Computational Electromagnetics
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FDTD: Materials and Boundary Conditions
Lecture – 13.08
Failure of ABC

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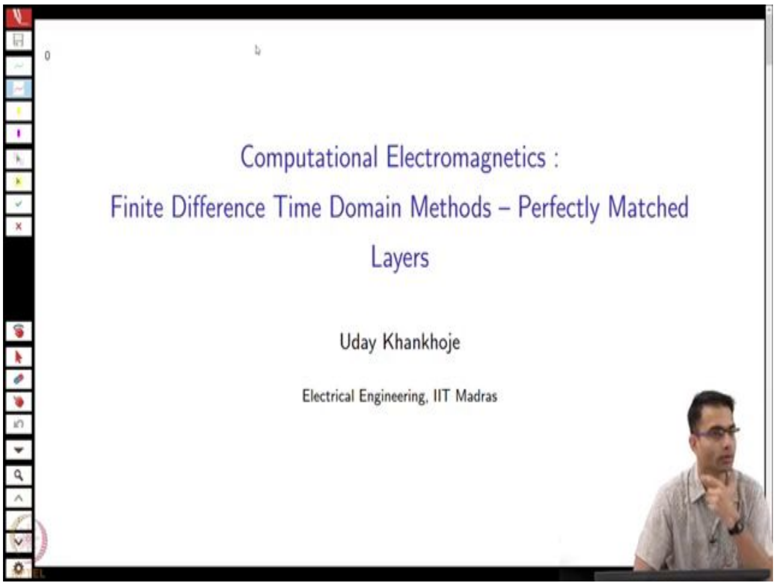

So, the next very important module that we are going to talk about is implementing first conceptualizing and then implementing what are called perfectly matched layers.

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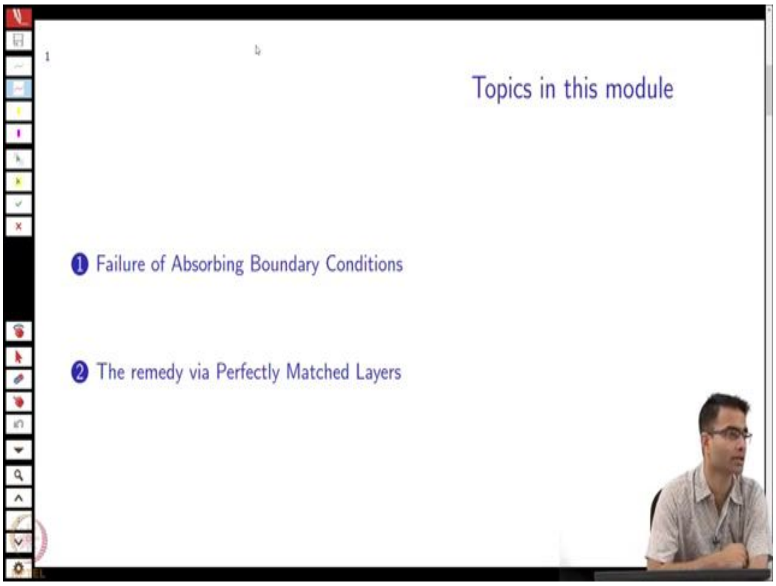

So, to motivate this first we will start with a why do absorbing boundary conditions fail and then the remedy ok, we have some motivation and then we come to perfectly matched layers. One thing I want to mention this perfectly matched layers, it is a recent sort of formulation 90's 1990's is been it was formulated it is not restricted to FDTD. Even FEM we can implement PML and people have done so ok. So, what I am telling you we will start with the theoretical development then implementation in FDTD, but we could also implement in FEM right.

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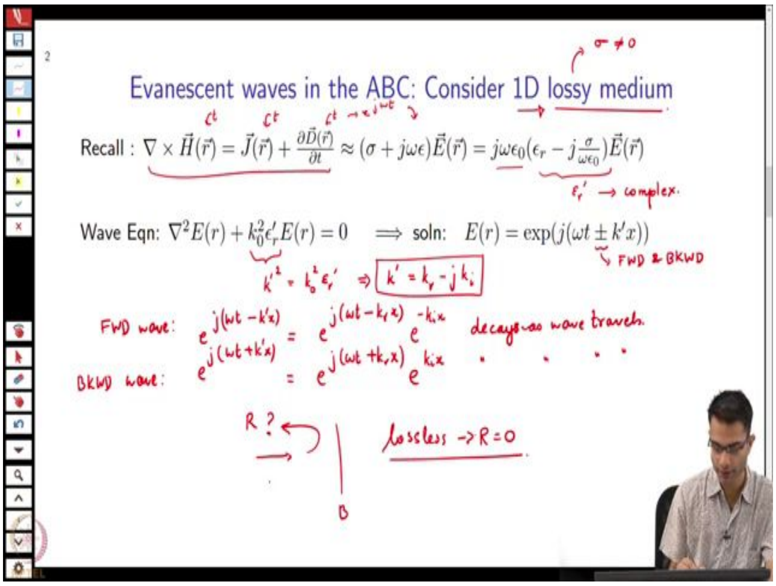

So, the failure of absorbing boundary conditions is what we start with right; so, let us jump in to this discussion by starting by considering a lossy medium ok. So, what is different is it is a lossy medium; previously when we had looked at absorbing boundary condition what did we start with? 1D vacuum. In 1D vacuum we impose the boundary condition and we found that absorbing boundary condition was perfect right, they give us no reflection what so ever.

So, we will keep the discussion limited to 1D, but now change it to a lossy medium that is the only difference and here we will find that the ABC is not able to deal with it ok. So, let us see how. So, lossy medium is characterized by a conductivity which is non-zero right and for now we will just took keep thing is really simple we will not even consider a dispersing medium, we will just consider a simple whatever we are doing $D = \varepsilon E$.

So, single frequency kind of a thing. So, you this is your Maxwell's equation everyone is familiar with $\nabla \times \vec{H} = \vec{J} + \partial\vec{E}/\partial t$. So, this should be a this should be a t also over here, but never mind similarly everywhere ok, so, it is understood. I replaced the current term by Ohm's law and I replaced by $\partial/\partial t$ by $j\omega$ standard right. So, all of this equation should have a t dependence and then we are doing the $e^{j\omega t}$ right.

So, that gives us this which all of us have seen during undergraduate courses and then the standard procedure from here is to try to extract the effective permittivity. So, what do I do? I pull out the $j\omega\varepsilon_0$ and whatever is left inside over here is in this bracket ok. So, what is this term look like it looks like effective relative permittivity. So, I will call that ε_r' which is; obviously, complex. So, this is just like our original Maxwell's equation, where supposing I put $\sigma = 0$ I get $\varepsilon = \varepsilon_0 \varepsilon_r$ relative probability ok, fine then what do I do?

So, 1D medium so, very simply I will take the two Maxwell's equations take the curl of the other and get a wave equation and this wave equation that I get is off this kind over here the only change that happens is that instead of k_0^2 I get an ε_r' whatever was the relative permittivity over there comes in over here ok. So, I am going to call this term over here as k'^2 we have derive wave equation many times. So, we would not derive it once again over here.

Student: (Refer Time: 04:22).

We are considering a homogeneous lossy medium. So, you can imagine like you know I have say water and I am doing FDTD within that and just 1D medium we are not even considering object right now. So, we want to see what is the impact of the ABC in a lossy medium we saw that in a loss free medium ABC was perfect no problem with ABC ok. When I solve this wave equation this is a solution that I get as before right, no surprises over here what is this k'^2 I have already mentioned over here $k'^2 = k_0^2 \varepsilon_r'$ right.

And so, from here I can get k'^2 as a square root of this expression square root of a complex number is going to give me a complex number ok. So, I am going to write this as $k_r + jk_i$. So, the solutions I said I mentioned are both plus and minus right. So, this is going to be give me a forward and a backward wave right. So, what is the forward wave look like? Same thing right. So, I am going to get $e^{j(\omega t - k'x)}$.

Student: Minus.

Mow let us expand that in terms of real and imaginary part. So, I am going to $e^{j(\omega t - k_r x)}$ e and then what happens next? Minus into yeah. So, what is that going to be? Should it be a plus or minus? So, $(-j) \times j$ is going to give me a $+1$?

Student: The plus 1.

So, we are just check this once again if this is what we will get yeah. So, what I will do is I am going to write this as a plus. It is up to me how to write, plus or minus, the sign will get absorbed into k_i ok. So, if I write it like this, what happens to this sign over here $-k_i x$ why would I do that? Right as the wave is traveling through a lossy medium, it should decay. It should not gain in amplitude right ok. So.

Student: (Refer Time: 07:29).

It was correct right.

Student: Yeah correct.

Because. no how j.

Student: Because a minus.

Or there is a minus also with the sign of with a sign of x.

Student: Yeah.

Right so, that became a plus and a minus sign yeah fine yeah right; so, too many minus sign. So, decays as wave travels and the backward wave $+k'x$. Does this wave decay? It had better decay. It is decaying right. Which wave which direction is this wave traveling? It is traveling in the negative x direction. So, x is decreasing over here right. So, this wave also decay as a travels. So, it's correct. Always check that you are getting a physically meaningful solution over here. One quick thing I want to point out that this k prime which I have written as $k_r - jk_i$. Both k_r and k_i will turn out to be positive numbers ok.

If I had the opposite sign convention for time if I had $e^{-j\omega t}$ what will happen? We will get a plus sign over here because your time convention does not change the physics of the problem, but the wave will decay ok. So, some places as I mentioned before will have a $e^{-j\omega t}$ convention in that case the effective permittivity will have a positive imaginary part ok.

So, now what we do? We have the situation we are considering is this is my boundary and I have a wave that is incident over here and we are asking that is there a reflection? When the medium was lossless, we saw that $R = 0$ we have already seen that. But now what happens? So, what do I have as the total filed over here?

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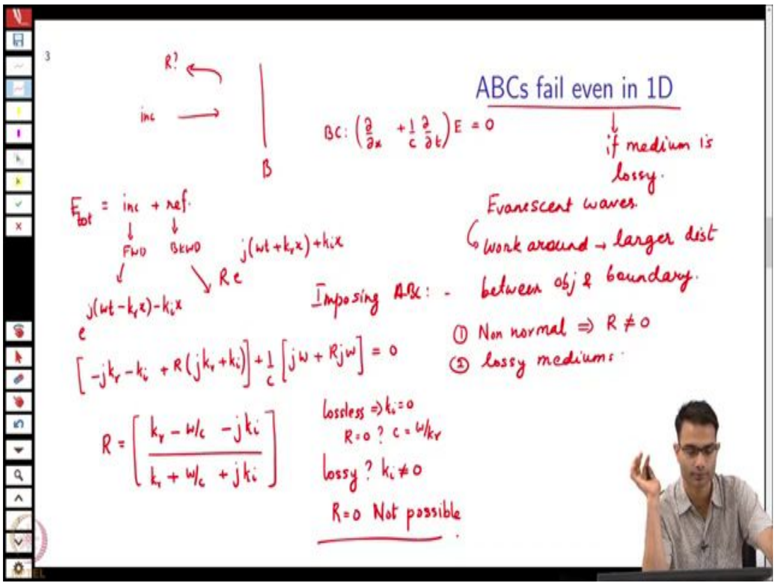

Let us come over here. So, this is my background this is my. So, the total field is going to be incident plus reflected right. We are not, so far, going into FDTD implementation just total field is incident plus reflected. And what was my the boundary condition that I was going to imply sorry employ? d/dx, was it plus or minus for the right hand boundary?

$$\partial E/\partial x + 1/c\ \partial E/\partial t = 0$$

That is what we are imposing ok. So, right so, these are the incident is forward traveling wave. Reflected is a backward traveling wave ok. So, I am going to write this as $e^{j(\omega t - k_r x) - k_i x}$ and what do I write the backward wave as R reflection coefficient $e^{j(\omega t + k_r x) + k_i x}$. Now I am going to put these two together and impose this boundary condition ok. So, the first so the derivatives now will happen with respect to space first 1D by dx.

So, what is the first term that I get I get a $-jk_r$ right then I get a $-k_i$ is $\partial/\partial x$ put on incident field plus R times now what will I get $jk_r + k_i$ this takes care of the $\partial/\partial x$ part, then comes 1 by c what is the time part over here? $j\omega + Rj\omega$ equal to 0 that is my, this is imposing ABC on, just imposing the ABC ok.

So, from this all what we can do is gather all the R's on one side and write the expression. So, what I get is. So, $k_r - \omega/c$ it is the reflection coefficient. So, what does that tell you? In our in our ABC what is the one knob we have in our hand to impose the boundary condition? Like in the 1D ABC or 2D ABC we have putting a $\cos\theta$ factor over the $1/c$. So, in some sense you can play around with that number c right we can multiply by some cos theta whatever. But now when you look at this expression what happens over here? First of all look at when $k_i = 0$ right.

So, lossless. What happens now? Can I make $R = 0$ yes, how right. So, if I put $c = \omega/k_r$ I am done. I get 0 reflection coefficient, but if I have a lossy that is k_i is not equal to 0, there is no hope. I cannot make this expression 0 how will I make this expression 0 there is always an imaginary part I can be the real part 0, but the reflection coefficient will always be there.

Student: Be with the (Refer Time: 14:31).

If.

Student: Be with those can omega by c equal to kr minus jki.

No omega by c is a real number how can omega by c equal to k r minus j k i?

Student: (Refer Time: 14:46).

Yeah. So, how can a real number be equal to complex number cannot be right. So, there is no value of c that I can play around with that can make this expression going to 0. So, $R = 0$ not possible ok. So, the conclusion is that even in 1D the ABC fail if the medium is lossy ok. So, what is the practical way around this situation? Supposing you had to live with it you had a lossy medium and you wanted to impose ABC what was the way around it? So, notice what are these waves what kind of waves we call these with a decaying part?

Student: Evanescent waves.

Yeah, correct say that again evanescent waves right. So, these are evanescent waves right so. So, what is the one property of evanescent waves that you can utilize even though the boundary condition is not perfect they decay with space right. So, you would have to put the absorbing boundary condition further away from the objects. So, that even though is imperfect the wave will hit it, but still it will decay ok.

So, the work around is larger distance between the object and boundary right as it is if I the moment I deviate from 1D anyway I am going to have a reflect loss I mean the reflection coefficient non-zero reflection that we have seen before right. So, to summarize there are two reasons that we say that the ABC fail. The first is of course that non normal incidence gives a reflection coefficient that is non-zero we already saw that and the second is lossy mediums ok.

So, this motivates us to go for something else we want to find a see there is another solution that does not increase the size of my computational domain a lot and gives me good accuracy ok. So, this part is clear.