Computational Electromagnetics

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Finite Difference Time Domain Methods
Lecture – 12.02
2D FDTD Formulation: Time Stepping

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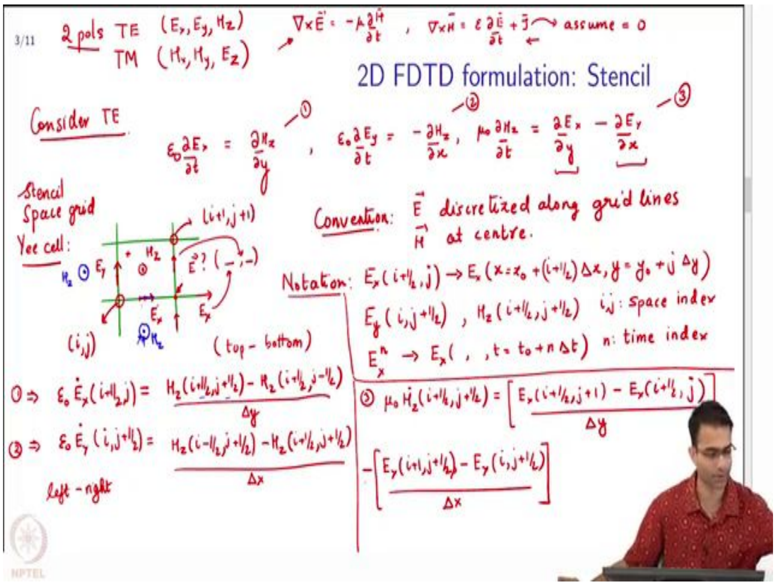

So continuing with our Finite Difference Time Domain 2D Formulation is what we are looking at what we have seen so far is how to deal with space right? Space special derivatives has replaced by finite differences in space across this stencil. So, if you look at your equations 1, 2 and 3 we have dealt with the right hand sides right, the next thing that is left is.

Student: Left hand side.

Left hand sides ok. So, let us let us come to that next.

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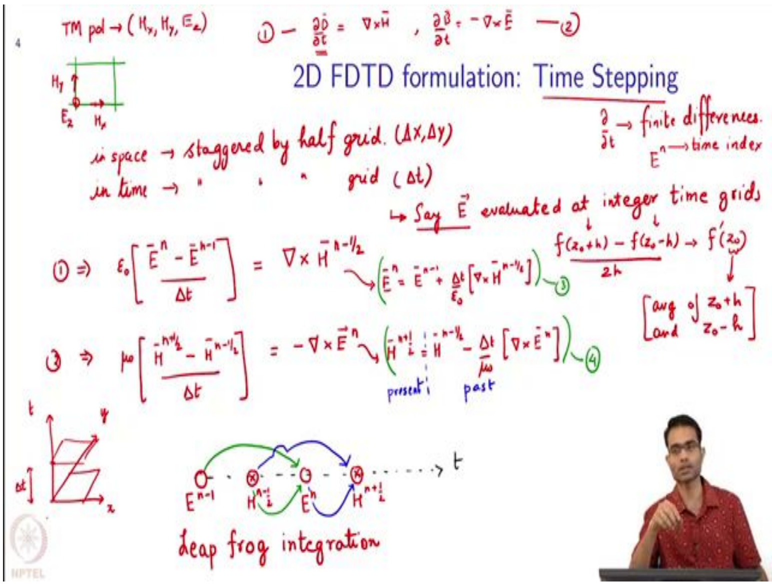

Before we move to Time Stepping, I thought at of just for sake of completeness I will also show you the stencil for TM polarization. So, far we spoke about TE polarization. So, for the TM polarization right what was the variables in TM?

Student: (Refer Time: 01:05).

H_x, H_y.

Student: E_z.

E_z these are the three variables that I have to think about right previously, we are doing TE polarization. So, we have seen that the logic has been to stagger all of these 3 variables by how much half a grid we should not all be at the same point in space they should be staggered; that was one consideration. Second consideration is we would like to have E at the grid lines where the grid lines are.

So, that it is easier to enforce boundary conditions. So, with these considerations its sort of very common choice for the TM stencil is as follows right. So, you can have your as before this can be your H_y ok. So, H_x is at the centre over here and where should you put E_z?

Student: (Refer Time: 02:01).

We could put it at the middle of the cell, but then the problem is boundary conditions enforcing. So, at the corner of the grid if I put it let us say here, it is still half a grid away from the other variables and its you know helpful in satisfying boundary condition; I could put it also in the middle of the cell there is no problem ok. So, it will be consistent ok, so this is the TM stencil that we will refer to when needed.

Now, we let us go back to the time stepping part right. So, in time stepping as always what we will do is replace this time derivative by finite differences right, what was the notation that we had for time? We put it as a superscript right so, it was written as E^n right so this was our time index ok. Now we said that in space the variables was staggered by half a grid right, staggered by. What about in time what should we do? Do the same thing? See we in the in this in these methods we are discretizing space and time so just as there is a $\Delta x, \Delta y$ in the stencil there is also going to be a.

Student: Δt.

Δt right so, that Δt the question can be should I have the variables being updated at the same Δt or some staggering over there as well. So, we will see that the same logic there is nothing special there is no special preference to space over time. So, whatever we do for space is what we will do for time. So, we will also stagger them by half a grid. Here it meant Δx, Δy; Here it meant means Δt. So, let us let us write it out so, let us take both are Maxwell's equations and try to write out what happenes right. So, very quick $\frac{\partial \vec{D}}{\partial t} = \nabla \times \vec{H}$, $\frac{\partial \vec{B}}{\partial t} =- \nabla \times \vec{E}$ .So, let us let us see let us look at let us take the first equation ok. So, $\frac{\partial \vec{D}}{\partial t} = \nabla \times \vec{H}$ is our first equation $\frac{\partial \vec{B}}{\partial t} =- \nabla \times \vec{E}$ is our second equation. So, $\frac{\partial \vec{D}}{\partial t}$ will make the same assumptions is about the medium linear homogeneous all of that.

$$\varepsilon_0\left(\frac{\vec{E}^n - \vec{E}^{n-1}}{\Delta t}\right) = \nabla \times \vec{H}^{n-1/2}$$

Now supposing I choose to evaluate look at look at for example, in this TM stencil H_x H_x is evaluated at one point in space what is the next point in space is it Δx later or is it $\Delta x/2$ later.

For H_x where will the next evaluation of H_x be? Δx right. So, when I look at for example, the electric field derivative over here the adjacent point so E, should be Δt apart or $\Delta t/2$ apart Δt apart same logic I am applying over here ok. So, let us say E is evaluated at integer points, integer time grids so, $E^n, E^{n+1}, E^{n+1}, \ldots.$

so, I am just writing right now in vector form right and if I have done this for the electric field then the magnetic field would have should have been evaluated at what time?

Student: (Refer Time: 07:15).

Should it be n? So, I am asking what I should write over here as the exponent not exponent superscript should it be n, n-1 or something else n-, half a grid point right this should be n-1/2 right because again what was the logical over here when I looked at the Taylor series and I found out $\frac{f(z_0-h)-f(z_0+h)}{2h} \approx f'(z_0)$ which is the average of this guy and this guy similarly here on the left hand side I have E^n and E^{n-1}. So, what is it a very good approximation of or something at n-1/2.

$$\varepsilon_0\left(\frac{\vec{E}^n - \vec{E}^{n-1}}{\Delta t}\right) = \nabla \times \vec{H}^{n-1/2}$$

What will be the now the rest is easy the what will be the second equation imply. So, $\vec{B} = \mu_0 \vec{H}$ in vacuum.

$$\mu_0\left(\frac{\vec{H}^{n+1/2} - \vec{H}^{n-1/2}}{\Delta t}\right) = -\nabla \times \vec{E}^n$$

So, now, let us just rewrite equation one over here. So, in terms of all the time indices which is the future most time index over here in equation one? n right E^n is the farthest I can go in time in this equation.

$$\vec{E}^n = \vec{E}^{n-1} + \frac{\Delta t}{\varepsilon_0}(\nabla \times \vec{H}^{n-1/2})$$

$$\vec{H}^{n+1/2} = \vec{H}^{n-1/2} - \frac{\Delta t}{\mu_0}(\nabla \times \vec{E}^n)$$

Well ok. So, let me start by saying that here this line over here we when we say that say that E is evaluated at integer time girds is a choice, I could have said E is evaluated at half integer time instance ok. Once I fix where E is evaluated H automatically becomes staggered by half grid right. So, I have got E evaluated at integer time instance and now if I want the value of E at half time integer then what should I do? Like earlier take the average if you want it at some other point interpolate so, the choice is up to you, but once you fix one everything else gets fixed question.

Student: (Refer Time: 11:37).

Yeah that is what I said it is my choice it is not a coming from physics or math it is a choice the convention, once I fix it, but then I have to be consistent with the rest. So, I could have said H^n then I am forced to do $E^{n-1/2}, E^{n+1/2}, \ldots$ Finally, remember that the discretizations are going to be so fine that literally will not matter.

So, now let us move attention to these two equations that I written over here. So, one of the sort of basic principles in FDTD is that once I get the once I make this grid I am going to evolve the field in space and time. So, space stepping and times stepping based on previous values of field I will get future values and. So, also for space so, if I were to make a timeline so let us make a you know time axis so this is I mean we should make it dotted time is what I am looking at and let us say one point over here is E^{n-1}. So, if E^{n-1} then let me draw one more point over here this will be E^n ok.

This is as time is progressive similarly in where should I draw the magnetic field values in between these right because it is half a grid. So, for example, this I will draw with a across this will be $H^{n-1/2}$ and then this guy will be $H^{n+1/2}$ ok. Now let us look at these two equations so this equation 3 and equation 4. So, what is equation 3 telling me, let us be very close attention to this. Supposing I know everything in the past and I want to evaluate the current value of electric field. So, past means the right hand side of this equation; that means, I know E^{n-1} I know $H^{n-1/2}$ ok.

So, to evaluate E^n what all do I need? E^{n-1}, $H^{n-1/2}$ right so, I can draw it like this that this value goes into here and so does this value ok. Let us draw it actually down here like this right now let us then go to equation 4 ok. Equation 4 is telling me what; equation 4 is talking about H n plus half right. So, you can think of this as this equal to sign is sort of dividing the present from the past. So, the right hand side requires what electric field at.

Student: n.

n magnetic field at.

Student: n-1/2.

n-1/2 right if I were to draw arrows over here what should I show you for example, this guy goes in here and this guy goes in over here. So, I am just graphically interpreting these two equations that there is a half grid and one grid away these both are required to evaluate the field at a given time instance. So, for example, E^n it is not enough for me to just know electric field at the past I also need to know magnetic field at half grid past.

So, this the way the diagram has been drawn it looks like a frog that is leaping in some sense right. So, that is why this scheme is called a leapfrog integration scheme leapfrog integration ok. So, this is a very very characteristic thing of your what you call finite difference time domain method where, there is this half grid is really what is bringing everything out ok. Now, we have seen in 2D it is possible to visualize space and time ok. So, right now for example, stencils which I have shown you we are looking at that top view, but now if I also wanted to visualize the time axis what can I do I can move it on the z axis right.

So, one common we have understanding things over here is you put time on the z axis, you call this let us see why and you call this x right this is your Yee cell at some time instant right, this can be your Δt can be this right. So, that is how it goes and so on. So, what is the FDTD scheme all about? It is using these update equations to time step the fields and space step the fields.