Computational Electromagnetics
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FDTD: Materials and Boundary Conditions
Lecture – 13.09
Perfectly Matched Layers (PML) Introduction

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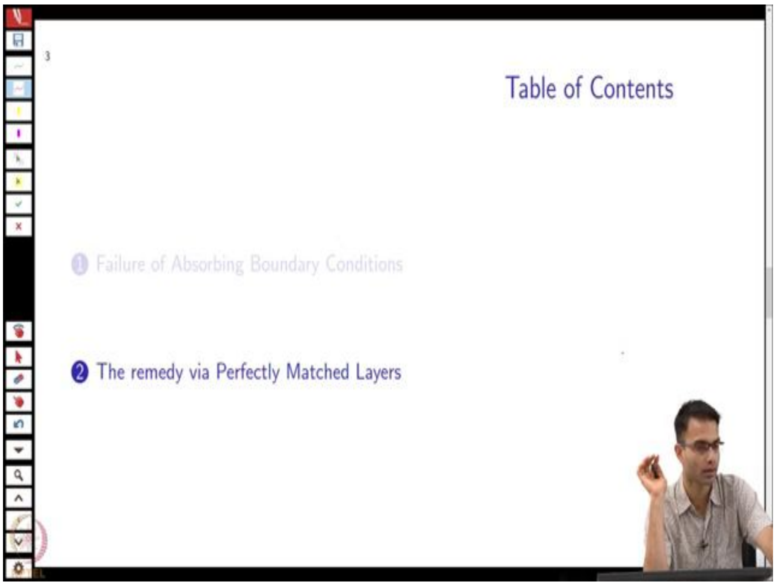

So, now let us see the remedy via this so called Perfectly Matched Layers ok.

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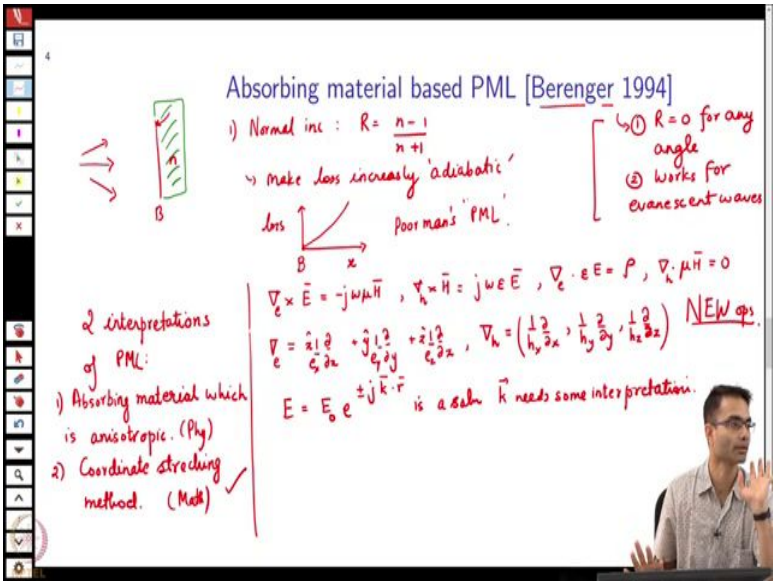

So, as I mentioned this is a relatively new development in the field proposed by Berenger in 1994 and subsequently many people have reinterpreted and reformulated the same idea ok. So, let us just sort of list out the two advantages that we will get from this ok. So, the first is going to be that it works it observes waves at all angles ok. So, $R = 0$ for any angle and the second one is: it works for evanescent waves.

So, in some sense, this is our wish list and nature is kind enough for us that this so called PML things it satisfies both these items of the visualist which is why it was a very good discovery in the field. Now, we will when we try to formulate this let us first before I formulated I want to give you a little bit of motivation ok. So, let us say that this is my computational domain over here and this wave is coming over here may be it's coming at this angle whatever variety of different angels and I am talking about let us say this boundary. So, these two boundaries are not important. So, waves are coming at all angles over here and we are finding that with ABC's they get reflected ok.

Now, if I wanted to not have that reflection one wave would be to observe whatever falls on it. If I can somehow observe what falls on it there will be no reflection right I mean this simple example if you go to a room and you speak the walls reflect, but if the walls had some absorbing material you will not be able to hear your echo that means there is no reflection. So, the idea is that maybe I can put some material over here that absorbs.

Now can I put an ordinary material? if I just put something with a large loss coefficient will it work? There is certain change in refractive index right. So, even for normal incidence the reflection coefficient is $(n-1)/(n+1)$. Now, if n is anything other than 1 there is a reflection even if it is complex right for losing medium the reflective index n becomes complex.

There is going to be reflection even though lose is there. So, how will you get around this, it does not seem that just pure absorbing material is not going to do it. Any other any creative solutions from here?

Student: (Refer Time: 03:20).

Very good right; so, one is if the loss. So, let us say normal incidence simple case reflection coefficient is $(n-1)/(n+1)$ this is n right. So, the suggestion is make n I mean make the loss increase slowly right what is a more technical word for slowly? Adiabatic we want a slow increase so it is called so, correct word is adiabatic.

So, in other words supposing this is the boundary over here right 0 and the loss part what you can do is you can increase the loss part like this right and the boundary sitting here this is your boundary. So, at the boundary, what is the value of n? 0 loss right. So, at the boundary the value of $n = 1$ right then I begin. So, the wave in some sense enters inside because there is it seems that there is nothing its slowly enters inside and by gradually increasing a loss factor you are able to gently absorb it ok. So, this is I mean some people call this a poor man's or poor women's PML ok.

Increase the loss gradually the reflection coefficient is greatly reduced ok. So, this is one; this is one reasonable way of implementing getting around the fact that all angles have some reflection. Now, we have arrived at the fact that I need some special kind of loss in this in order to satisfy my wish list right. So, you can if I tell you that the PML is a special kind of absorbing material you can see the motivation, we have already come across just by simple common sense we have come up with one kind of absorber in which where the loss increases gradually right. So, there are two interpretations of PML in the literature ok.

So, one of them is going to be what is called as extending our logic we will call it an absorbing material ok, but not just any absorbing material that material will turn out to be anisotropic. So, I am just giving you information right I am not deriving it for you ok. So, anisotropic material right now we have only been dealing with isotropic material where whichever direction you go the loss is the same, but it turns out that if I create different losses in different directions I am beginning I end up with a material that is a very good absorber the second interpretation is a more mathematical interpretation which is called the coordinate stretching method. Both of these give the same values of your ε and μ in for the PML ok. So, it is the same principle seen from two different waves; the first wave is the physics based wave the second wave is a more math based wave ok. So, this is physics this is math ok. Now which interpretation now we are going to choose?

Student: (Refer Time: 07:48).

Ok right. So, we are going to choose the math based interpretation because the math gets easier if I choose this interpretation right, the physic physics based interpretation is very appealing. But the math becomes more complicated because the moment I have anisotropic medium then I have to start dealing with tensors and all of that right, but this coordinates stretching is a very beautiful and simple way of arriving at this thing ok. So, what we do in this coordinate stretching approach is going to be very interesting.

So, what is our usual Maxwell's equation again this beauty of this method is to start with starts with Maxwell's equation. So, let us just write down our usual Maxwell's equation right. So, $\nabla \times \vec{E}$ right and let us keep matters simple we will assume $j\omega t$ time dependence right. So, this becomes $-j\omega\mu\vec{H}$. $\nabla \times \vec{H}$ I will write as $j\omega\varepsilon\vec{E}$ right and similar the other two equations ok.

Now here itself is where the coordinate stretching approach presents a very interesting way. So, what it says is let us reinterpret the del operator itself ok. So, let me write down the other two equations. So, that you know that I am going to do it to all of these equations ok.

Student: Per me.

Per me.

Student: The material.

The material.

Student: (Refer Time: 09:30).

Yeah.

Student: (Refer Time: 09:31).

No all of that is absorbed to ε. So, ε can be complex and μ also can be complex right. So, what I do is I redefine this operator itself ok, in a way that I will describe right now. So, what is this? So, ∇_e. So, let us write down ∇ first. So, what is ∇ ?

$$\nabla \equiv \hat{x}\ \partial/\partial x + \hat{y}\ \partial/\partial y + \hat{z}\ \partial/\partial z$$

Student: (Refer Time: 10:03).

Right each of these three yeah. Now what is ∇_e right as the word coordinate stretching suggests each of the coordinates is going to gets stretched by some number which I choose and what is that number?

$$\nabla_e \equiv \hat{x}\ (1/e_x)\ \partial/\partial x + \hat{y}\ (1/e_y)\ \partial/\partial y + \hat{z}\ (1/e_z)\ \partial/\partial z$$

it look strange, but that is going to be the approach. So, I am redefining the operator in this. So, I am now left the physical world away I have entered a mathematical world where I have created a new operator called ∇_e which agrees with physical world for values of $e_x = e_y = e_z = 1$.

This is small $e_x\ e_y\ e_z$ these are scalars they have nothing to do with any physical constraints similarly this ∇_h I just write it in slightly smaller a notation $1/h_x\ 1/h_y\ 1/h_z$ these are the new operators ok. So, if I use if I take these operators and get try to solve wave equation from here do you think the form of the solution. So, we know for the wave equation is a solution is $e^{\pm jkr}$ time has been dropped out of here.

If I take this interpretation over here and I rederive the wave equation what you think will happen, will I still get a wave like solution this intuitively I should get a wave like solution except what will happen this $1/e_x^2$ will come somewhere $1/h_x^2$ will come somewhere. So, I will get a wave like solution, but that wave will have that that factor of e_x or h_x whatever in the exponent right. So, you can work it out I am just giving you it is very simple ones just put this out take a 1D wave problem put it in you will find that I still get a wave like solution ok.

So, we can say that $E = E_0 e^{\pm j\vec{k}\cdot\vec{r}}$ is a solution now if this is going to be a solution. Then this $\vec{k}$ needs some interpretation ok. So, we should expect that somehow these e's and h's will appear inside this $\vec{k}$ somehow where else right it has to go some were there is no way for any of it cancel off ok. So, it has been a little vague so far, but now what we will do is drive home the point let us take a 1D example so.

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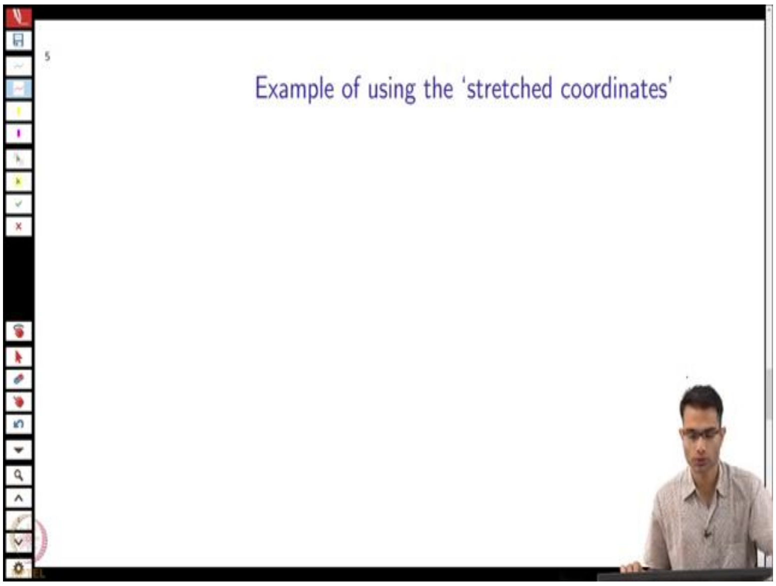

Student: Is there a (Refer Time: 13:40).

Is there a relation between e_x e_y e_z you should ask more is the relation between e_x e_y e_z h_x h_y h_z the answer is yes we get into how will that how will those relations come? I mean when we start putting in Maxwell's equation wave equations those relations will come. So you should remember one thing. This is not what nature as given as we are doing these manipulations to get these desired objectives out of it that is. So, remember it is an engineering tool kind of a thing that we are doing.

We are scaling the coordinates within the operator within the operator.