Computational Electromagnetics
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FDTD: Materials and Boundary Conditions
Lecture - 13.1
Dealing with non-dispersive dielectric media

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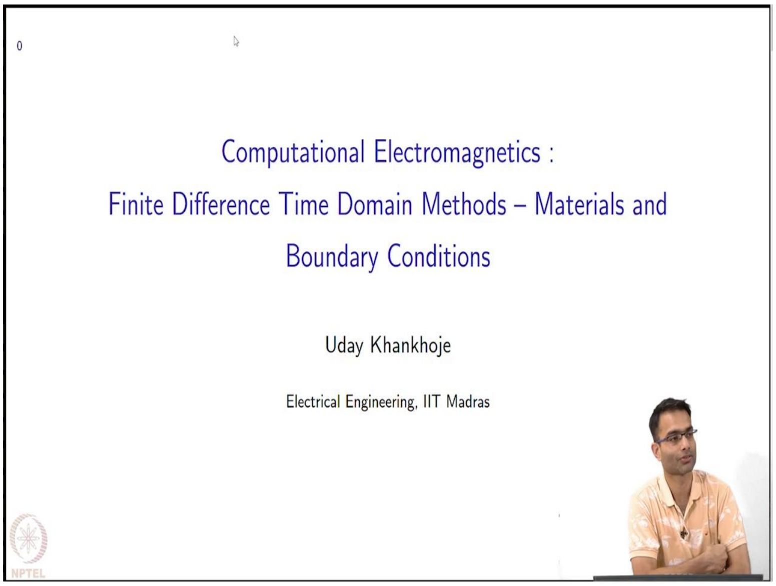

So, in this module we are going to graduate from looking at just vacuum, we are going to make the first steps to look at the situations where there are realistic materials and also boundary conditions how we will impose boundary conditions right. So, we are building the FDTD in complexity step by step. So, far in vacuum looked at the main thing how do you set up the computational cell, how do you look at stability, how do you look at convergence and accuracy all of those things right. Now let us look at materials ok.

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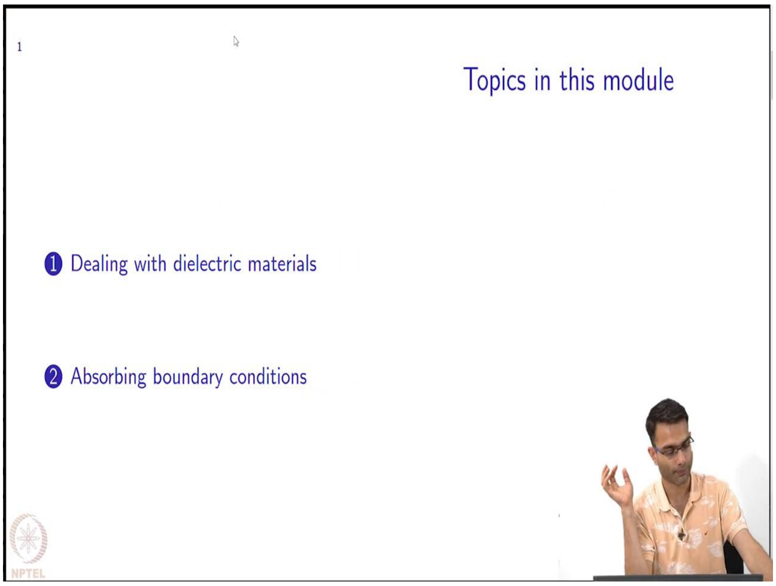

So, we will start with dielectric materials ok.

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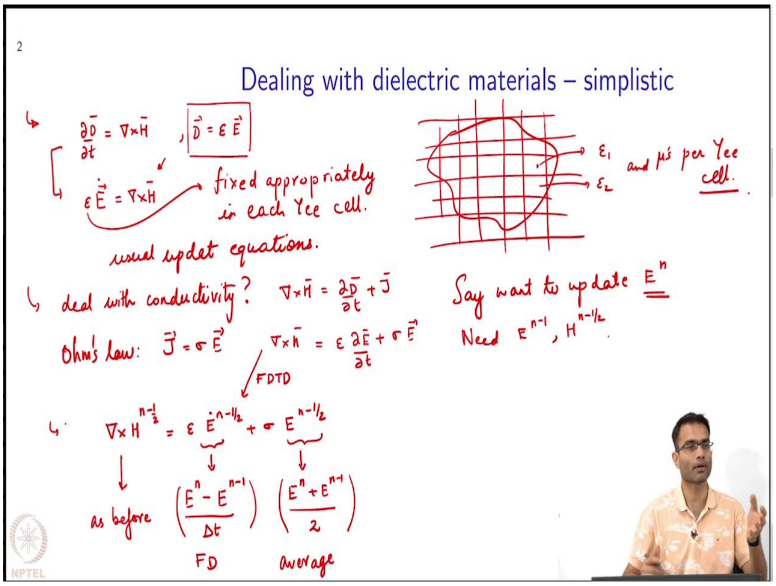

Now, your Maxwell's equation says for example, $\nabla \times \vec{H}$ right and supposing I have a dielectric material. So, what is a constitutive relation I used, how would I introduce the permittivity of the object in here?

Student: Epsilon.

ε needs to come in that right. So, what I would do is put $\vec{D} = \varepsilon\vec{E}$. Now one of the main things that I want to sort of challenge in this section is our use of this equation. So, what we will find out today is that our use of this equation is actually been very very fraud throughout. So, we will try to put this on a proper plotting now ok. So, for now let us just assume that this equation is correct.

So, then what would happen is that you would simply have this equation would become $\varepsilon\dot{E} = \nabla \times \vec{H}$ correct and that is how we deal with in any Yee cell I assign the permittivity to be $\varepsilon_1, \varepsilon_2$ whatever right. So, this is then fixed appropriately ok.

Student: Every cell we have computational domain.

Yes ok. So, supposing this is your computational domain right. So, what a Yee cell is going to fracture this whole thing into a square grid that is how we do it and now this over here this has some epsilon 1 ok. So, first of all I what I do is I do a piecewise constant approximation of epsilon. So, wherever there is a square grid I assume the ε is constant over there. So, this is some epsilon 1, this is some ε_2 and so on right.

So, on this grid once I fix the values of ε then I use these update equations with the appropriate value of epsilon or mu right I can I will also replace $\vec{B} = \mu\vec{H}$ and put those values of μ right; so, ε and μ's per Yee cell and usual update equations ok. Now, what if my material also has non-zero conductivity? How do I deal how do I how will my update equations change them?

Student: Lossy thing.

Yeah there is a lossy thing. So, how do I deal with it?

Student: Complex permittivity.

Complex permittivity is one thing, but when I do complex permittivity the implicit assumption there is that I am going to I expand the fields as in the $e^{j\omega t}$. If I do not want to do that, I want to stay in the time domain then how do I deal with?

Student: We introduce new components.

Introduce?

Student: New components.

Introduce a new component. Yeah how do I introduce it? So, let me just write this. So, deal with conductivity first of all.

Student: (Refer Time: 04:21).

So, there is a $\vec{J}$ term first of all this Maxwell's equation is a complete or there is something missing in it?

Student: J.

The $\vec{J}$ term is missing right. So, what actually is there is that $\nabla \times \vec{H} = \partial\vec{D}/\partial t + \vec{J}$.

Student: Yes sir.

And J can replaced by?

Student: Sigma.

And that is what law?

Student: Ohm's law.

Ohm's law right so, Ohm's law right. So, then this equation will this update equation becomes

$$\nabla \times \vec{H} = \varepsilon\, \partial\vec{E}/\partial t + \sigma\vec{E}$$

Now let us put this into our FDTD notation. Supposing so, I am going to remove the vector sign it is understood from now by now. So, $\nabla \times \vec{H}$ supposing I am updating the electric field at every integer instance that is what we are doing.

So, then the magnetic field is updated at half integer right. So, for example, I will write

$$\nabla \times \vec{H}^{n-1/2} = \varepsilon\, \dot{E}^{n-1/2} + \sigma\dot{E}^{n-1/2}$$

I have just written down the whole I mean whole equation in discrete form if the left hand side is at time n minus half right hand side should also be at time n minus half. Now this is not a problem because I am implementing this by finite differences. So, what is this actually amount to? $(E^n - E^{n-1})/\Delta t$, right. Finite difference in time and the approximation is most accurate at $n - 1/2$. So, its consistent right. You agree with me so far.

So, what is the problem now over here? I was storing electric field only at integer time instance, but now this equation is requiring that I also needed at half a time instant. So, what should I do? Average it, right. I do not want to be storing electric field at halftime integer instance also. So, the best thing to do is to replace this term by $(E^n + E^{n-1})/2$ right. So, this is average and this is finite difference and this is as before.

So, this is how you would deal with a material that has conductivity right now in your in your LeapFrog system what all do you need. So, supposing say you want to update E^n. What all do you need to know from this equation? I need to know?

Student: Let us.

Let us start from the farthest back in time I need to know E^{n-1} and I need?

Student: $H^{n-1/2}$.

$H^{n-1/2}$ ok.

So, I can do, my update equations are slightly modified from before this because I have a σ term has come over ok. So, there is just some small thing to keep in mind. So, this is your simplistic way of dealing with dielectric materials questions ok? Now what we could do I mean we can simplify this expression that I have this update equation I can simplify this typically I want to update for E_n. So, I will bring E_n to one side and everything else to the other side right. So, I can do that simplification.

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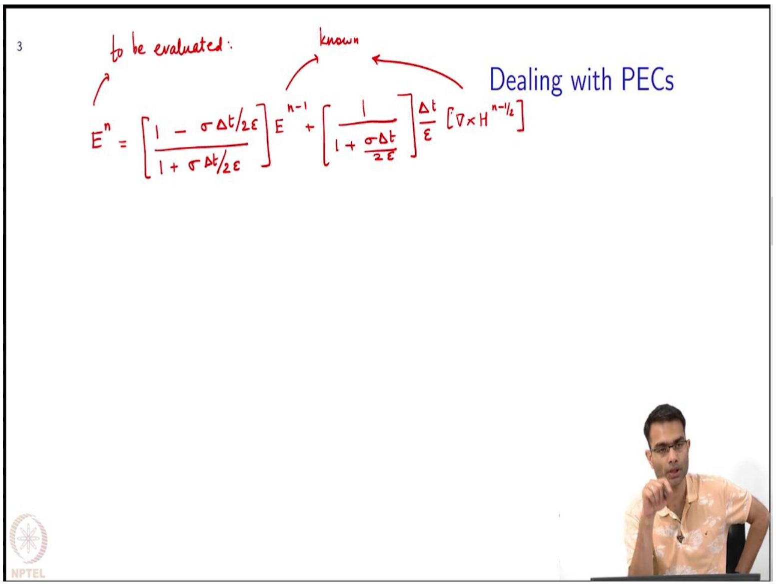

So, for example, what I will get. So, try to work it out. So, E^n I have a E^{n-1} and $H^{n-1/2}$ right those are the 2 terms over here. So, the first step that I get is 1 I will just write the final answer which simple manipulation gives you. You can confirm that that is also what you get ok. So, this is our update equation. In this, plug in the values of σ and ε. So, remember here in what we have done your ε here is referring only to the real part and sigma is the conductivity that is giving the loss part.