Computational Electromagnetics
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Finite Difference Time Domain Methods
Lecture - 12.08
Accuracy Considerations Higher Dimensions

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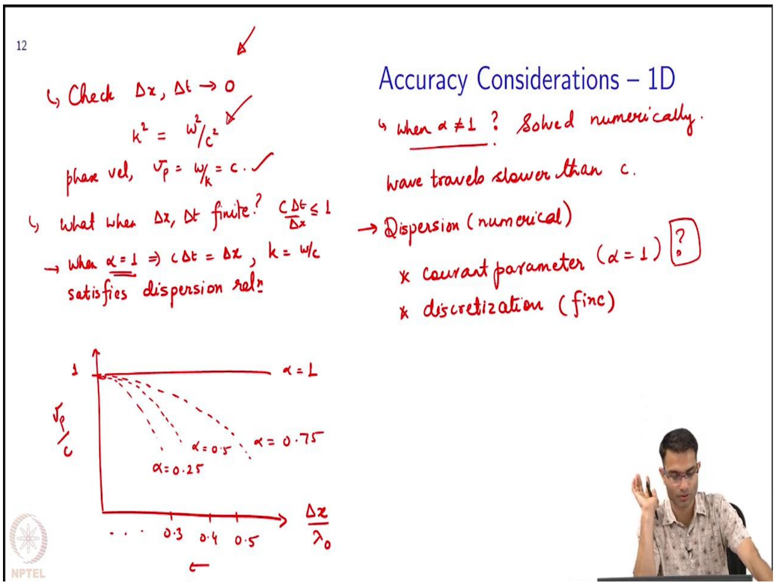

Let us why I have put a question mark over here any guesses? So, what I mean the recipe of fixing $\alpha = 1$ is fine there is no problem with it in the 1D world. In the one dimensional world I can fix you know $c(\Delta t/\Delta x)$ to 1 no problem.

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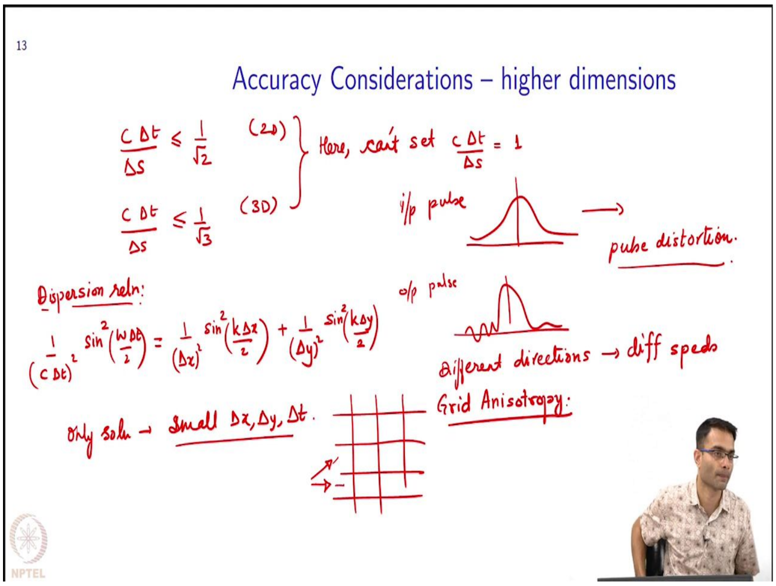

What happens when I got to higher dimensions? Was that what was that relation between I mean if you remember was $c\Delta t/\Delta s \leq \frac{1}{\sqrt{2}}$ in the case of 2D

in 2D and in 3D this was $1/\sqrt{3}$; now I cannot choose $c\Delta t = \Delta s$ this has to be less than 1. So, here cannot set $c\Delta t = \Delta s$ why? Because if I do it what happens I would not get a wave like solution only it is a unstable situation. So, then what? Then you have to deal with it the only fix to it is the second solution right and the second solution was by making your discretization fine right.

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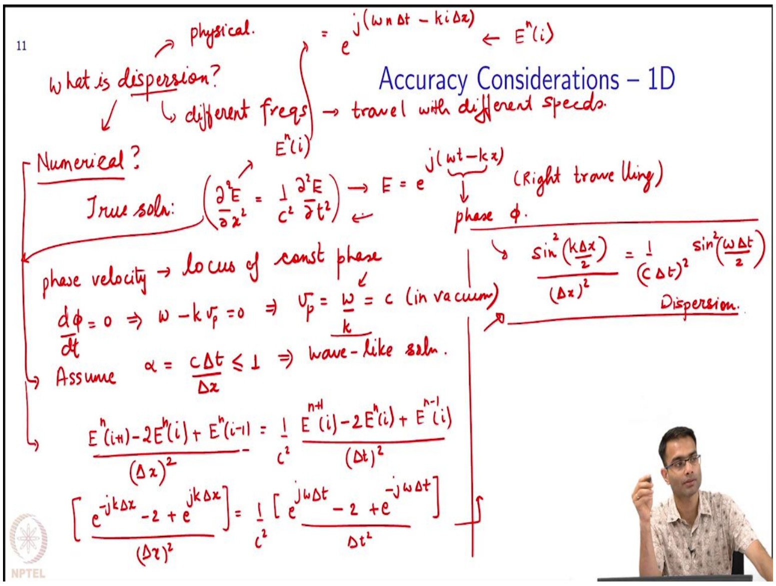

So, let us I mean the same discrete version right which we had. So, if you look at this equation over here we started with this wave equation over here. In the case of 2D what will happen, I will have a second derivative of y also ok. So, these dispersion relation how do you think it will get modified? If I wanted to write this dispersion relation for.

Student: (Refer Time: 02:16).

Right exactly. So, this dispersion relation will become using finite difference time domain I am going to get. So, I will write

$$\frac{1}{(c\Delta t)^2}\sin^2(\omega\Delta t/2) = \frac{1}{(\Delta x)^2}\sin^2(k\Delta x/2) + \frac{1}{(\Delta y)^2}\sin^2(k\Delta y/2)$$

I will get 2 terms.

Right, one from Δx and one from Δy so, now you would have to I mean the same analysis that we did in the previous slide we would have to repeat it to generate the dispersion diagram over here. What is dispersion? Just the relation between ω and k. Wherever in optics or electromagnetics you here the word dispersion it means what is the relation between frequency and wave vector that is what is meant by dispersion ok.

So, this is a problem. So, the only solution is small Δx Δy Δt there is no fix to distinct. So, what is a very so, now, that we have to live with it in 1D we can fix it put $\alpha = 1$ it is allowed it gives me a stable solution in 2D and 3D I have no way around it.

So, what is sort of one very obvious consequence of this is think of let say you know laser pulse I want to simulate a laser pulse and it goes and hits some electromagnetic resonator or cavity or something ok. As this laser pulse travels let say it is travelling in free space right it is going to numerically disperse because, of pulse in time is made up of several different frequencies. But, now each frequency has its own velocity right; if you look at this curve over here well not in this curve, but for different every different ω will have a different k.

Student: Sir, you please solve delta is equals to (Refer Time: 04:27).

Yeah, once I fix a discretization I will change the relation between ω and k at different omegas. So, what happens is suppose I send a input pulse like this nice Gaussian pulse we send like this outward happens is you will get distorted right. So, you may the output pulse by output I mean the pulse scene after some space and time right.

So, this may look something why, because the constituent frequencies that went up to make that pulse are travelling at different speeds now, if they all travel at the same speeds I would get the same pulse right. So, this is what is called pulse distortion. This pulse distortion for example, will happen in an optical fibre because glass is a dispersive medium you have optical fibre glass every frequency travels in a different speed.

So, this anyway happens physically, but if I was simulating free space it should not happen, but it will happen in FDTD. So, again you are the way to beat it is only one small Δx Δt ok. So, pulse distortion is what happens and it is given in higher dimensions there is another thing that you have to leave it and again the solution for that is making Δx Δt small right. So, imagine 2D grid like this, I send a wave I send a pulse like this it distorts.

Now, I send a pulse like this some other direction what will happen, because this discretization is finite the approximations are now slightly different right. So, now the wave travels at different speeds depending on which direction it is travelling in the grid. So, this wave and this wave even if I choose a single frequency pulse I send the single frequency tone as it is called it will travel at different speeds here and at different speeds over here.

Student: (Refer Time: 06:36).

There will be.

Student: 45 so.

If I do not take this to be 45 degrees take this to be any angle. So, the approximate I mean because I am using finite differences to approximate the wave equation its more or less accurate depending on how well aligned you have with the grid right. So, this is so, different directions different speeds.

Student: Sir, you happens (Refer Time: 07:08).

Yes, the medium is the isotropic is a statement of the physical properties of the medium air is isotropic medium, but by virtue of me discretizing space and time where who is discretizing space and time not nature we are doing it in our computer. The moment we do it we have created anisotropic materials where none existed right. So, different speeds I am want to say I have already given the word away this is called **grid anisotropy**. So, any real world simulation has all of these errors in it there is no such thing as a perfect simulation; perfect simulation happens when Δx Δy Δz Δt is all so tiny right.

So, that is why I mean one part of real world computational EM is writing your code very good, but after that you have to you know bench mark it with known solutions you take the simple thing like this you write a 2D FDTD code you think everything is great then you run a pulse in different direction then you see oh its behaving so differently. Then the next phase is in finding out what are the parameters Δx Δt or Courant parameters such that the dispersion is acceptable let us say it is only 1 percent you can live with that it is its you know 20 percent you cannot live with that.

So, you keep reducing your I mean making your discretization finer until you have reached some threshold. So, what can that threshold be; for example, my simulation domain is some fixed amount over here and as the wave travels from the left to the right the dispersion effects will grow more and more. As I reach the rightmost like the two farthest points on the in the simulation domain I can fix before hand and say the dispersion form these two points should be no more than so much. I decide that before and then once I obtain that then I do the rest of the simulations with those parameters. So, this actually takes almost as much time as code writing in the first place ok.

Student: Courant (Refer Time: 09:15).

While you could correct for dispersion effects if it were a very trivial simulation now if they were only vacuum sure you can correct for a, but if it were only vacuum what is the need to do FDTD you know it is a plane wave everywhere. The more difficult things happen when there is some medium some aircraft or whatever is there right. There it is not possible to correct for these things. Because you are what is your bread and butter finite differences how much more corrections will you do in that? Again those corrections will be direction dependent. So, you will introduce more effect, more problems by trying to correct it. So, you have to live with it there is no other records

Student: (Refer Time: 10:02).

So, integral equations so, its slight its tempting to look at integral equations and say now this problems were there, but integral equations we formulated at a single frequency yeah at a single frequency this problem I mean now. So, in a integral equation and I have to do Fourier transform to look at pulse distortion.

Student: (Refer Time: 10:24) single frequency (Refer Time: 10:25).

Yes yeah. So, that is yeah. So, good point integral equations are therefore, the much more accurate ways of solving computationally in problems FDTD is like your brute force guy he gets you the answer after lot of effort and still not so good.

Student: That is saying that worst.

Yeah.

Student: So, it is like a continuous frequency (Refer Time: 10:48).

Yeah.

Student: As.

So, if you take a Gaussian pulse in time it is Gaussian in frequency.

Student: And some other hand continue the lecture video of the discretisation.

Ha.

Student: (Refer Time: 11:04).

Well I do not have to worry about that I have discretized Δx Δt once I set my source the wave travels. That the update equations take care of it.

Student: Integrally.

For the integral equation then I have to discretize in frequency solve for each frequency put it together

Student: That this discretization frequency could say the answer.

Yes. So, discretizing frequency will bring its own errors because depending on how fine you do it in frequency that accurate is your solution in time right. So, then you have tradeoff how much time do I want to devote to computation. So, I mean integral equations at least in the optics community are not very popular right, people will do FDTD and pay the price in discretization you want more accurate just crank up Δx Δy .

Student: (Refer Time: 11:54).

Yeah I mean numerically once we have put it along to the computer we are no longer going to see $e^{j(\omega t-kx)}$ they are going to see numbers complex numbers then you can fit whatever thing you want to do it, but you will find that even a 1D problem if alpha is not equal to 1 right it is not travelling at the speed of light.

Student: (Refer Time: 12:17).

Even exactly that is what we have saw over here. Even when there is no medium if you look at the dispersion relation; even when there is no medium when Δx and Δt are finite.

Student: So, if you said by alpha equal to 1D and times.

Yes, you should not say any distortion in 1D if I said alpha equal to 1, but other places I have to live with it ok.

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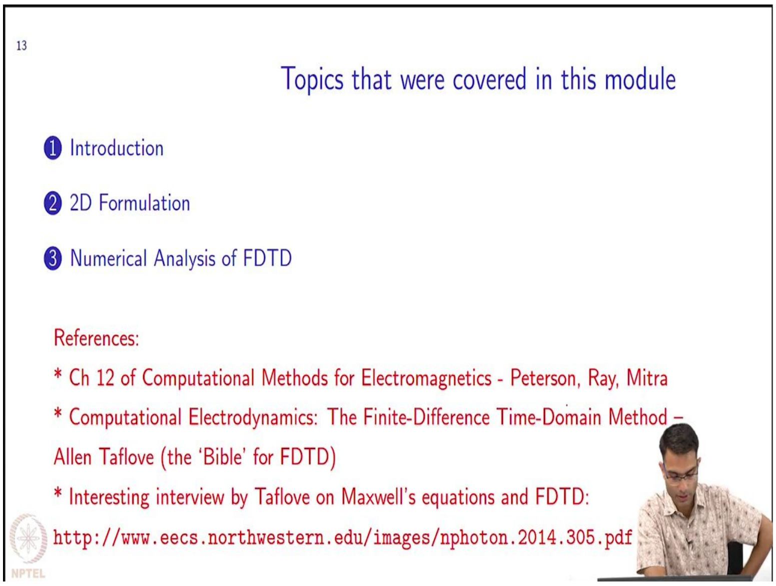

So, these yeah so, these are the sort of references for this chapter 12 of Petersons book which is very nice reference there is the so, called Bible of FDTD it is a book by Taflove which was one of the first few books written and I found a interview written by I mean given by Taflove on Maxwell's equations and FDTD. So, you can go and read this interview, he talks about I mean a little bit of history of Maxwell's equations and where he thinks it is going in so on and so on, do read this thing alright.