Computational Electromagnetics
Prof. Uday Khankhoje
Department of Electrical Engineering
Indian Institute of Technology, Madras

Finite Difference Time Domain Methods
Lecture – 12.05
Stability Criteria – Part 2

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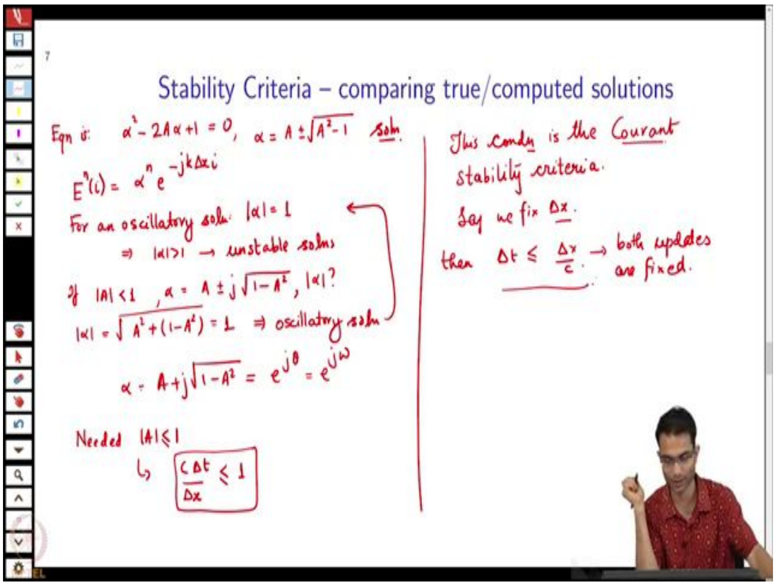

So, what we did is we sort of gave a very precise mathematical way of arriving and this constraint. There is actually a, there is an intuitive way of arriving at this which is more sort of more fun than what we did right.

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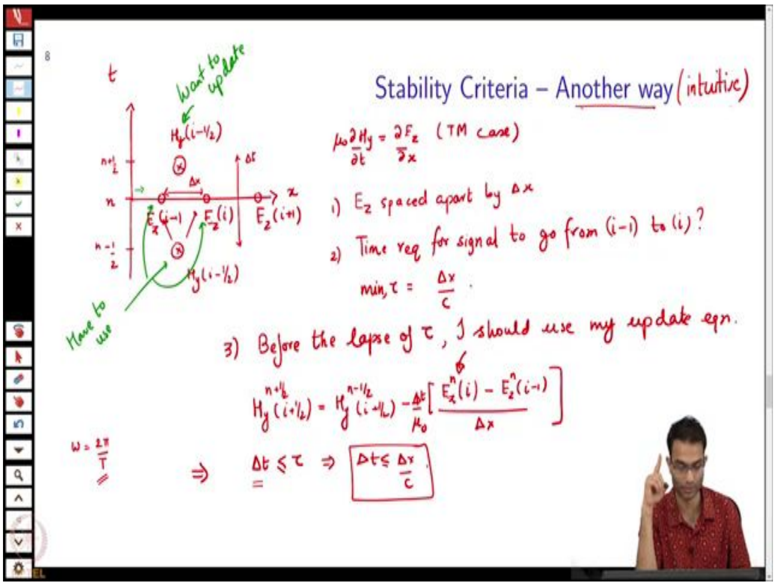

So, for doing this intuitive way of arriving at the stability criteria what I will do is I will draw a simple 1D grid right LeapFrog scheme is what we have been talking about. So, our grid this is my space and this is my time. On this I am considering, so, this is my $E_z(i-1)$ this is my $E_z(i)$, this is my $E_z(i)$. What other variables I need, $\vec{E}$ and $\vec{H}$ ok. Remember, in time also the variables are staggered by half a grid and in space also right.

So, this is going to be my $\vec{H}$ let us call it some H_y ok. So, $H_y(i-1/2)$ and correspondingly there will be an $H_y(i-1/2)$ minus sorry this will be i yeah this will also be a $(i-1/2)$ ok. In terms of time since this is say n, this is $(n+1/2)$ and this is $(n-1/2)$ ok. So, I have taken just E_z and H_y ok. So, the TE polarization I am just looking at one of these equations ok.

So, so, I mean the differential equation for this was

$$\mu_0 \partial H_y/\partial t = \partial E_z/\partial x$$

and this is the TM case. In TM case, this is this is the equation that I had the right E_z, H_x, H_y. So, equation for H_y was the following. What do we do now is. So, this is my Δx is this and Δt is whole thing over here ok.

So, let us write it in stepwise form. So, E_z is spaced apart by Δx now already know that what is the next thing? So, what is the time required for the signal to go. So, time required physical time required for the signal to go from $i-1$ to i. Imagine there is a physical wave that is propagating in space and time what is the minimum time required for the wave to go from $i-1$ to i. So, what is the distance?

Student: (Refer Time: 03:51).

Distance why is it 1?

Student: Statistics.

Distance is Δx. What is the speed at which this wave can go as fast as possible c right. Now, the third point we can say is that minimum is. So, I am calling this minimum is tau this value of $\Delta x/c$ is the minimum time required for the signal to go from here to here right from left to right ok. Now we can say that before this before the lapse of τ I should use my update equation. What was my update equation?

So, just look at this equation on top over here I can write

$$H_y^{n+1/2}(i+1/2) = H_y^{n-1/2}(i+1/2) - 1/\mu_0 \times [E_z(i) - E_z(i-1)]/\Delta x$$

This was my update equation right. So, what all values are involved in evaluating $H_y^{n+1/2}$ its $E_z(i-1)$, $E_z(i)$ and H_y at the same location in space, but a different time location previous time location ok. Now, this.

Student: Minus.

Minus.

Student: Delta.

There is a Δt over here. This is the equation. So, now, before this period of interval τ goes I should use my update equation why because, if I do not use this update equation then this guy over here $E_z(i)$ would have got a new value. So, I am using the update equation, but the value inside it is wrong, I would be using the past value of E_z over there I do not want to do. So, before the wave has physically gone I want to quickly use my update equations while my variables are still faithful to the physics my variable says $E_z(i)$ $E_z(i-1)$ all at some time instant n.

So, when I substitute the value inside these things it should be actually the value of the field at those times in points in space and points in time correct. Not if I wait for more time I will use this formula, but the numbers that I put inside will be wrong because there will refer to old wave has already travelled physically. So, I want to do in some sense it sounds very grand when I said I want to do a faster than light calculation before the light gets there I want to use this update equation right. So, that basically says that the times. So, this implies at the time step should be greater than or less than this tau? Less than tau right and that is exactly what we had earlier ok.

Student: $\Delta x/c$.

$\Delta x/c$ yeah I mean it is a very fine.

Student: (Refer Time: 07:27).

Yeah. So, yeah, the time step has to be. So, I mean yeah we usually thing that c is the very large value. So, $\Delta x/c$ is going to be a very small value right, but that is not exactly the right way to think of it. The correct way to think of it is to imagine there is a wave at a constant frequency ω right. So, that ω I can write as $2\pi/T$. So, what we should actually be looking at is the comparison between ω and this time period T. That will not turn out to be such a drastic number ok.

For example supposing you are in optical frequencies; optical frequencies what is omega ok? Forget optical frequency let us say you have at microwave 1 gigahertz, 1 gigahertz is 10 to the 9 right. So, what is the time period there?

Student: 10 to the minus 9.

$10^9/c$ is going to give you what? Again small numbers right. So, Δt looks like a very small number, but when seen in relation to the time period of the wave it is not going to be some microscope in fact ok. So, this is the intuitive way of arriving at this stability criteria.

Student: There is depends (Refer Time: 08:46).

Yes, the third part again. So, first part is clear space the E_z the part by Δx, then we are saying that in this equation when I want to update H_y ok. So, I want to update want to update this guy. What all do I have to use? So, I have to use this guy, this guy and this guy as per the formula. Now when I am going to use these equations, these had better represent my current estimate of these quantities sounds obvious, but of course, I they should be current.

But if I wait for a time greater than delta x by c then the wave has which is let us say in one case travelling from left to right the wave has already travelled from $E_z(i-1)$ to $E_z(i)$ has physically travelled, but I did not since I have not yet done a time update my variable still contained the old value in the location of E_z it still has an old value because I have not done the time update.

So, when I go to use the equation I am using an older outdated version of E_z in order to prevent that I shall quickly do the update before the wave actually reaches this. So, when I use the equation I am using the actual value that I know my best estimate. So, it says delta t should be less than tau ok. So, depending on your.

Student: How does (Refer Time: 10:19).

So, this is the sort of a thought experiment this is only a thought experiment right the wave as travelling through free space we are just saying how quickly can I simulate everything such that I am being faithful to the variables that I am writing on my wave of course, I am not sitting there and waiting for like travelling right. So, these all what Einstein who the call Gedanken thought experiments fine.

So, what is nice about this stability criteria being derived in this what I will call an intuitive way is that it is very easy to generalize this idea to higher dimensions 1D is fine a question you can ask is in 2D is the Courant stability criteria the same right. So, here you will find factors of root 2, root 3 etc.