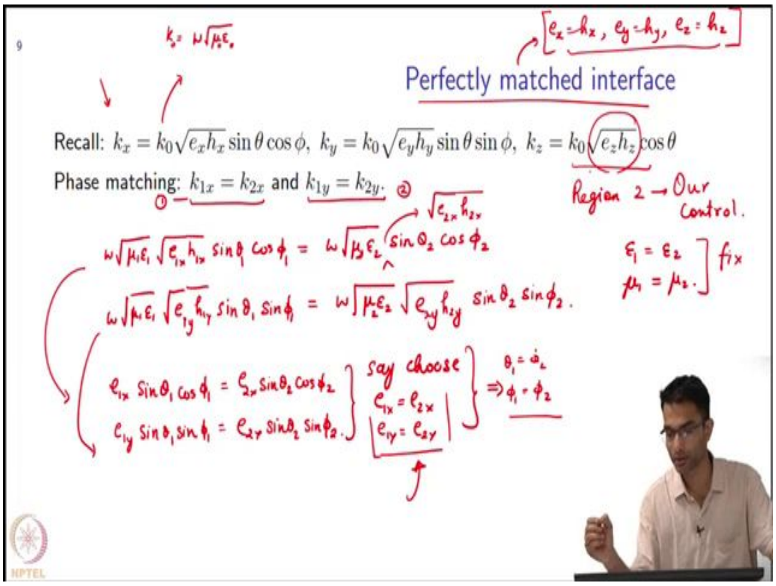
Computational Electromagnetics
Prof. Uday Khankhoje
Department of Electrical Engineering (EE)
Indian Institute of Technology, Madras

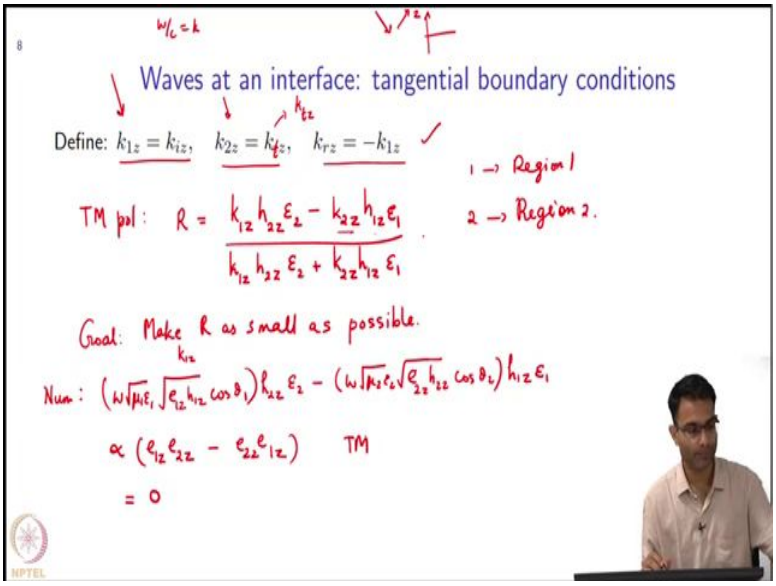
FDTD: Materials and Boundary Conditions
Lecture – 13.14
PML theory – Summary

(Refer Slide Time: 00:14)



Now let us go back over here supposing we choose we make a further simplification choose $e_{1x} = e_{2x}$ and $e_{1y} = e_{2y}$ right that is the one way of making sure I mean. So, what will this together imply $\theta_1 = \theta_2$ and $\phi_1 = \phi_2$ right. Now once, but I have not said anything about e_z right now the final thing that is left is to look at the reflection coefficient. So, let us look at the reflection coefficient over here what is this reflection coefficient look like?

(Refer Slide Time: 01:07)



So, $k_{1z}h_{2z}$ is there and $k_{2z}h_{1z}$ is there. $\varepsilon_1 = \varepsilon_2$ so, forget it. So, let us just look at the numerator right. Now what was k_{1z}? Here right k_{1z} has its expression over here. So, let us write it down over here.

$$(\omega\sqrt{\mu_1\varepsilon_1}\sqrt{e_{1z}h_{1z}}\cos\theta_1)h_{2z}\varepsilon_2 - (\omega\sqrt{\mu_2\varepsilon_2}\sqrt{e_{2z}h_{2z}}\cos\theta_2)h_{1z}\varepsilon_1$$

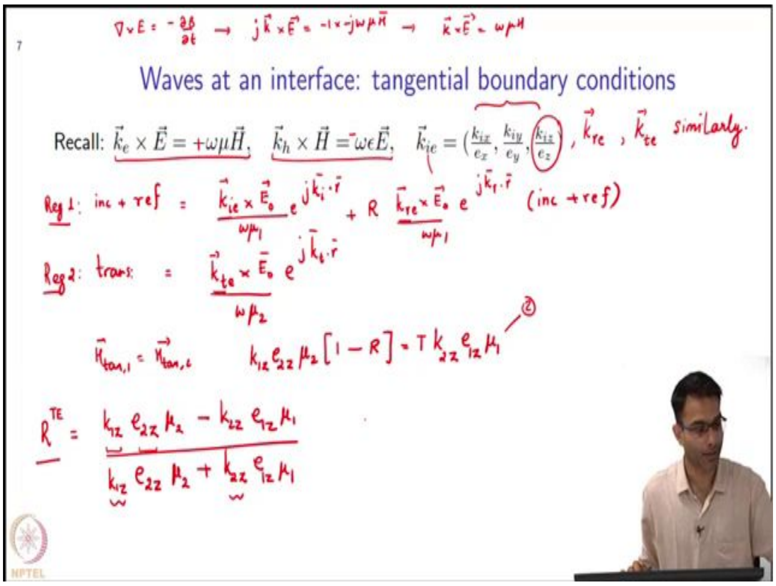
By choosing $e_{1x} = e_{2x}, e_{1y} = e_{2y}$ I get to fix the angles to be equal right. So, now in this expression what can what all comes out common right. So, this so, this simplifies to. So, let us just write (since $h_{1z} = e_{1z}$, $h_{2z} = e_{2z}$)

$$\alpha(e_{1z}e_{2z} - e_{2z}e_{1z})$$

Which is what? 0! Did I have to specify a particular angle? I have managed to make sure that this reflection is 0 at every angle right where is the trick then where is the catch I have made almost everything equal.

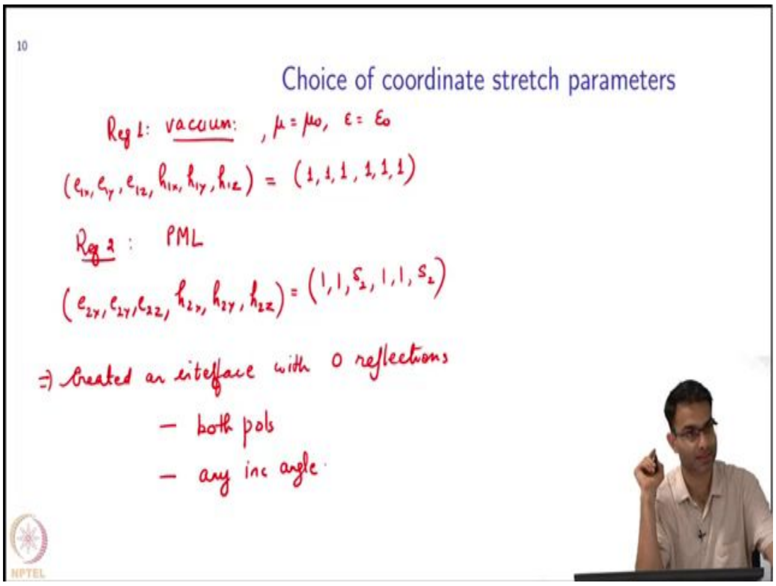
$\varepsilon_1 = \varepsilon_2, \mu_1 = \mu_2$ that help me to get rid of all of these factors over here, but where is the how is the medium 2 different from medium 1 I fixed even over here I even fix these guys to be the same where is the free knob. If everything is equal then there is no interface the z part right I have not said anything about the z part because I got $e_{1z}e_{2z} - e_{2z}e_{1z}$. So, whatever value they are it cancelled off. So, this was in the case of TM.

(Refer Slide Time: 03:58)



If you look at the expression for TE same thing you will notice happening over here $k_{1z}e_{2z}$ over here. So, I am going to get this equal to 0 the reflection coefficient for TM and TE polarization. So, is polarization independent reflection coefficient at any incident angle and how did I get that any incident angle? By this choice of stretching parameters right.

(Refer Slide Time: 04:30)



So, let us write down our choice of stretching parameters right. So, if region 1 let us say its vacuum. So, $\mu = \mu_0, \varepsilon = \varepsilon_0$ and $(e_{1x}, e_{1y}, e_{1z}, h_{1x}, h_{1y}, h_{1z}) = (1, 1, 1, 1, 1, 1)$. So, everything has to be 1 because I am trying to model a physical medium this is the inside of my computational domain.

Now, when I come to region 2 is where the fun lies right this is my. So, called perfectly matched layer L is for layer. So now, what choice do I have? So, $(e_{1x}, e_{1y}, e_{1z}, h_{1x}, h_{1y}, h_{1z}) = (1, 1, s_2, 1, 1, s_2)$. So, by doing this what have I done I have created an interface with 0 reflections not just I mean under two cases for both polarizations and any incident angle right it is almost like magic.

Its ok? So, the question is about where do these parameters go in the simulation before. So, we will have a another set of discussion on implementation details there we will talk about it, but at a high level what will happen is the update equations have to be modified because I have modified Maxwell's equations. When I modify the update equations these parameters enter inside the simulation, but physically what has happened physically I have actually created some kind of a anisotropic medium which is accomplishing this magic that is what has happened ok.

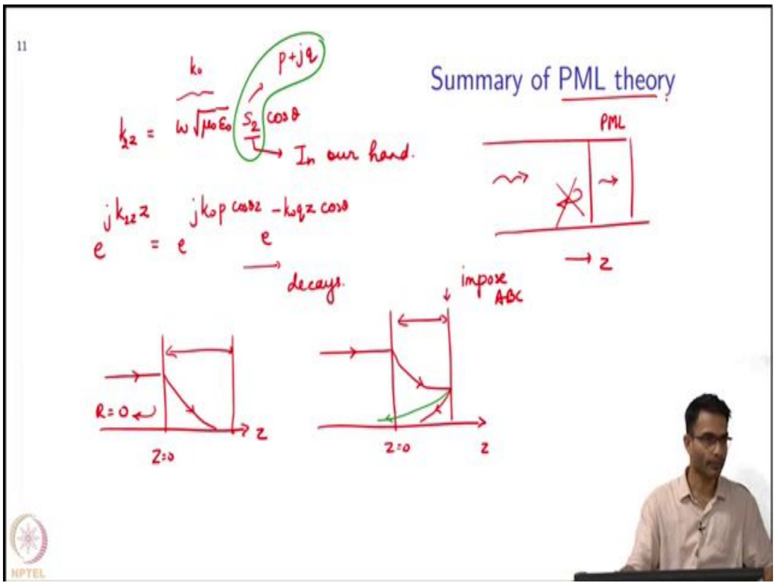
Student: The opening set of Maxwell's equations which have somewhere.

Yeah.

Student: Is that variant we just 1 inside and s_2 outside.

s_2 outside yeah exactly yeah. So, this is this is what we do now so, far I have not told you anything about what this s_2 should be ok. So, that is actually something that is you know in our hands.

(Refer Slide Time: 08:24)



So, let us sort of summarize this PML theory over here this s 2 parameter over here where does it appear? It is this guy right. So, when I write down over here $k_z = \omega\sqrt{\mu_0\varepsilon_0}s_2\cos\theta$. s_2 is in our hand. Now a wave that is travelling a wave has travelled from a simulation domain over here let us call this the z axis a wave has travelled like this is and there is no reflection as we have seen there is no reflection.

So, that wave has to go somewhere. So, it has now its travelling in the in the in the PML layer it is travelling and its propagation constant along the z direction has this k_{2z} over here and a what do I want for this. I mean a even this PML layer that I have introduced over here this has to it has to be a finite layer right. So, I mean I have to stop it at some place. So, how do I how do I mean how would should I choose this s_2 ?

Student: Such decay.

Such that it decays; that means, s_2 should be no s_2 should not be close to 0.

Student: Take.

s_2 should be complex right because the how do I write the I mean the wave vector over here $jk_{2z}z$ that is what I would have other terms are also there right. Now if I make this complex right if I make this as $s_2 = p + jq, k_0 = \omega\sqrt{\mu_0\varepsilon_0}$ that is what happens right $e^{jk_{2z}z} = e^{jk_0 p\cos\theta}e^{-k_0 q}$; that means, as the wave is travelling in plus z over here it decays.

So, what has happened is that it's entered inside at z=0 is the interface right at z=0 is entered now as z increases its value decreases right. So, if I were to plot as a function of z what the PML is actually accomplishing let us say that this is z=0 and this is like the amplitude of the wave this enters inside it begins to decay.

Student: Decay.

This is my PML right. So, it has decayed and there is no reflection right. So, R=0 ok, but what do you have to ensure? We have to ensure that the width of this PML layer which is in your hand had better be large enough that the wave decays inside the wave supposing it does not supposing you know you have only so much RAM on your computer and you can only fix you know thin layer.

So, then what can happen is let us redraw it over here shorter. So, it came over here it decayed then what, then first of all. So, z=0 first of all I need to now impose a boundary condition here otherwise how will I terminate the wave. So, what boundary condition can I do?

Student: Robin boundary.

Robin boundary condition what did you say yeah our usual ABC absorbing first order ABC I can apply it over here that works anyway for I know it does not work for evanescent waves, but still it does something. So, I can either I can impose a ABC over here because I have to I have to specify something. So, there will be some reflection now as this some reflection this will further decay.

Student: (Refer Time: 13:00) add a (Refer Time: 13:01).

Add a.

Student: The radiation boundary.

The radiation boundary condition, where we replace this partial the special derivative by the plane wave thing the; what we have we use in the case of FEM we use in the case of FDTD right, so, decays over here. In case this layer is not is not is very thin then what will happen is you may have a wave that enters back into the computational domain if the layer thickness is thin right.

So, it is important to understand supposing I code up perfectly a PML and I find that there is a wave still coming from the PML its it could it, its not necessary that my coding was incorrect it may be just that the layer thickness is so, small that I am getting a reflection from the backend of the PML right. So, both cases are possible depending on depending on the thickness of the layer and this how I set this parameter right.

So, lot of the design goes into how do I set this people as j q term ok. So, this we did not take any recourse to FDTD for this.

If you make p very high.

Student: q.

Sorry q is what you want to make very high right the loss part may not p.

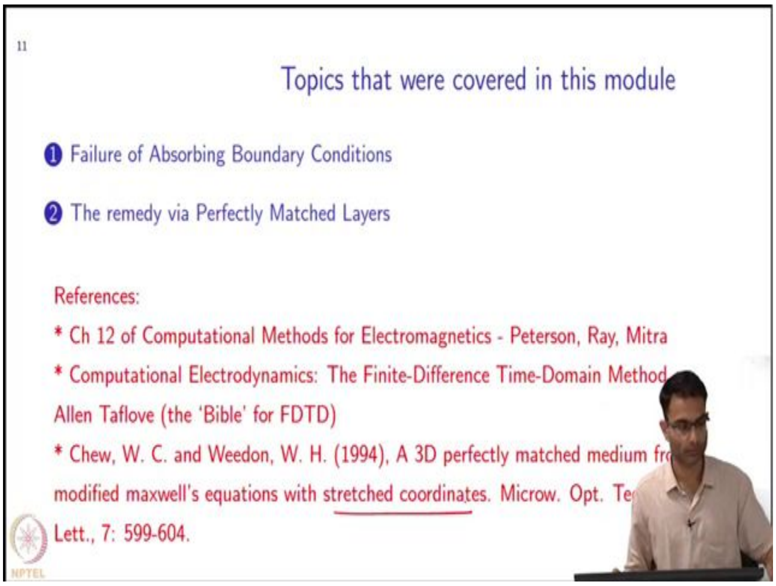
Student: The basic very well did not (Refer Time: 14:23) because.

Yeah. So, the question is yeah. So, theoretically it seems that if I make q to be very large then it will decay very sharply, but there are certain numerical issues that happened when I do that. In the perfect world this is great right, but even when I go the moment I implement it numerically there are still some reflections that happened this abrupt change never works. So, actually many people what they do is even q is may is introduced in a adiabatic wave.

So, this saw I mean you get a hang of this when you code it up theoretically I can show this some graphs, but it is not interesting ok. So, that sort of brings to a close the idea of PML theory as I said in the very beginning you could have done this in 2 ways one is to stay within the realm of physics have two different materials one normal material one anisotropic absorb it gives you the same results. Our approach was by using coordinate stretching; coordinate stretching was interpreted as a anisotropic absorber ok.

And we found that by playing around with just 1 parameter and getting a very beautiful result no reflection and any polarization at any angle ok. So, because of this elegance of this approach it is become standard in both FEM and FDTD.

(Refer Slide Time: 15:56)



So, the paper main the main paper which are referred to for explaining the philosophy of PML using stretched coordinates is this 94 paper by Weng Cho Chew. So, do read its very readable right based on whatever we have done you should be able to read the paper in one shot ok. So, in subsequent modules we will look at how do we actually implemented, but the hard part is over I ok.

Student: (Refer Time: 16:23).

We will.