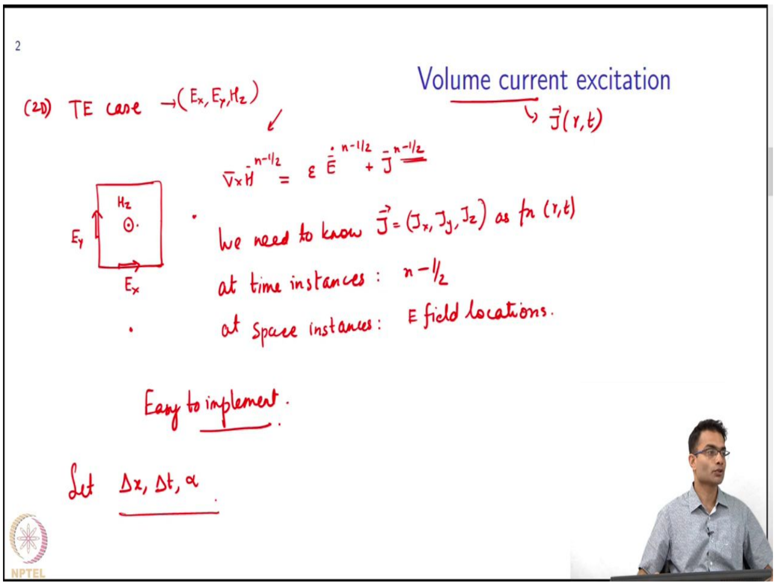

So, this is an easy part right this is coming straight out of Maxwell's equation ok. So, a volume current excitation is simply given by term that we all know J at some point in space and time ok; so, for example, if I take the TE case right 2D TE case. So, TE case what are my variables? Now (E_x, E_y, H_z) always confusing right so, let us write down what the stencil looks like.

This is my E_x, this is my E_y and my H_z is at the center and what was our update equation? I had $\nabla \times \vec{H}^{n-1/2}$, we have already we have just recently written this. So, we can use you know

$$\nabla \times \vec{H}^{n-1/2} = \varepsilon \dot{\vec{E}}^{n-1/2} + \vec{J}^{n-1/2}$$

So, in terms of if I take the components over here what happens? So, this is a vector remember these are all vectors. So, how would you implement it? So, we need to know $\vec{J} = J_x\hat{x} + J_y\hat{y} + J_z\hat{z}$ also, but J_z will actually go into the other polarization see we have written.

So, if I know this as a function of $\vec{r}$ and t, then wherever my update equations are I just simply add this value of $\vec{J}$. what is the most natural place, a natural position in space and time where I should add this or incorporate this into the update equations? Along with $\vec{J}$

actually sorry along with $\vec{H}$. So, I mean, first of all we should know it at half integer time instance right, but if I am, if you are given, if you give me $\vec{J}$ as a function of space and time then wherever it appears I just substitute it right.

So, you are right. So, this is at time instances n minus half, but you are right that at the space location it is going to be the electric field. Because here when I have a space derivative of magnetic field, I am going to get the finite differences right. So, H_z from here or left and right whatever; so, it is going to be more accurate at the $\vec{E}$ field locations right at space instances. Yeah electric field derivative at $n-1/2$ captures everything right.

So, this is nothing very difficult over here. Now it seems that that is all there is to it just put in the value of $\vec{J}$ and you are done ok. Now the question interesting question is, is there going to be any constraint on this J? So, in our FDTD simulation let us for now forget PML and all that right. What are the parameters that you have to set to run your simulation correctly and get the correct answer? What do you have to decide before you run the simulation? Courant stability factor right; so, I have to set delta x delta t and alpha right; alpha should be such that its stable right this is what I have to set.

Now, the question is, is there any relation between how fast a J or any relation between the current term and these parameters or does not look like? It should not have frequency components right very good right.

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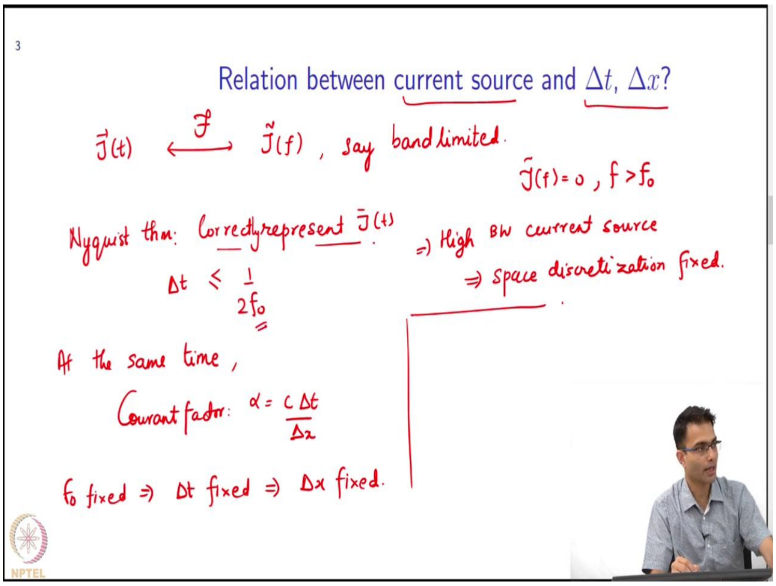

So, is there a relation between the current source and Δt, Δx right? So, your J, let us just take it as a function of time right, we will have some Fourier transform right. So, if I write down the Fourier transform over here, it will have some $\tilde{J}(f)$ and let us say that it is band limited let say it allows only a maximum frequency variation up to some number right. So, say it is band limited. So,

$$\tilde{J}(f) = 0,\ f > f_0$$

Say f_0 is the maximum frequency. Then by the Nyquist theorem right in order to correctly represent this $\vec{J}(t)$ is there a constraint on the time samples? Yes right. So, what should the time sampling be? So, Δt is there and twice right. So, that is going to be $1/2f_0$. Now, this delta t should what should be greater than or less than this? Less than this right I have to sample at least the Nyquist rate in order to correctly rep correctly represent this thing over here and at the same time we have our Courant factor right. So, Courant factor what does it say? $\alpha = c\Delta t/\Delta x$, this is bounded depending on the dimensionality.

So, given a current source I get a limit on Δt that, Δt cannot be any greater than $1/2f_0$ then. So, the Δt is fixed now you have also got fixed your α right. So, therefore, your space step is also fixed right. So, f_0 fixed implies Δt fixed implies Δx fixed right because in the FDTD space and time are very strongly related, you cannot independently do whatever you want right.

So, if I have a very high bandwidth current source right. So, implications is that if I have a very high bandwidth current source, space discretization must be very fine right it sounds counter it sounds like what is the relation, but you can see the relation is via the courant parameter right space discretization. So, you have to do this calculation beforehand because once you coded up your update equations your alpha is fixed. So, the moment you give me Δt, my Δx is fixed right.

Now if I am going to change my current source and make it to have higher bandwidth what will happen? All your update equations will run, stability will be there because you have fixed alpha correctly you will get some answer and that answer will be wrong. Because your current source only is not being implemented correctly right its not there is not enough resolution in the time step there to capture the full characteristics of the current source right. It will capture a band limited version.

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(Refer Time: 10:04) version right yeah. So, all these kinds of things will happen and it will be very hard for you to debug what exactly happened because you skipped signals and systems. So, this is a very important aspect in setting these parameters correctly.

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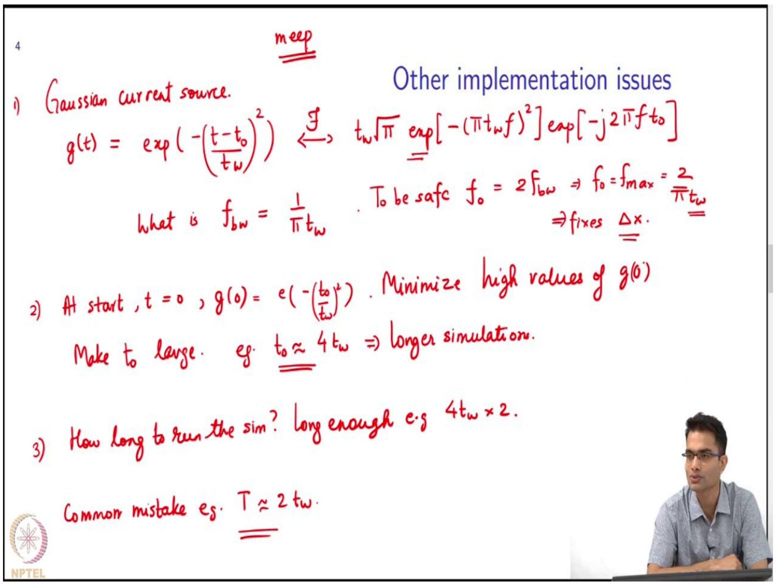

So, let us take an example, a popular kind of implementation of a current source ok. So, you use a Gaussian current. Gaussian current source is very popular ok.

$$g(t) = \exp(-((t - t_0)/t_w)^2) \leftrightarrow t_w\sqrt{\pi}\exp[-(\pi t_w f)^2]\ \exp[-j2\pi f t_0]$$

So, for example, it is a Gaussian forget the space dependence ok. So, by controlling this t_w, I can make I can play around with how sharply the current sources are right. So, typically, supposing you wanted to implement a current source which was on at some particular time instance.

If you implement it as a step function what is the problem? Supposing there is an on off I mean there is a current source you want to implement and you implement it as a delta function or as a step function. 0 for so much time, then on and then 0 what will happen? If the frequency components are so large that I will I mean what we just saw previously, my f_0 becomes very large. So, instead of doing that I implemented in a smooth way.

So, the Fourier transform of this is Fourier transform of a Gaussian is a Gaussian that is one of the reasons you choose a Gaussian way to implement a source right. So, I will just write down the Fourier transform. This is in the frequency domain ok.

So, which of these terms is giving me the sort of 3 dB bandwidth? I mean how do I know the width in the frequency domain which term is going to or what is the width what is the bandwidth roughly right? So, it is going to be $1/\pi t_w$ right after that it begins to drop off in amplitude that is what we are looking at right there is a carrier frequency over there also.

Because at $1/\pi t_w$ what happens to the value of this exponent? It drops by $1/e$ right that is how we define bandwidth roughly right. So, that is um. So, what we can do is to be on the safe side? We can set f naught the maximum frequency maybe $2f_{bw}$ right. So, if I for a if I filter this signal in frequency domain at $2f_{bw}$ I have captured most of the signal right that is a safeguard ok.

So, that implies that f_{max}. So, its $f_0 = f_{max} = 2/\pi t_w$. So, this is my maximum frequency and this fixes indirectly this fixes Δx right. Just as we did over here, once I fix my f_0 I get my Δt, I use the Courant parameter to fix Δx ok. So, this is just an example.

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It is a very popular way of modeling a current source that is the reason we are doing it ok. So, you can control how many frequencies are in this in your current source by playing around with t_w right. You can implement a very broadband source or you can imply implement a narrow band source we are playing around this t_w right because narrow in time is wide in frequency and vice versa. So, I can so, it is very very popular right you will not want to implement it via step function, you will use something like this ok. So, this is showing you how to sort of the issues that you have to keep in mind right.

So, in other words to fix your Δx you need to keep an eye on this t_w parameter that is the implication. So, this was one. Another aspect is what happens when you start the simulation right. So, at the start at $t = 0,\ g(0) = e^{-t/t_w}$.

So, again here is a factor suppose your simulation starts at time $t = 0$ suddenly you have a very large value of current right that is a source for a lot of numerical instability, and this will give rise to higher frequency components right. Because as far as the simulation is concerned at $t = 0$ let us see set everything to 0, then next time instant Δt suddenly you are setting $g(0)$ to be so, so high right. So, this in turn may give rise to higher frequency components right. So, what is the fix for this? If you wanted to minimize if you want to minimize high values of $g(0)$, simplest fix is increase t_w right..

So, make t_w large right. So, for example, t_w is going to be fixed. So, not t_w, t_0 right. So, t_0 for example, I can fix to be say $4t_w$, I do not want to play with t_w for this; t_w is what is controlling the characteristics of my source narrow band wide or wide band or whatever, but t_0 supposing I fix to $4t_w$ then what happens is this is e^{-16} which is a very low number. So, my simulation starts very gradually right I do not get encounter very high frequency numbers or whatever right. What is the price I am paying? There has to be a price by doing this.

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Discretization is how does that change with t_0? Discretization is fixed the moment I fixed t_w, what is the price that I am paying by setting t_0 equal to this? My simulation time has increased, I have to start all the way from when the source is very low in value, I could not have started just at the peak. I have to go back to the tail of the Gaussian wait for it to build up, then I get a smooth frequency distribution everything right.

So, the price is longer simulations and the final sort of thing to keep in mind is, we have not spoken about how long to run the simulation for right. How long should it be? So, similarly you should go for an equal amount on the other side of your t_0 right. So, you could go from $4t_w$ before the pulse maxima, and another $4t_w$ after the pulse maxima right.

So, the answer is long enough example $4t_w \times 2$. So, a common mistake could be right. So, common mistakes to avoid is an example I set the total time equal to let us say $2t_w$ right its everything is coded up your time step is correct space step is correct everything is correct, but you forgot to run the simulation for long enough. So, you just simulated some very small part of the pulse and you have not excited because those frequencies were not generated you have not actually been not able to study your problem properly right.

So, these are all the further aspects that you have to keep in mind after you have coded up perfectly right. So, even a perfectly working code may give you wrong answers, if you do not, for example, fix your t_0 correctly or T correctly which the simulation will not take simulation will give you some number ok.

So, there is a lot of research that has been done into how to define the current source as a function of time which I am not going into, but a lot of work has been done and this software which I have mentioned to you earlier Meep as many different ways in which you can choose the current source best suited for your application we will come to it later. So, this is what would be called a hard source right is it fine? Ok.

So, we will close it over here.