Computational Electromagnetics
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2D Finite Element Method
Lecture - 11.09
Computing Far Field

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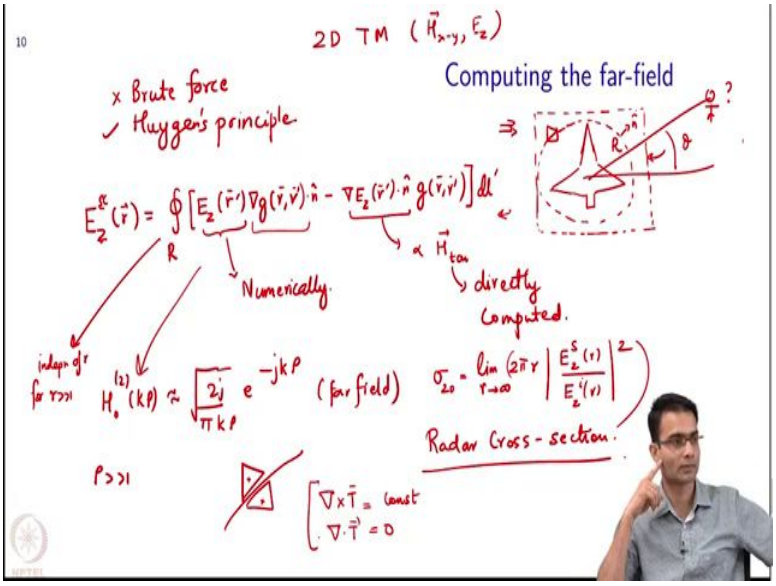

So, finally, we got a system of equations $Ax = b$ and we have solved it right. So, now the usual problem is to find out the far field right. So, again just give a schematic over here this was my let us say my aircraft and let us say this was my computational domain right and I want to find out the and this is being eliminated by some let us say radar field over here and I am standing over here and I want to find out what is the field over here.

So computing the far field is there what kind of strategy do you think will have to do? Should I expand my computational domain to go all the way to the receiver? is that necessary? No. So, should I do instead?

Student: The incident wave.

Incident wave is some wave is given to you. Now I want to find out the far field what I have found out so far, let us say I solved my $Au = b$. This is my computational domain. So, there are you know triangles everywhere and so on. What I have got as a at the end of this calculation what have I got used. So, I know the field everywhere inside the domain, but now I want to find out the field somewhere far away. What do I do?

Student: Everything we can do in that way.

So, one so, that is what one thing that has been suggested is that when if you want the field somewhere far away you somehow I have to expand this domain to include this fellow. That is what is called a brute force way of doing it. is that a smarter way of doing it?

Student: Because there will be shaded this everything in that way.

Brute force way is one way suppose I want to find out radar cross section; that means, my observer is where? Infinity. Can I possibly mesh so much? So, what was what was the very start of this course we have talked about Huygens principle. What is the Huygens principle tell you? If I know the tangential field on a close control they act like. Secondly, sources and I can propagate them find out the field everywhere.

So, how will apply this principle here? Do I know the fields on a close contour and closing the object I do. So, I mean, for example, this boundary itself is a closed boundary in closing the object ok. So, I can use this boundary I know I have calculated the fields over that I can use that to apply Huygens principle and find out the field anywhere that is one thing.

Now, we had a discussion on boundary conditions and we have seen that the for example, the radiation boundary condition we have used its in exact. So, the error of the boundary condition grows as I get closer and closer to the boundary as I move further inside gets better I am just giving you a giving information we will not actually calculate it. So, what would be better is that if you in your mesh itself you had a nice set of edges like this which define a boundary ok.

So, when you mesh your when you use your CAD software instead of define a let us say circle or something around it and what meshing software will do is it will respect that edge. So, it will align the triangles to be in such a way that you can get a nice smooth set of edges.

So, there will be U_1, U_{15}, U_{25} whatever those are the use that are sitting over here on this contour.

So, you now know the field on those points ok. So, brute force is a bad idea. Instead we will do Huygens principle ok. So, for example, we already this is an expression which is we have calculated several times the field that any r as per Huygens principle. So, E_s is scattered that is what I want. What is that integral? It is a contour integral over, let us call this you know, a contour R. What are the terms that will come here?

Student: (Refer Time: 05:14) in 0.

So, this is scattered only. So, it's the $E_{total} = \phi_{inc} + \phi_{scat}$. So, remove the ϕ_{inc}. So I am only calculating the scattered field. So, the first term so, remember this is going to be g and ∇g those are the 2 terms.

Student: Yeah right.

Right so, first one will be

$$E_z{}^{sc}(r) = \oint_R [E_z(r')\ \nabla g(r,r') \cdot \hat{n} - \nabla E_z(r) \cdot \hat{n}\ g(r,r')]dl'$$

Let us call dl prime because I am going over this is my contour right. We are still in the TM polarization that is why I have written this is E_z. Do I know Green's function that is a question? yeah. So, in this case once since I have taken the contour to be if you go back and recall your discussion of the Green's function I divided space into 2 volumes V_1 and V_2 in Huygens principle for finding out the field in the outside region which Green's function do I need the outside region or the inside region? The outside region. In this case what is the outside region? Free space. So, I know this g.

It is simply my Hankel $H_0^{(2)}$ whatever like. So, the good part is I know the Hankel function further what else can I say, I can make all the standard approximations that I had done for your Hankel function which is what that $H_0^{(2)}(k\rho)$ right. So, for $\rho >> 1$, we have approximated this as $\sqrt{2j/\pi k\rho}\, e^{-jk\rho}$. So, all of these approximations we can substitute inside over here.

In this so, finally, before we end this discussion, in this expression do we know everything? Have we calculated everything? Let us go step by step. $E_z(r')$ or let us start from the right hand side let us make it simpler. Remember this is 2D T M polarization which means H in a x y plane and E_z these are the 2 variables 3 variables. Do I know this term?

Student: Yeah that is the H tangential.

Yes we have shown that this is proportional to H_{tan}, have I calculated H_{tan}? That was exactly my variable in the problem.

Those are tangential edges right. So, that is another nice part about the edge base FEM that I got straight away this thing. So, I know this. What about this guy? Do I know this guy?

Student: Maxwell's.

I do not know it, but can I calculate it? How?

Student: Maxwell's.

Maxwell's equations. Means taking $\nabla \times \vec{H}$ will give me $\vec{E}$. How do I take $\vec{H}$ now? I know $\vec{H}$ as numbers I do not know it as.

Student: Numerically.

Numerically right. So, now I know everything inside this. So, numerically I have to take the numerical $\nabla \times \vec{H}$ and that is why it is important for mesh $\vec{H}$ to be very fine. So, that the numerically approximation is more or less correct right and once I have got this then my favorite expression is for your RCS what was it equal to $\lim_{r \to \infty} 2\pi r\, |E_z{}^s(r)/E_z{}^i(r)|^2$.

Student: While computing do you put r to be a large number or (Refer Time: 10:00).

Do you put r to be a large number? Well if you this was part of one of the assignments earlier. If you calculate this expression this would turn out to be independent of r for r greater than 1 for very large. So, it does not matter.

Student: (Refer Time: 10:22).

You do not have to put anything this expression the expression for RCS this expression for RCS will turn out to be independent of r. It will only depend on this θ.

Student: Is they identical to your $g(r, r')$?

Yeah in the Huygens principle we have $g(r, r')$, but then you have to use these approximations and when you use this approximation that value of r cancelled off you do not need to substitute anything, if you have substituting it you are doing something wrong.

Student: Can we take the curl?

Yeah you can take the curl. Yeah I mean that is exactly what we are going to do.

Student: But you said numerical.

So, what I mean by numerical curl over here is yes $\vec{H}$ is in terms of the U s and $\vec{T}$ s and $\vec{T}$ is in terms of x and y.

Student: Yeah.

Right I will get some expression for it now if you look at well we are running off time this is one edge this is one triangle and this is another triangle if you look at these Whitney basis functions that we have if you actually go to calculate $\nabla \times \vec{T}$, you should do it you will end up getting a constant because of the form of $\vec{T}$. So, what happens is that you get one constant here and another constant here.

So, the $\nabla \times \vec{T}$ is actually not continuous for this basis function that is one defect of the basis functions. So, what will actually have to do is find out the curl here, find out the curl above and take an average of it. So, that is what I mean by numerically find out. Yeah, in the end, we have a smooth function of $\vec{H}$ everywhere in the plane. Yeah that is a good point and another very interesting thing about these basis functions if you do this you get a zero for the basis functions we are using. So, we will work this out in an assignment.

Student: (Refer Time: 12:20).

Take any one of them a single basis function has these properties, $\nabla \times \vec{T}$ is constant, $\nabla \cdot \vec{T}$ is zero alright.