Computational Electromagnetics
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FDTD: Materials and Boundary Conditions
Lecture - 13.07
Implementing ABC in FDTD

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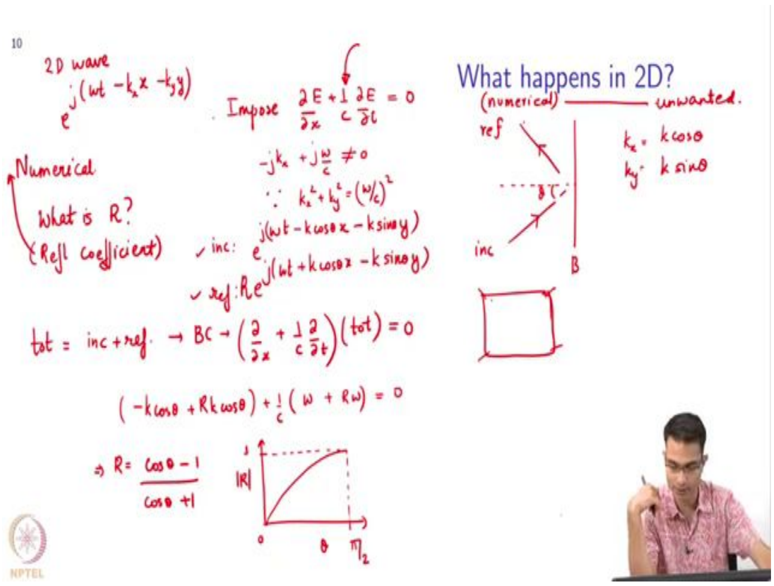

So I clear. So, that brings us to the next question of how do I actually implement this in FDTD ok?

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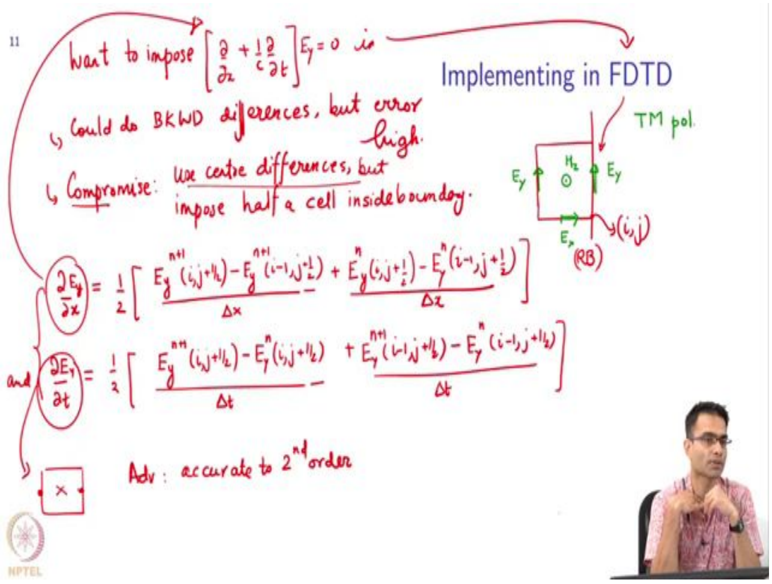

So, let us take a definite case again this is my boundary and I am showing you one Yee cell ok. So, just make it bigger. So, this is a right boundary I means there is nothing beyond this boundary and I am taking a simple 2D TM polarization case ok. And what do I want to do is want to impose which component of electric field should this we apply to in the identities which is the field on boundary E_y right E_y is the variable that is lying on the boundary. So, I have to impose the boundary condition on E_y ok. So, we want to impose this in FDTD ok. So, you see the challenge now.

Student: E_y is tangent to the boundary.

E_y is tangent to the boundary, but imagine a plane where that is travelling towards to the right boundary the k vectors going to be perpendicular to the wall, but where was the electric field. Electric field is transverse right consistently right. How would you do this? See what you are trying to do you are trying to impose this condition at which point over here.

$$[\frac{\partial}{\partial x} + \frac{1}{c}\frac{\partial}{\partial t}]E_y = 0$$

So, there is a space derivative and there is a time derivative, how will I calculate the space derivative by finite differences can I do it? So, one possibility is that I can take a backward difference instead of a centre difference; centre difference depends on some value to the right and some value to the left and subtract those two.

Here I cannot do that because it is the end of the domain I have nothing to the right of this. So, I cannot do centre difference. So, one possibility is I do a backward difference in space I can still do centre difference in time right, but what is the disadvantage of doing a backward difference error is error increases. As it is this is the boundary condition going to introduce the error then on top of my imperfect way of doing finite differences will give you further error. So, I could do backward difference, but error is high ok.

So, the compromise that we do is this use centre differences. So, use, but I cannot do centre differences. So, because there is nothing there is no variable store to the right of this boundary. So, then I impose it use centre differences, but impose this half a cell inside the boundary if I impose this so, at for example, at the location of x, z. So, I am going to impose this boundary condition technically it should be on the boundary, but doing that has this problem of backward difference.

So, I say let me just apply this boundary condition in the middle of the cell rather than at the end of the cell. So, that I have a value on the left and the right I can do centre differences. So, that is a compromise solution it is not a golden rule that you have to do this if you really want to do you could do backward difference also both will have some tradeoff in terms of the error.

You will have to update the the E_y value yeah. So, this whole implementing in FDTD is how do I update E_y on the boundary. So, we are that is what we are trying to arrive at.

Student: Backward.

Yeah. So, I will tell you what I mean. So, dE_y/dx that is the first term to apply right how do I implement this. So, I am saying instead of taking the backward difference I take centre difference centre difference I can only take between the values that I already have right. So, E_y on the left and E_y on the right and subtract these two, but what I do is dE_y/dx I am going to take the average over 2 time instance; 2 time instances.

So, I am going to find out the finite difference not at the time instance n, but also n-1 and take the average to reduce this error because I am I am imposing it in the middle of the cell not at the boundary. So, I want to minimize that error so, I take an average. So, its going to be the average over here.

$$\frac{\partial E_y}{\partial x} = \frac{1}{2}[((E_y{}^{n+1}(i,j+1/2) - E_y{}^{n+1}(i-1,j+1/2))/\Delta x) + ((E_y{}^{n}(i,j+1/2) - E_y{}^{n}(i-1,j+1/2))/\Delta x)]$$

$$\frac{\partial E_y}{\partial x} = \frac{1}{2}[((E_y{}^{n+1}(i,j+1/2) - E_y{}^{n}(i,j+1/2))/\Delta t) + ((E_y{}^{n+1}(i-1,j+1/2) - E_y{}^{n}(i-1,j+1/2))/\Delta t)]$$

So, with the result both of these expressions this and this are being evaluate are most accurate here right in the middle of the cell. The first one; obviously and the second one also right I am taking the average of something from here and something from here. So, its more accurate in the middle of the cell. So, that is the meaning of this compromise I am using a centre difference I am interpreting it as a centre difference, but it is being by doing this averaging it is being imposed half a cell inside the boundary.

Student: (Refer Time: 10:33).

So, advantage was accurate to second order right because its a centre difference ok. So, this is what we will substitute for E_y into this boundary condition and in any sort of what you call implementation we want to get an update equation order right. So, what will we do?

Student: It is a centre difference.

It is a centre difference; centre differences is accurate to second order in h the spacing between a point that is what I mean and forward difference or backward difference are only accurate to first order it is the power of the error we have done that it in the beginning in finite difference ok. So, now, what do we do we will take these guys each of this expression. So, I take this guy take this guy and substitute them here right and what will that give me, it will give me some update equation right.

What is the term that I need to update from these various terms which is the term that I really need to update which I do not know I mean everything else I know for every E_y inside the domain I have my usual FDTD update equation for what term I do I do not have an update equation so far. The E_y on the boundary right so, that is $E_y^{n+1}(i,j+1/2)$ ok.

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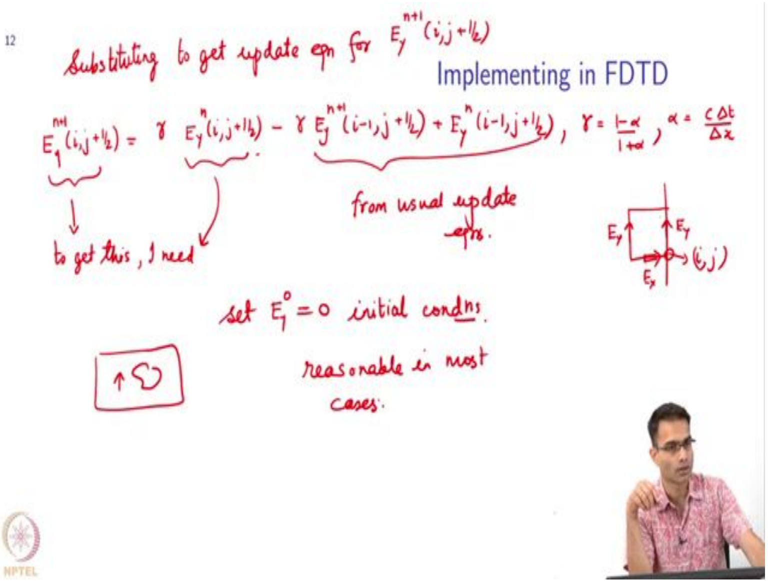

So, substituting to get; so, what do I get? So, I will just I will write down its very simple right there are how many 4 terms and 4 terms 8 terms right. So, what will be the key? keep the term that you want on the left hand side move everything else to the left hand side ok. So, I will write down the final expression. So, there is a term gamma comes which captures a lot of the constants I will just write down the final expression and then tell you what this is.

Student: When we consider (Refer Time: 13:45).

I am looking for $E_y^{n+1}(i,j+1/2)$ yeah that is what I want because that is the guy on the boundary. $\gamma = (1-\alpha)/(1+\alpha)$; α is my usual courant parameter.

$$E_y^{n+1}(i,j+1/2) = \gamma E_y^{n}(i,j+1/2) - \gamma E_y^{n+1}(i-1,j+1/2) + E_y^{n}(i-1,j+1/2)$$

So, this is an update equation purely in terms of E_y s at various different locations in space and time right. So, in order to get this I also need at the same point over here I need this right.

So, this is the same point in space. I need the electric field on the boundary at a previous time instance to updated at the present time difference right the only thing that difference between these two guys is n and n+1 ok. The other guys these are being updated by how? They are on the interior right i-1. So, this is from usual update equations ok.

So, how can we update this there is a little bit of trick here now right I mean little bit of catch I need to know this guy to begin with only if I know it at a previous time instance can I use this update equation in the future. So, how will I what is the way out of this problem, how will I know it in the beginning?

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No.

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The problem is that in order to get the field on the boundary at some time instance I need to know the value of the field on the boundary at a previous instance. So, how do I know it in the very first place at the beginning of the simulation what do I know it to be?

Student: 0.

0 right. So, typically what has happened is let us say this is my computational domain over here this is some antenna some object over here at time t=0 none of the fields have reached the boundary right we are stepping in time. So, the field is slowly hit the object and then its travelling outwards. So, at time t=0 very safely I can set the E_y s on the boundary to be equal to 0 right. So, set $E_y{}^0 = 0$.

Student: (Refer Time: 16:46).

Initial conditions on the field I can set to 0.

Student: (Refer Time: 16:53).

I mean that is a typical way they may if you have a specific reason to set it to something else you could do that, but if my I am turning my source on at time t=0 right then of course, filed has not reached anywhere I will set everything will be 0 right.

Student: (Refer Time: 17:10).

Yes. So, actually when you do these calculations you can store the field values at every time snap you can see very beautifully field is travelling outwards in whatever direction. So, you can visualize multiple scatterings is happening and then finally, reaching a steady state value in your FEM and integral equations you do not get to see that beauty you just get final steady state solution and which is reasonable in most cases reasonable ok.

So, what we have what we have looked at is how to take your absorbing boundary conditions and implement them in FDTD and for that we need to take care of our very initial conditions right.

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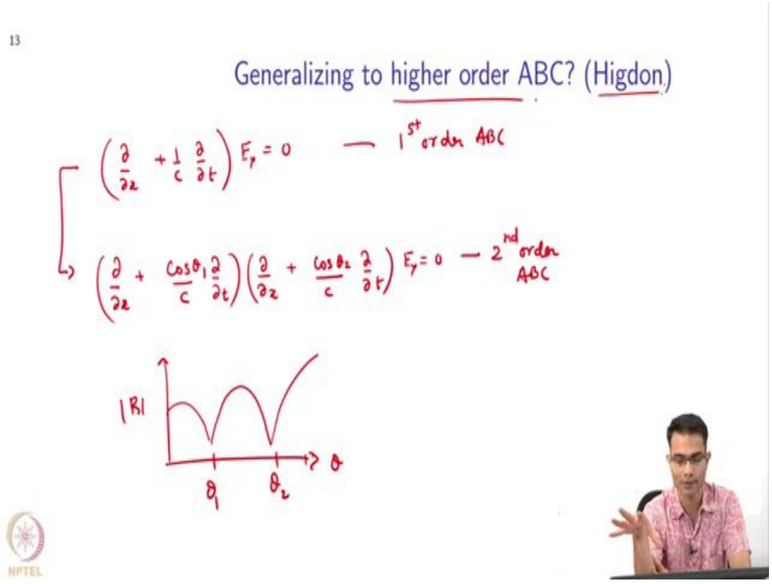

So, if this is clear then we need to conclude this module will I just mention one very simple way of making your boundary condition accurate for higher order ok. So, what was the boundary condition that we had imposed first $[\frac{\partial}{\partial x} + \frac{1}{c}\frac{\partial}{\partial t}]E_y = 0$ this was 1st order ABC now supposing I say I want the reflection to go to 0 at not just one angle, but two angles.

So, the key is to play around with this expression this is what I am imposing. So, I generalize these two I will just write the expression and then we can work it out later why this is correct one expression and another since its 2nd order they will have to be two terms like this.

$$(\frac{\partial}{\partial x} + \frac{\cos\theta_1}{c}\frac{\partial}{\partial t})(\frac{\partial}{\partial x} + \frac{\cos\theta_2}{c}\frac{\partial}{\partial t})E_y = 0$$

So, we are not actually going to implement this in this course, but I just want to give you the high level idea.

So, if you put this put your do the same analysis that we did right toward I mean wave travelling at some non-normal direction and calculate the reflection coefficients what you will find is that if I plot the reflection coefficient over here versus θ_1 and θ_2 are over here what happens is that you get nulls like this ok. So, these are called Higdon boundary conditions named after the scientist who ok. So, that concludes our discussions of higher order I mean how to implement 1st order ABCs and just high level idea of what to expect in a higher order.