Computational Electromagnetics
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FDTD: Materials and Boundary Condition
Lecture – 13.12
PML - Tangential Boundary Conditions

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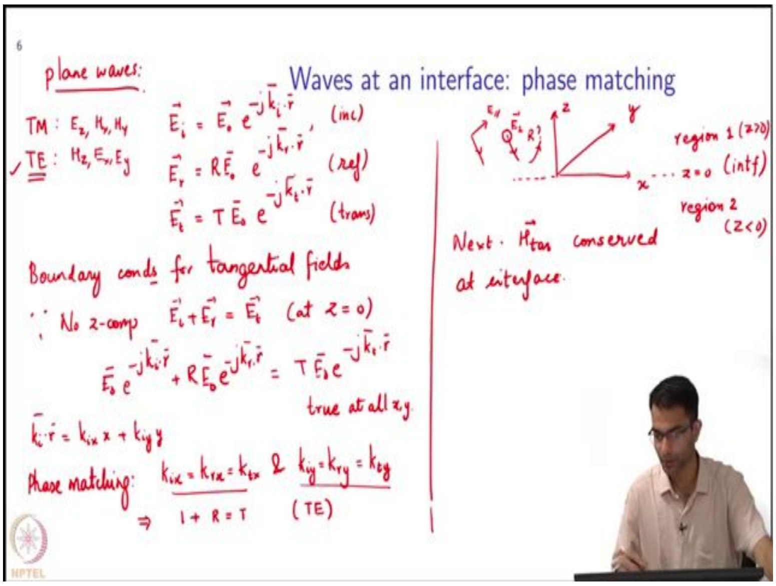

So, now, let us look at the H tangential, here is where the derivation differs a little bit from your high school derivation. It should because, I have redefined my Maxwell's equations with the coordinates stretching.

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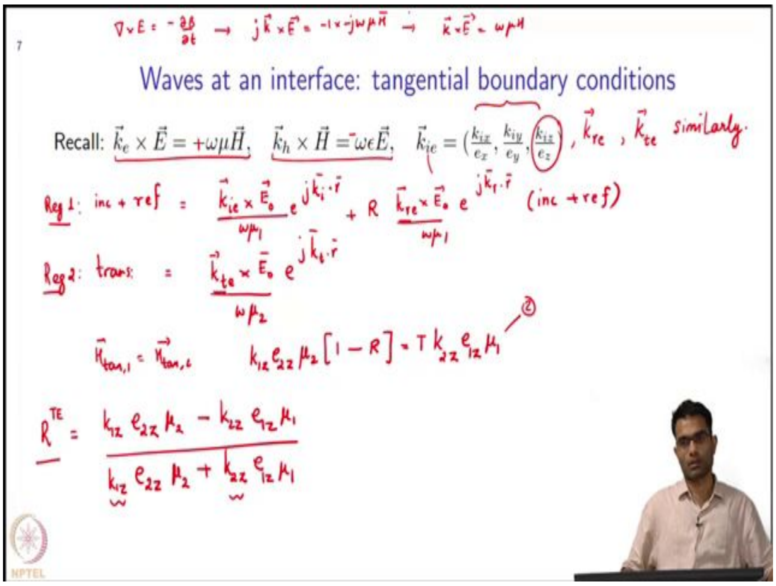

How did and how would did we do that, remember we had this equation, this was $\nabla\times\vec{E}=-j\omega\mu\vec{H}$, that ∇ became replaced by ∇_e and that ∇_e got replaced by $-jk$ right so.

Student: (Refer Time: 00:51).

For a plane wave it is exactly true, for a plane wave it is exactly true right. So, I mean we had done this earlier also, if you look back over here right.

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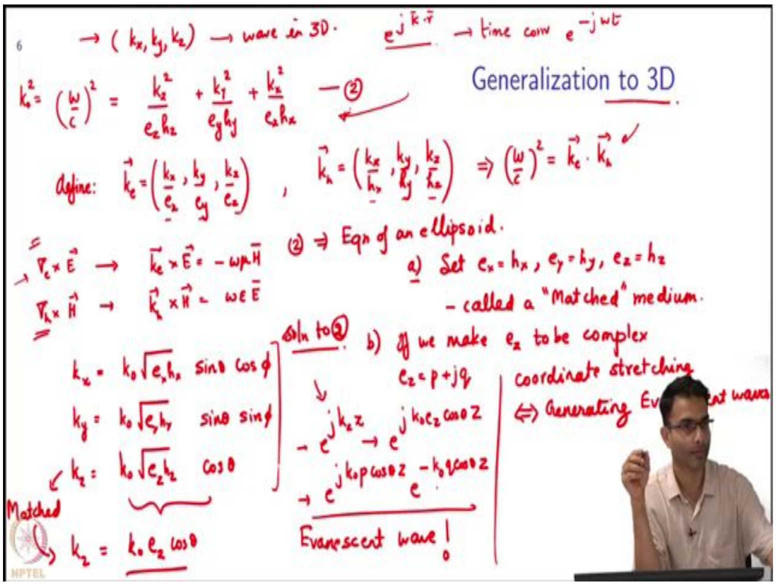

This is this is familiar. So, $\vec{k}_{ie}$, denotes the incident over here. Similarly, there will be a $\vec{k}_{re}$ and there will be a $\vec{k}_{te}$. Is that fine?

Yeah, you should check on these the signs of these two equation ok, they may be off by a minus sign but, we will just we will go with this over here because we will just verify it as we go, anyway.

So, using these equations, I can get my I have my electric field here, I have defined my electric field, from here I can get the magnetic field right. So, when I get the magnetic field from here, I am going to get so, in region 1. So, region 1 is the top part, where I have a incident plus reflected right, that is what I have. So, give me the electric field and I can give you the magnetic field that is going to be the approach ok.

So, this is going to be equal to $\frac{\vec{k}_{ie}\times\vec{E}_0}{\omega\mu_1}e^{j\vec{k_i}.\vec{r}} + R\frac{\vec{k}_{re}\times\vec{E}_0}{\omega\mu_1}e^{j\vec{k_r}.\vec{r}}$

Incase if you forgot how this came right. So, we can actually just derive it over here. So, $\nabla \times \vec{E} =-\frac{\partial B}{\partial t} \rightarrow j\vec{k} \times \vec{E} =-1 \times -j\omega\mu\vec{H} \rightarrow \vec{k} \times \vec{E} = \omega\mu\vec{H}$.

Alright very good so, this is our incident plus reflected field, for the magnetic part and in the region 2, I have only a transmitted field ok. So, this should be very easy to write down.

$\frac{\vec{k}_{te}\times\vec{E}_0}{\omega\mu_2}e^{j\vec{k}_t.\vec{r}}$. Now, I can I mean I have everything right. So, what do I need to do next, this H that I get, I mean once I get, once I have this, k in general has 3 components and my E has x, y components.

So, I can take this cross product, vector cross product, from that which term do I have to keep because, I want to enforce $\vec{H}_{tan,1} = \vec{H}_{tan,2}$. So, I should drop out which part? So, when I take this cross product over here, I will get a very simple expression.

$k_{1z}e_{2z}\mu_2[1-R] = Tk_{2z}e_{1z}\mu_1$

So, I have got my 2 equations in 2 variables right, this was my first equation over here,

$k_{ix} = k_{rx} = k_{tx};\ k_{iy} = k_{ry} = k_{ty}$

So, I mean you can trust me, this is you can see the algebra is pretty simple over here.

So, putting it together,

$R^{TE} = (k_{1z}e_{2z}\mu_2 - k_{2z}e_{1z}\mu_1)/(k_{1z}e_{2z}\mu_2 + e_{1z}\mu_1)$

Now, you notice that there is a, I need to say a little bit more about what are these, what is this k_{1z} and k_{2z} right. So, it is just and it is straight forward.

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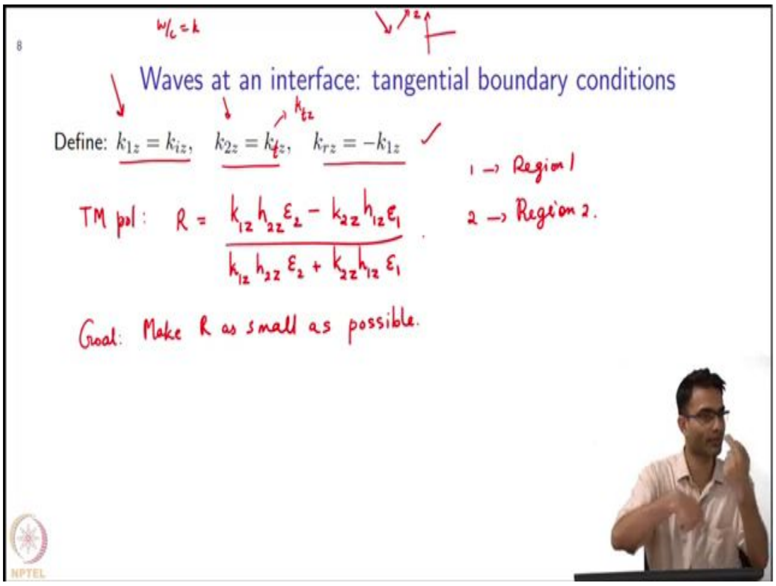

I am defining k_{1z} as, the z component for the incident wave vector right, k_{2z} Transmitted wave vector right so, that is k_{tz} ok. So, $k_{rz} = -k_{1z}$ is the part that is telling me that the reflected wave is travelling in the opposite direction as the incident wave right.

So, these are the 3 definitions that we need ok. So, this formula makes is what I get ok, similarly in the TM polarization, that is also remaining right. So, TM polarization, you would take the different conventions for electric field and I will again I will just write down the final expression ok, you will notice a symmetry between these expressions.

$$R^{TM} = (k_{1z}h_{2z}\varepsilon_2 - k_{2z}h_{1z}\varepsilon_1)/(k_{1z}h_{2z}\varepsilon_2 + k_{2z}h_{1z}\varepsilon_1)$$

I can write the transmission coefficient also but, it is not very useful because, my goal is to make R as small as possible ok. So, is it I mean is it; are you all comfortable with what we did?

Student: (Refer Time: 12:31).

Well that is depends on the medium you know.

Student: So with the vector is (Refer Time: 12:42) and we did the same as (Refer Time: 12:45).

The wave vector and transmission along the z direction will not be the same right, it depends on the propagation in the medium, if I have different epsilon mu here and different epsilon mu here, why will the z components be the same, they may not be right, it depends on the medium properties and in case you forgot this was the relation followed by the dispersion relation over here, this is what these k's are following right. That is how we set up the new Maxwell's equation as we call them right. So, I have got the R^{TE}, which has e and μ right.

So, notice I mean what is the what is the additional part or what is different here compared to what we knew forever, the new part is the introduction of the e's, if I make the e's equal to 1, I get back my high school formulas. Similarly, if I make my h is over here equal to 1, I get back my high school formulas, that is the only thing that has changed. So, whatever formula you had right that is this formula but, my knob engineering knob in my hand is this is now, new factors of e's and h's, small e's and h's ok.

Student: So one like to do (Refer Time: 14:02).

Yes 1 and 2. So, 1 is region 1 and 2 is region 2 ok. So, that is that is only the only different thing is presence of the small e's and small h's. Now, we have to do something interesting with it yeah.

Student: (Refer Time: 14:22) say k one is a (Refer Time: 14:25).

k_{1z} is the incident waves z component of the I mean, the z component of the incident wave vector, that is over here.

Student: So, in the vectors we have the only the (Refer Time: 14:42) wave vectors in e_{z1} and e_{z2}.

Yeah, the z components, they are whatever appearing only and the medium properties of course, right $\varepsilon_1, \varepsilon_2, \mu_1, \mu_2$.

The region 2 properties will influence the reflection coefficient.

Student: So, but there is no, I think there is no reflected coefficient.

k is a wave vector.

Student: Sir.

Right so, if I have a plane wave travelling in this medium for example, if you take not this stretched coordinates but, a normal medium rights over there I know that $\omega/c = k$, any wave travelling has to obey this, whether it is incident or reflected it does not matter. So, this is that k vector, the wave vector that has to be obeyed by any wave.

Student: (Refer Time: 15:50).

The reflected wave will have a negative component because, this travelling in the opposite direction as the incident part right. So, supposing I if I take a 2D view, this is the z axis over here and incident wave comes over here and the reflected wave goes in this direction, what is different between these two wave vectors, the z component, the x component in fact, does not change, that is your Snell's law, angle of incidence equal to angle of reflection, how will that come, $k_{ix} = k_{rx}$ and $k_{iz} = -k_{rz}$, that is how the wave turns around right.

Yeah, it is an our choice, I mean we are doing a general derivation, we are not restricting ourselves to magnetic or non magnetic.

Student: So the (Refer Time: 16:37) that is magnet.

We will see it is up to us, I have kept it as a general what happens you take stretch coordinates and do Maxwell's equation; this is μ_1, ε_1. Now, medium 2 is in our hands.