Computational Electromagnetics
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2D Finite Element Method
Lecture – 11.11
Summary of FEM Procedure

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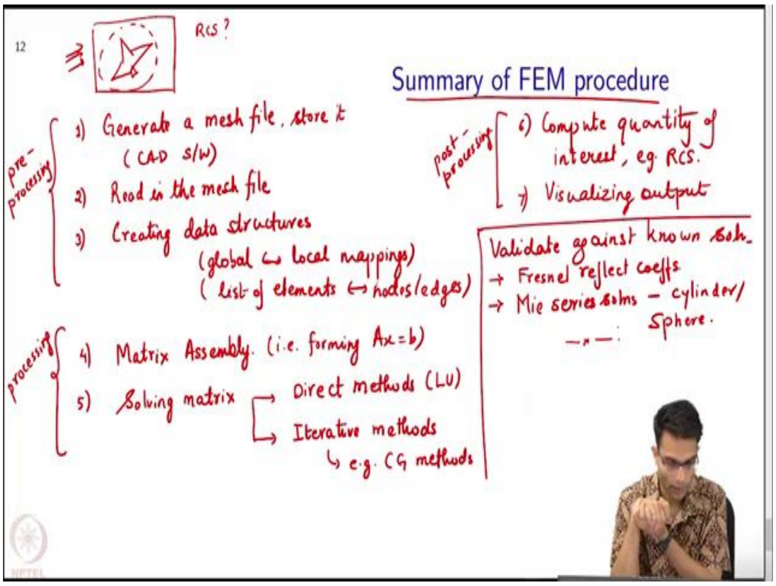

Let us look at the Summary of the FEM Procedure, what are the various steps that we have to do? Ok. So, what do you think could be step number 1 ok, so, for definiteness let us assume problem this is my aircraft over here, and I want to calculate RCS: Radar Cross Section of this aircraft you are given some incident radar field over here ok. What is step 1? You are given this problem let us you know a aircraft designer or ship designer you are given this problem what would be step 1? How do I start solving this problem?

Student: Total field for information equation.

Well before that even before I think really basic

Student: Break into triangles.

Break into triangles so right. So, I do I how do I break into triangles what is the tool that I need for that?

Student: (Refer Time: 01:15).

Right so, I need to generate a mesh file right so, that is CAD software autoCAD is a popular one in open source there is over Gmesh is one popular software. So, there are many ways of doing it right. So, generate a mesh file store it. So, for complicated objects this is itself for big task imagine you are given let us say like you know some you know MiG-21 or something you need to have a realistic 3D model of this in the computer and then you have to for the fracture it ok.

A real life object will have some very tiny tiny things here and there which need not be meshed. So, you should smooth over it. So, meshing this is not a trivial thing by any means ok. So, even CAD software is generally very sophisticated or you have some you know complicated electrical circuit maybe you do not want to discretize the resistor very finely order it so, this is a complicated step.

What would be step 2? The step 2 is once I have read in this made once have made this mesh file, now I start writing my code will I understand this I mean what is of interest to me in this mesh file what I am looking for?

Student: Boundaries.

Boundaries not just boundaries the location of the triangles, the nodes of those triangles right that is of interest to me because based on that like we did in the previous slide I can calculate the matrix. So, it looks it looks trivial, but this is an important step. Read in the mesh file; as a computational EM engineering you have to write this wrapper to read this mesh file because, it will be in whatever format CAD software did CAD software is general CAD software for any problem they may not even know or care that you are using its computational here.

So, you have to write wrapper that will correctly read in the data in a pass the coma whatever all of that. So, its a fair amount of work its not trivial having done that you have read in all of these what you do next what would you do next? So, once I read in these things I mean what would I read in to.

Before I make the coupling coefficients I need to create efficient data structures to store of all these elements rights so, this is creating.

Student: Store the mapping.

Store the mapping rights so, this has all of the global to local mappings list of elements, two nodes, edges, you have to create all of these right. Supposing like in the previous example you give a triangle now once you given triangle you need to know which are the nodes in it.

So, you need to have a of accessing given element number find out which nodes are there you have to create the data structure no one is going to make it for you right. So, these 3 steps are what come under the category of preprocessing ok. So, now, all your data is in your program. Now we can start now we can start working in it.

So, what is the next step? What we just discussed in the previous slide that is calculating the matrix elements it is called matrix assembly. So, that is i.e. forming Ax=b. Next obvious step once Ax=b what you do? Solve it right. What kind of complications might be here? So, here is where this is actually this is in itself for separate course in numerical linear algebra. So, here you have a choice you can either do direct methods.

So, what are direct methods things which you have already seen so far. Gaussian elimination and LU decomposition all of those things, but you cannot keep doing this as the size of the problem becomes large and large. In fact, it may not be even the possible to store the matrix explicitly, then what is the alternative direct approach are iterative methods.

So, typically when your problem becomes a 3 dimensional problem, doing this LU decomposition itself becomes very tedious. So, you have to switch to what are called iterative methods any example of the most popular iterative method?

Student: Newton.

Not Newton, conjugate gradient if you heard about conjugate gradient method is a way of solving system of equations ok. So, there are there are families of these things example.

Student: (Refer Time: 07:15).

Various methods are there. Conjugate gradient methods as I mentioned this is whole course in itself of different methods depending on the elements. So, these are the this is what is called the processing ok. What is next step? You have found out x, now actually calculating the quantity of interest. So, in this case it will be RCS rights. So, for RCS I will need some kind of contour around the object then I will apply Huygens principle based on the values have calculated of x on that and find out the quantity of interest right.

So, compute. If you are doing a circuit simulation here is why you would calculate things like impedance of whatever right based on what you have calculated. This is ok, this is one thing another optional thing can be which is very powerful is visualizing the output ok. So, so if I visualizing the output you have got the mesh you now visualize the values of x that you have calculated visualize it on the mesh in 2D, 3D whatever right that gives you a lot of intuition into whether your problem is correct has been solved correctly or not ok. And there are good software in the open source for doing this is called post processing ok.

So, is that all are we done or if there is some important thing now imagine that you are you have this task you have given to calculate RCS or some object and you come start step 1,2,3,4,5,6,7 and you declare success and victory after that no why?

Student: (Refer Time: 08:29).

It might be how do you know it is correct or not.

Student: (Refer Time: 09:33) check the RCS.

Well you do not know the correct RCS.

Student: Experiment gives.

Supposing you have experiment.

Student: I do numerical convergence.

Numerical convergence is something that you have to do any way, but it may be that you have converged numerically to the wrong answer; I mean you have coded a consistent set of equations which are wrong like let us say you forgot the ${k_0}^2$ term altogether. So, it will give you numerical convergence, but you have solved Laplace equations, instead of Maxwell's equations equation. So, what would you do now very critical step in a any computation.

Student: I will create a (Refer Time: 10:13) role.

Right so, what you need to do is validate your code against the known solution right. So, this is extremely important; validate what kind of examples of known solutions can you think of? Ok. So, basically we want up what we what is EM problem whose solution I know. So, then I code up and I simulate and I should get the known answer. So, what is the simplest EM problem you can think of where you know the answer analytically?

Student: (Refer Time: 11:01).

Something simpler than sphere and cube, what is a problem that even the class 9 or 10 student knows the answer too.

Student: (Refer Time: 11:13).

Not point suppose a.

Student: Wave through free space.

A wave through free space ok, but there is no calculation involved over there what is what reflection coefficient if I have a slab of glass then every school kid has done. What is the reflection coefficient what is the transmission coefficient? There is a formula in terms of the refractive index and its how many dimensional problem? You can make it a 1 dimensional problem, just normal reflection and transmission I can calculate that if I were writing a 1D code, I know the answer you know reflection coefficient is $(n_2 - n_1)/(n_1 + n_2)$ I should get the same answer there right. So, I can now make that 2D right incident at non normal right.

So, Fresnel reflection coefficients this can be one way of testing whether your code is giving the correct answer any other solutions you can think of.

So, it turns out there are very very few problems in electromagnetic where there is a known analytical solution and one is this Fresnel reflection coefficient, the others are scattering from infinite cylinder and a perfect sphere these are called Mie series solutions cylinder and sphere. These have known analytical solutions we can they look ugly because there are in terms of Bessel function and Hankel functions and all that.

But you can code it and superpose your answer with this analytical answer and that tells you that you know whether your code is correct or not sure short way. Then you take a leap of faith replace that object by your object and hope everything else for that right having the luxury of validating against experimental data may not always be there if you have that is great, but then you have to have confidence that your experimental configuration does not have unaccounted errors themself.

So, typically you would first validate against some known object then use that to refine your experimental setup and back and forth the real world is always very messy. So, this sort of summarizes the process flow these the FEM has a lot of coding details that are there particularly in the pre processing step if you do not do it right your code can be very inefficient.

In making these data structures accessing of these data structures should use object oriented programming all of those details everything that you have learnt it comes into play over here. So, yeah so, that brings us to an end of the FEM we will not go to 3D FEM, but the idea is same right shape functions, weak form, matrix assembly, calculating the quantity of interesting this is a process flow.