Computational Electromagnetics
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FDTD: Materials and Boundary Conditions
Lecture -13.13
Perfectly Matched Interface

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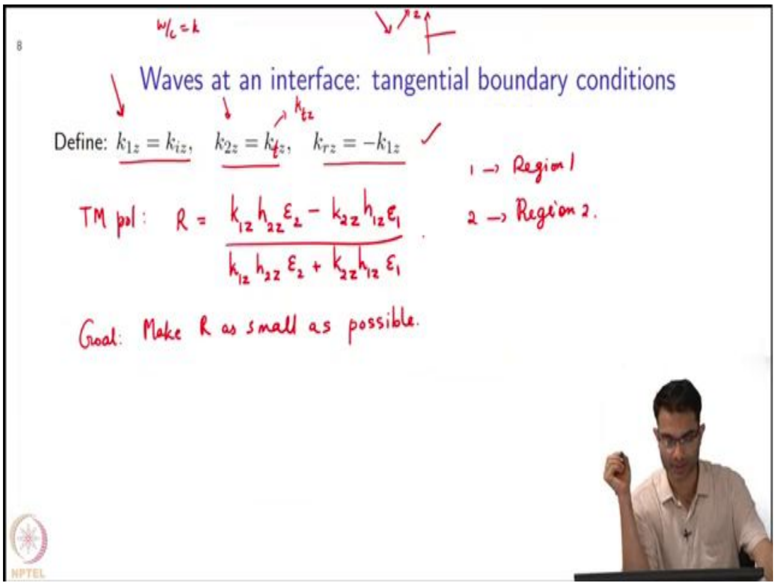

So, let us see what values we can set for it ok. So, now, proceeding further towards our actual objective, what we did right now was the general derivation.

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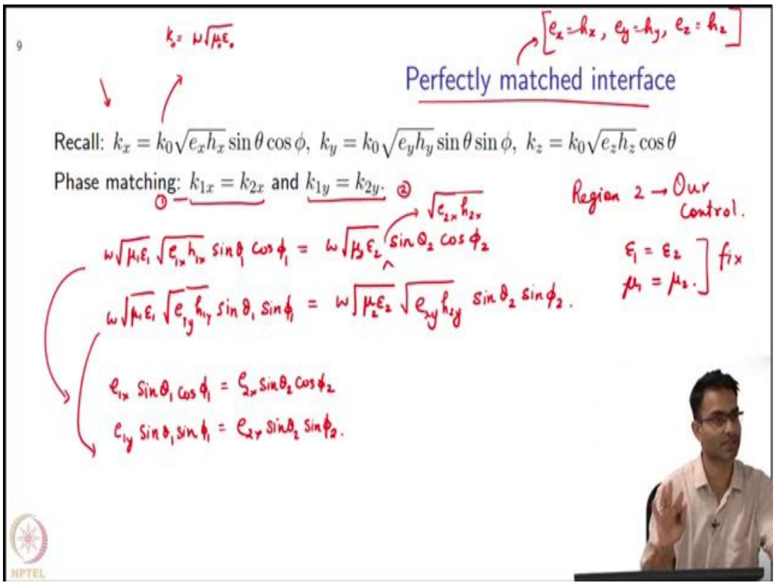

So, now we are going to look at what is called a perfectly matched interface and this is by means of revision. So, what was our $k_x = k_0\sqrt{e_x h_x}\sin\theta\cos\phi$, we had defined it in this way this was in the previous few slides remember this came from the what was the motive, motivation?

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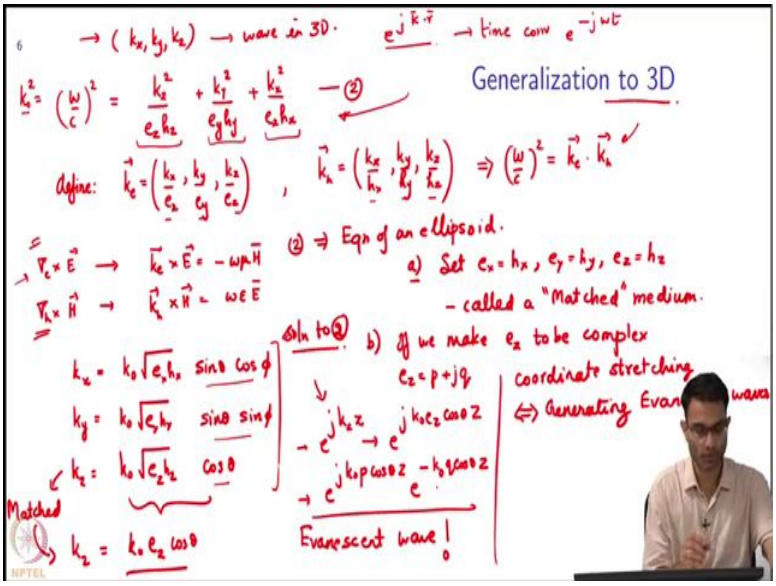

We can show you that equation once again equation 2 over here that was a equation of a ellipsoid. And the solution to the ellipsoid will give me over here the length of the semi major semi minor axis are like this, and in parametric form I got a ellipsoid right.

So, and this $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$. So, I am just rewriting this for your convenience this is my $k_x = k_0\sqrt{e_x h_x}\sin\theta\cos\phi$, $k_y = k_0\sqrt{e_y h_y}\sin\theta\sin\phi$ and $k_z = k_0\sqrt{e_z h_z}\cos\theta$. And what is this k_0 in general going to be $k = \omega\sqrt{\mu\,\varepsilon}$. So, this is this is in general k ok. So, we will just keep this in mind because there are 2 regions have different values of μ_1 and ε_0 ok. So, now phase matching condition required this and this right because the phases I have to rotate at the same speed ok. So, what does this tell us? So, we will just substitute over here.

$$\omega\sqrt{\mu_1\varepsilon_1}\sqrt{e_{1x}h_{1x}}\sin\theta_1\cos\phi_1 = \omega\sqrt{\mu_2\varepsilon_2}\sqrt{e_{2x}h_{2x}}\sin\theta_2\cos\phi_2$$

We are calling this perfectly matched; perfect matched means what? We have taken it is our choice we take $e_x = h_x, e_y = h_y, e_z = h_z$ reduce the number of (Refer Time: 02:38) variables right.

So, what are these just so, that you do not get confused what are these θ's and ϕ's over here? Do they correspond to any physical angle? They could right i mean just take the simple case $e = h = 1$ everywhere that is your free space right. So, $k_z = k_0\cos\theta$. So, if I have a wave vector between angle theta with the z axis, giving me the z component of the wave vector right.

Now, region I and region II will the waves be travelling at the same angle? No right. So, I have that is why I have put θ_1, ϕ_1 in region and θ_2, ϕ_2 in region II. I am not getting into what the precise values are because they are not important ok. So, this is the first condition; what about the second condition its going to give me?

$$\omega\sqrt{\mu_1\varepsilon_1}\sqrt{e_{1y}h_{1y}}\sin\theta_1\cos\phi_1 = \omega\sqrt{\mu_2\varepsilon_2}\sqrt{e_{2y}h_{2y}}\sin\theta_2\cos\phi_2$$

So, what why is this thing called a perfectly matched interface and the reason is very very literal perfectly matched. So, perfectly matched meant these things that is what we have defined in a few slides ago, but perfectly matched interface. If you did not know anything about fancy PMI and I told you interface is perfectly matched what would you think?

Student: (Refer Time: 05:07).

Impedances are matched right. So, in other word what is the simplest way to match impedances make $\varepsilon_1 = \varepsilon_2, \mu_1 = \mu_2$ let us suppose we do that right. So, what so, these equations they give further simplify.

$$e_{1x} \sin\theta_1 \cos\phi_1 = e_{2x} \sin\theta_2 \cos\phi_2$$

$$e_{1y} \sin\theta_1 \cos\phi_1 = e_{2y} \sin\theta_2 \cos\phi_2$$

Any further thing, that we can do? So, what is our goal? Our goal is what is the goal of PMI? Why are we doing all of these?

Student: R=0.

R=0 right.

If I fix $\varepsilon_1 = \varepsilon_2, \mu_1 = \mu_2$ does it mean that there is no interface. If there is no interface then R=0.

Student: Yeah.

Right so, we have to see that right, but remember I mean this is now we have gone to a non physical situation because I have introduced these e's and h's right so, and they have made an appearance in the reflection coefficient. So, it is not clear whether just $\varepsilon_1 = \varepsilon_2, \mu_1 = \mu_2$ is enough to guarantee that these a reflection coefficient is 0 ok.

Student: (Refer Time: 07:40) properties of the material have changed.

The properties of the material have changed via the small e's and small h's.

Student: So, there an interface.

There is an interface yeah. There is an interface it is because, now I have expanded my idea of an interface itself.