Computational Electromagnetics
Prof. Uday Khankhoje
Department of Electrical Engineering
Indian Institute of Technology, Madras

FDTD: Materials and Boundary Conditions
Lecture – 13.15
Implementing PML into FDTD - Part 1

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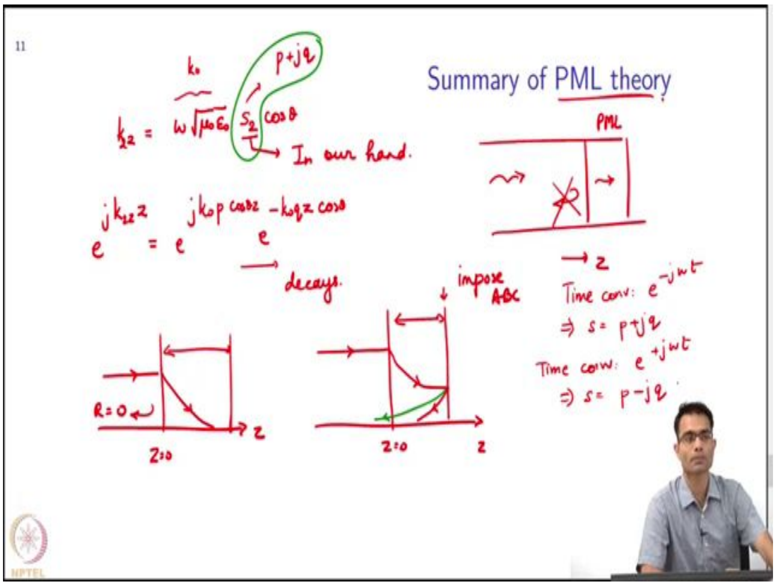

Right so, this is what we are seen so far of a sort of summary of the PML theory right. Conclusion was that just by changing the coordinate stretch parameters I can mimic a lossy medium with 0 reflections that was our. One thing I wanted to point out right in this derivation that we did we assumed a time convention in this derivation of $e^{-j\omega t}$.

So, our time convention was $e^{-j\omega t}$. This led us to choosing our s of the form $p+jq$ right because its $e^{-j\omega t}$. So, the space term has a $+jkx$ right. Now if I were to have the opposite time convention $e^{j\omega t}$ then my s would be $p-jq$ ok.

So, these are some of the very simple implementation issue you should keep in mind otherwise you will find that nothing works in your code all right. So, what we have discussed about PML so far is independent of how I will implement it right. So, finally, after having done all of this theory the payoff is how do I actually implemented in my method. So, that is what we are going to talk about next right.

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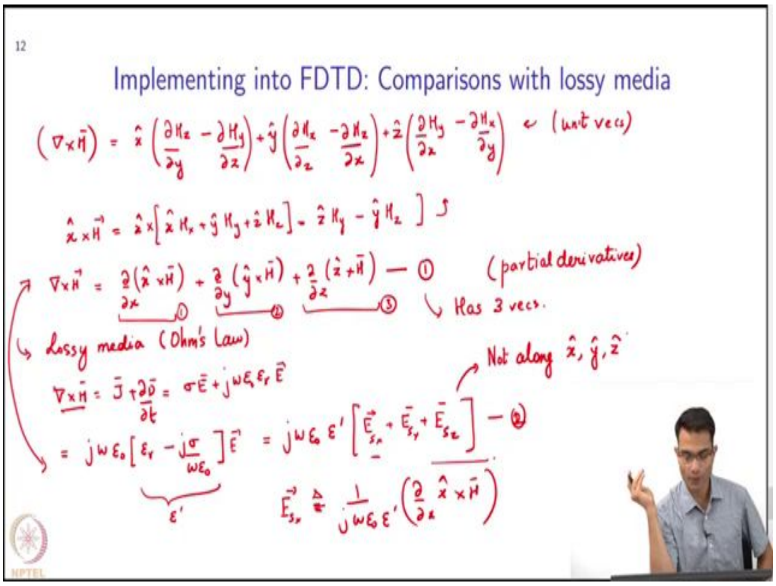

So, implementing it into FDTD and turns out its actually very simple just depends on us looking at Maxwell's equation a little bit differently; why because that coordinate stretching parameter it appeared in Maxwell's equation just by relooking right. So, again a simple relooking is what will help us.

So, let us do a little bit of a trick ok. So, our usual Maxwell's equations let us just rewrite them in a way that is useful. So, for example, the curl of H right; so, let us expand it out. So, the $\hat{x}$ component what all terms are going to be there that is a first term then I have a $\hat{y}$ term which is going to be $(\partial H_x/\partial z - \partial H_z/\partial x)$ and then will have a $\hat{z}$ component it should have a $(\partial H_y/\partial x - \partial H_x/\partial y)$. This is a usual way in which we write the curl right.

Now, if you remember when we had redefined our Maxwell's equations we had got these coordinate stretching parameters. Were the accompanying the $\hat{x}$ $\hat{y}$ $\hat{z}$ or were the accompanying something else when I redefined ∇_e or ∇_h? How did I redefine those?

Student: (Refer Time: 03:21).

Yeah, but it accompanied which term? The partial derivatives term right not the unit vector terms right. So, for example, so, what I can do from here is extract out the common partial derivative terms ok. So, to see that let us have a look at one other thing. So, supposing I ask you what is $\hat{x} \times \vec{H}$ ok; so, $\hat{x} \times \vec{H}$ will be. What do I get from here?

There is only a $\hat{z}$ and a $\hat{y}$ right. So, $\hat{z}$ is going to be H_y and I am going to get a $-\hat{y}\, H_z$. Do you recognize a term like this up here somewhere in the expression above? Does this sort of appear somewhere? What is I mean yeah. So, what is common to these guys? So, what is common over there the partial derivative with respect to x.

Student: x.

$\partial/\partial x$ is appearing everywhere here right. So, I can rewrite this like this. So, $\nabla \times \vec{H}$ I can rewrite now as a

$$\nabla \times \vec{H} = \partial/\partial x\ (\hat{x} \times \vec{H}) + \partial/\partial y\ (\hat{y} \times \vec{H}) + \partial/\partial z\ (\hat{z} \times \vec{H})$$

No approximations, this just rewriting. So, over here it was written in terms of the unit vectors $\hat{x}$ $\hat{y}$ $\hat{z}$ now I am rewrite. So, this is a unit vector and this is in terms of I have grouped the partial derivatives right. So, why do I why do I do this because this is now in a form that my coordinate stretching will given easy interpretation coordinate stretching will just multiply the $\partial/\partial x$ by a $1/H_x$ or $1/E_x$ right.

So, this is allowing us to see that now. So, let us look at, for example, to make the compare to make the implementation into FDTD we will use something which you already know which is how to deal with lossy media ok. So, when say let us just put this equation 1 we will put it on hold and we will see the comparison next look at lossy media and we will use something very simple we will use our Ohm's law we have already done that before Ohm's law is?

Student: J.

$\vec{J} = \sigma\vec{E}$ right; so, over there what did I have? So, $\nabla \times \vec{H} = \vec{J} + \partial\vec{D}/\partial t$. We know this and this became. So,

$$\nabla \times \vec{H} = \vec{J} + \partial\vec{D}/\partial t = \sigma\vec{E} + j\omega\varepsilon_0\varepsilon_r\vec{E}$$

which I rewrote in I can write this in terms of a complex permittivity right. So, I can write this as

$$\nabla \times \vec{H} = j\omega\varepsilon_0[\varepsilon_r - j\sigma/\omega\varepsilon_0]\vec{E}$$

So, this we have already seen we can call this as ε'. You might have guessed why we are taking recoats to a lossy media because we have already given the interpretation that PML with coordinate stretching is like a lossy an isotropic media right. So, we can use something that we already know towards over here ok. So, now, $\nabla \times \vec{H}$ by Maxwell's equations is going to be equal to. So, I have this relation over here. I can now use this equation 1 that I have written over here right. So, all of these terms accompanying the electric field are scalars right.

So, this vector over here, will you agree that these each of these 3 terms is a vector right direction is not clear I mean it depends on where H is right. So, I can rewrite this whole thing in terms of 3 vectors like this I can write this as $\vec{E}_{sx}$, $\vec{E}_{sy}$, $\vec{E}_{sz}$. So, what am I saying I am saying that equation 1 has 3 vectors right it has 3 vectors each of these terms is 1 2 3 each of these 3 is a 1 vector right and $\nabla \times \vec{H}$ finally, by Maxwell's equations is is going to be equal to this term the electric field also has 3 vectors.

So, I am defining these 3 vectors. So, $\vec{E}_{sx}$ for example, what is the definition? This is the new definition I have introduced. This will be equal to $\vec{E}_{sx} = 1/j\omega\varepsilon_0\varepsilon' (\partial/\partial x\, (\hat{x} \times \vec{H}))$ by definition this is my definition can I do that? So, I have defined I have decomposed the electric field into 3 vectors $\vec{E}_{sx}$, $\vec{E}_{sy}$, $\vec{E}_{sz}$ huh.

Student: (Refer Time: 09:32).

So, good point. So, these vectors $\vec{E}_{sx}$, $\vec{E}_{sy}$, $\vec{E}_{sz}$ they are not along $\hat{x}$ $\hat{y}$ $\hat{z}$ they are not they are along 3 arbitrary directions, what are those 3 arbitrary directions? They are given by the unit vectors $\hat{x} \times \vec{H}$ right I can decompose a vector into any 3 orthogonal directions that I want.

So, these are not along $\hat{x}$ $\hat{y}$ $\hat{z}$ 3 arbitrary directions which are defined as $\hat{x}\times\vec{H}$ $\hat{y}\times\vec{H}$ $\hat{z}\times\vec{H}$ ok. So, so far I have not done anything new except just reinterpret Maxwell's equations and lossy media right there is no mention whatsoever of PML ok. So now, next is we will look at how to implement PML. So, can you anticipate what will happen?